Foundational Physics

for the HSC

Fourth Edition **2026 Syllabus**

Glenn Y.W. Kim

Contact
For further questions please contact Glenn Y.W. Kim at glennkimyw@gmail.com.

Contents

Preface

This textbook was written because I could not find one that focussed on interesting unorthodox questions, with thorough solutions so that antisocial students did not have to fear asking anyone else how to solve a question.

Foundational Physics for the HSC is primarily intended for Year 12 students studying the New South Wales HSC Physics course who want questions that require possibly a more refreshing perspective on familiar concepts. Though, it may also interest those just starting out in physics.

The book was started in late 2024, and finished in the middle of 2025. There may be errors hidden in the document due to my lack of time, money and also care. If you find any of these errors, it would be appreciated if you contact me through the email address glennkimyw@gmail.com.

I would like to thank the many people who helped me in writing this book. In particular, I am grateful to Clara Walker, without whom this textbook might never have reached the internet. I also thank Samuel J. Rofail for identifying numerous errors and suggesting many improvements since the First Edition.

Foundational Physics for the HSC is no revolutionary textbook, but it does hold some of my more interesting ideas that I've had commuting on public transport.
I hope the textbook helps, or at least does no harm.
That's a good place to start as any.

- Glenn Y.W. Kim

Attributions

Fernando, J. (2007). Mano. In Wikimedia Commons.
https://commons.wikimedia.org/wiki/File:Mano.svg

Langendorf, R., Schneider, S., & Klein, P. (2022). A typical Hertzsprung–Russell diagram. In ResearchGate.
View on ResearchGate
Adapted.

The Quantum Atlas, & Feng, A. (2019). Electron orbitals. In Wikimedia Commons.
https://commons.wikimedia.org/wiki/File:Electron_Orbitals.png
Adapted.

Some derivations were inspired by the approaches presented on LibreTexts.
https://libretexts.org/

Using this Textbook

This textbook can be navigated easily using the embedded links in the table of contents. The coloured boxes denoting questions and solutions also have embedded links which will take the reader between the two.

Questions that sit comfortably outside the NESA-supplied syllabus have been marked with an asterisk (*). For the HSC Physics student that will never touch anything resemblant of physics again after their exam, there is no need to attempt such questions. But for those that are interested, or those who would like to pursue science after high school, I recommend attempting these questions as they can act as a transition to problems that can be seen at a slightly higher level.

There is one question marked with a double asterisk (**), and such a question would be appropriate for those who would like to be the life of the party, at a party of socially lost university physics students.

As for the primary purpose of this textbook, it is to provide additional resources to HSC Physics students. However, the textbook adopts a more problem-solving-oriented approach with the questions, and thus naturally escapes the typical atmosphere of faithful HSC style questions. However Prometheus did not remain faithful to Olympus, for he believed the fire of the gods belonged to mankind.

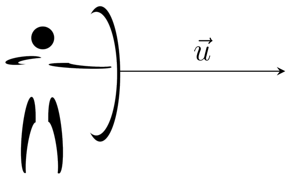
Advanced Mechanics

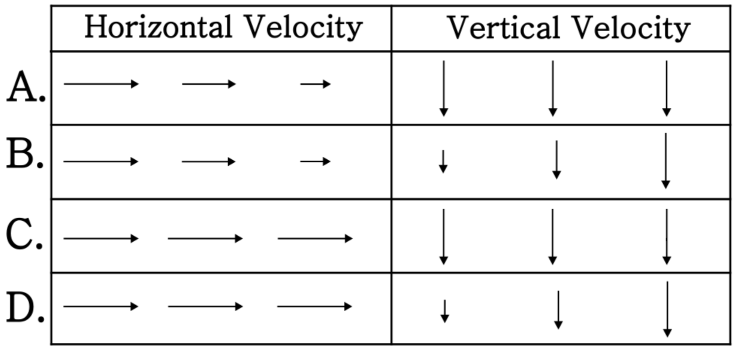
Projectile Motion

Individual Components of Projectile Motion

Question 1

An arrow is shot as shown:

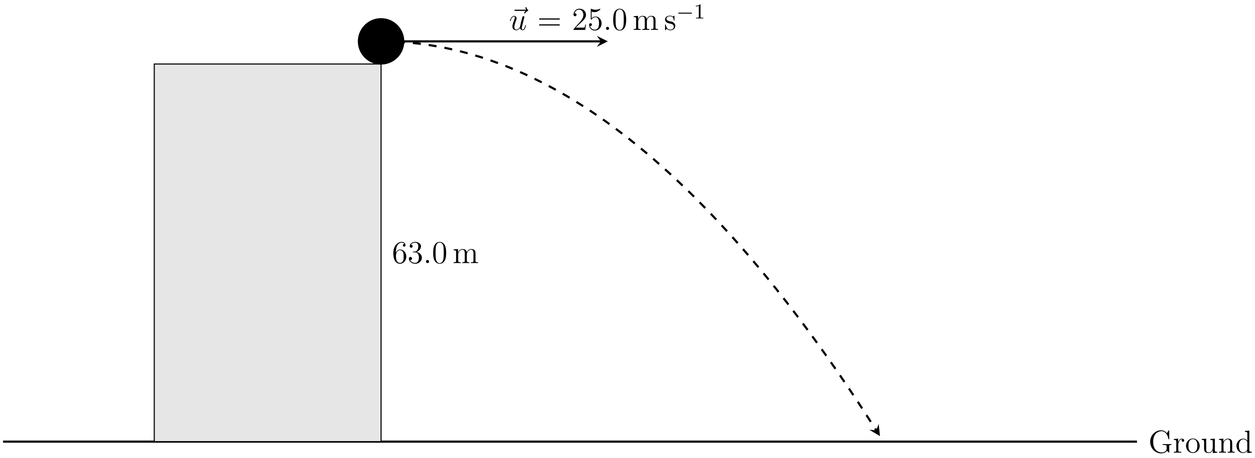




Which set of vectors
best describes the scenario?
(Ignore air friction)

Question 2

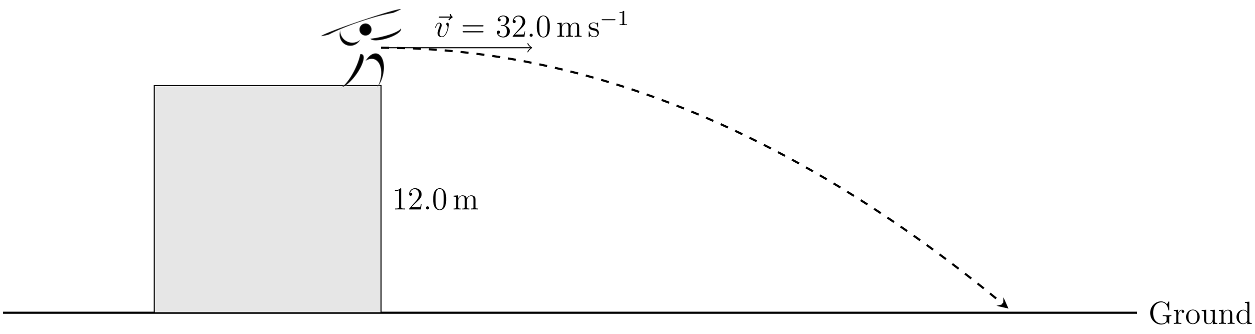
A basketball is rolling off a 63.0 m high cliff. The ball is rolling at a speed of $25.0\,\mathrm{m\,s^{-1}}$ before rolling off as shown:



What is:
(a) the time it takes to reach the ground,
(b) the range, and
(c) when will it be 20.0 m above the ground?

Question 3

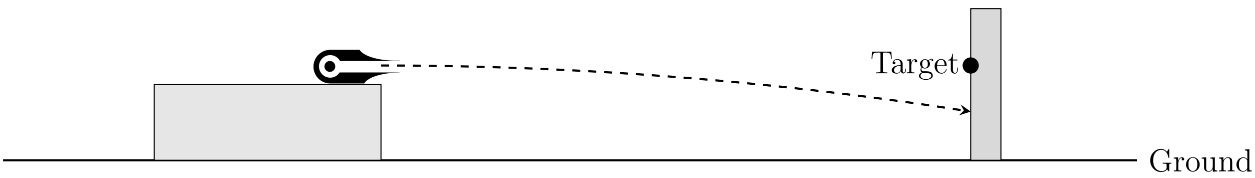
Emmy throws a javelin directly forward from the top of a podium 12.0 m tall at a velocity of $32.0\,\mathrm{m\,s^{-1}}$ as shown:



What is the acute angle the javelin makes with the ground?

Question 4

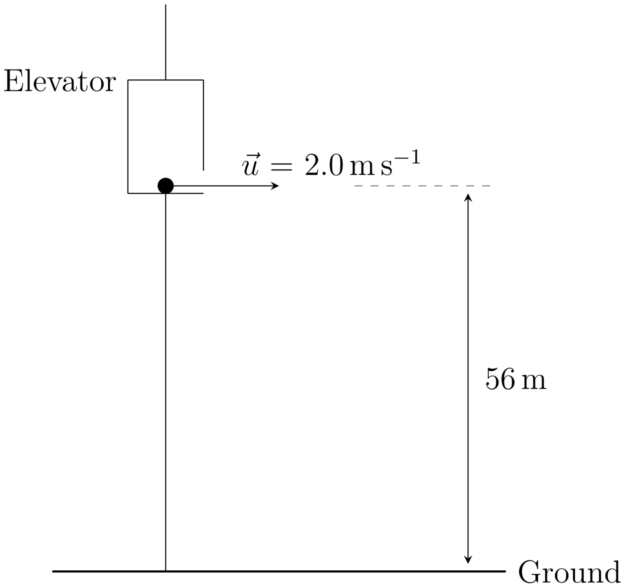
Emmy operates a rail gun and shoots a projectile at a speed of $16\,\mathrm{km\,s^{-1}}$ directly at a target that is at the same elevation as the rail-gun's barrel. The projectile strikes 176.4 m below the target.



How far away is the target from the rail-gun?

Question 5

An old elevator is descending at a constant speed of $4.2\,\mathrm{m\,s^{-1}}$. There is also a rubber ball on the ground of the elevator that is rolling towards the wall at $2.0\,\mathrm{m\,s^{-1}}$. However, the rubber ball goes through a hole in the wall of the elevator and rolls off and falls. If the vertical displacement of the rubber ball from falling off the elevator is $56\,\mathrm{m}$, what will be its horizontal displacement?



Question 6

The same elevator from **Question 5** is now ascending at a constant speed of $12\,\mathrm{m\,s^{-1}}$. A lead ball that has 10 times the mass of the rubber ball rolls through the hole and falls with a vertical displacement of $56\,\mathrm{m}$. However, its horizontal displacement is exactly the square of the horizontal displacement of the rubber ball from **Question 5**. What is the lead ball's furthest horizontal displacement in the condition that it is exactly $1\,\mathrm{m}$ above its starting position?

Projectiles Launched at an Angle

Question 1

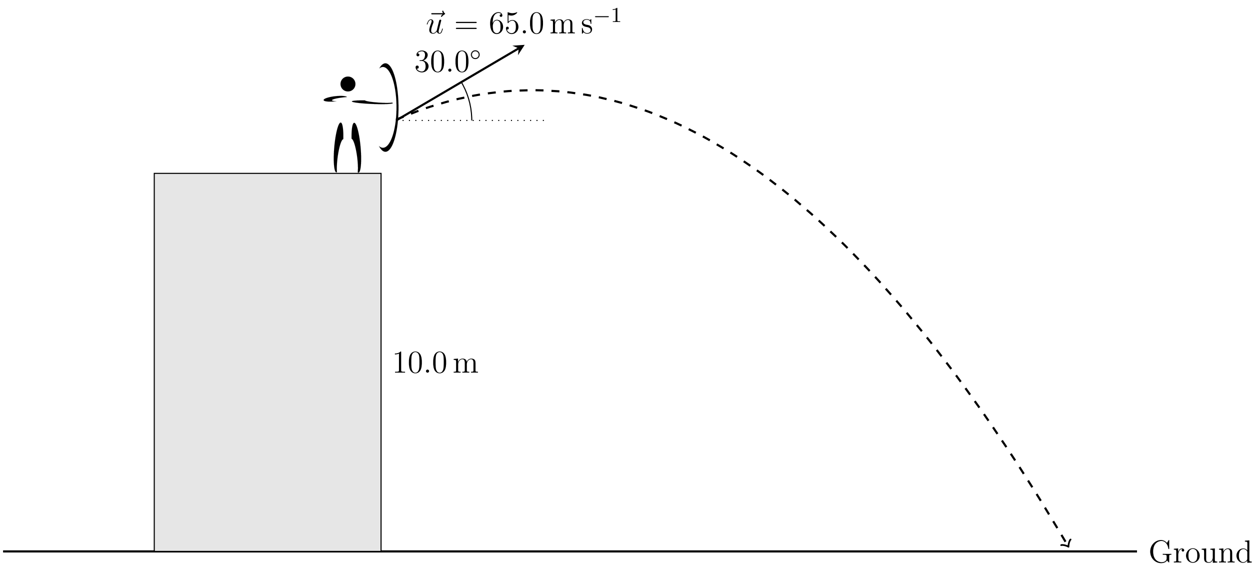
Emmy kicks a soccer ball at an angle of 45° to the horizontal at $24\,\mathrm{m\,s^{-1}}$ on flat ground. What is:
(a) the time it takes for the ball to reach its maximum height, and
(b) the vertical displacement at its maximum height?

Question 2

Amanda then kicks the soccer ball considerably harder than Emmy from **Question 1** and sends the soccer ball flying so that it impacts the ground 280 m away from its original position. It is known that she has hit the soccer ball at the same angle as Emmy. What was the magnitude of the initial velocity of the soccer ball?

Question 3

Emmy shoots an arrow at $65.0\,\mathrm{m\,s^{-1}}$ at 30.0° to the horizontal while standing on a 10.0 m tall building as shown.



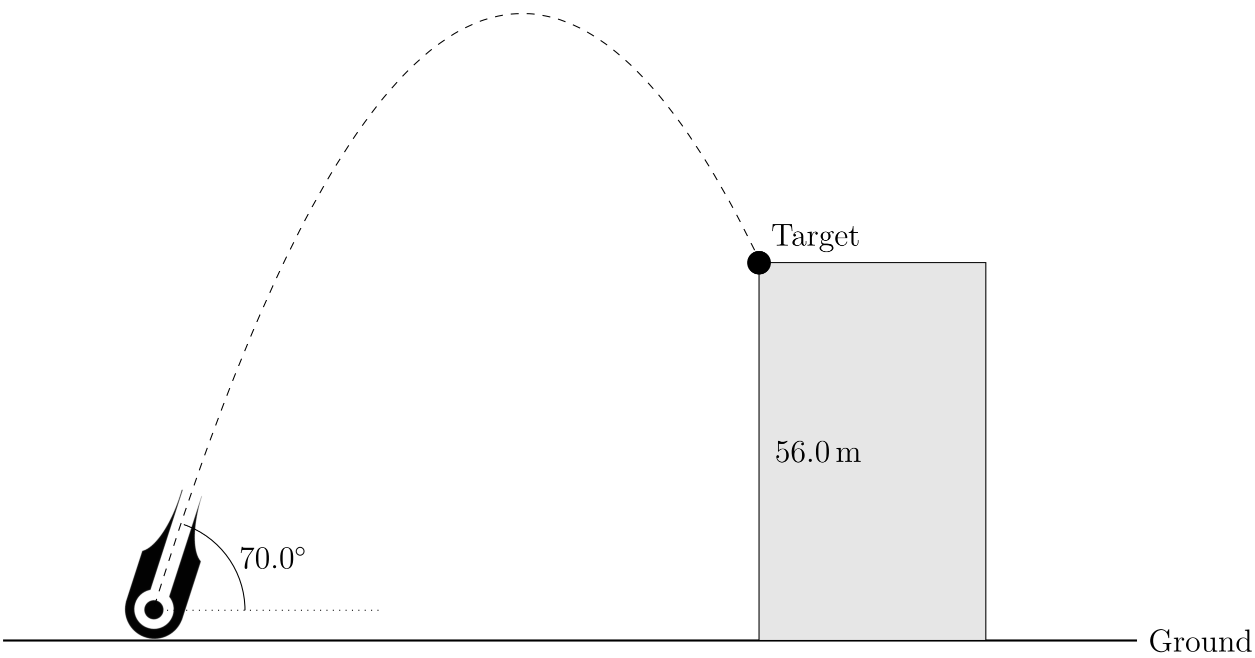
What is the range of the arrow?

Question 4

Emmy from **Question 3** then steps off the building and shoots the arrow with the same angle and the same relative velocity to himself when she was exactly 6 m above the ground. Now, what is the range of the arrow?

Question 5

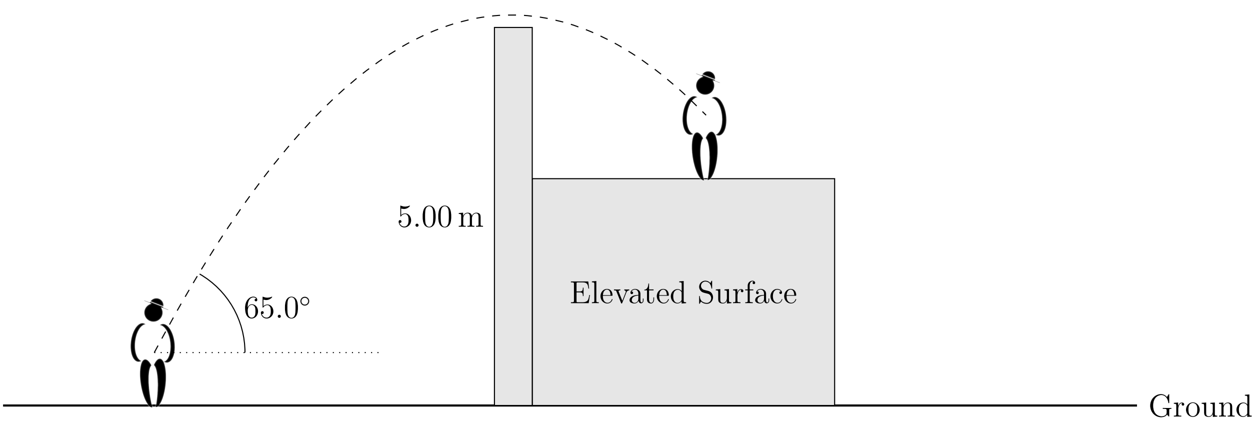
Emmy is operating a modern ballista to shoot a spear at a target as shown:



If the horizontal distance between the target and the ballista is 124 m, at what speed must Emmy shoot the projectile to hit the target?

Question 6

Emmy is throwing a tennis ball to Amanda as shown:



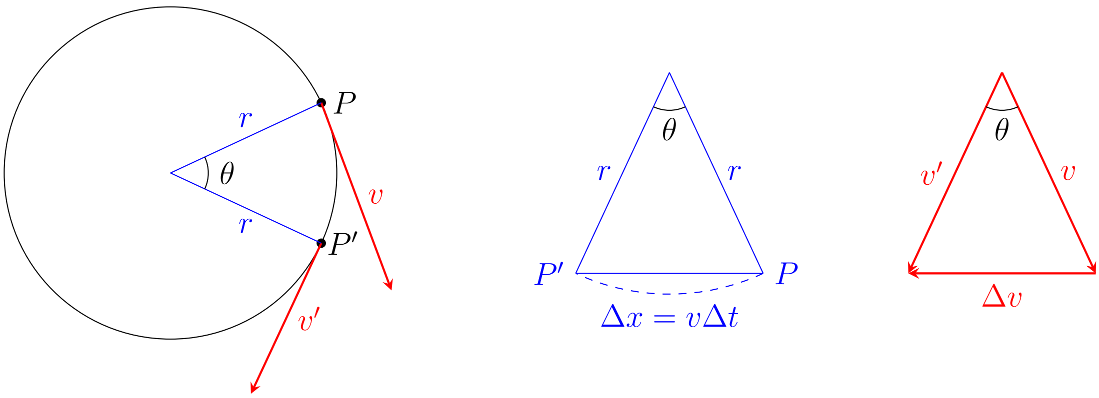
Amanda catches the ball, and it can be seen that the ball reaches its maximum height just as it passes over the 5.00 m tall wall. If Amanda catches the ball when it has a vertical displacement of 3.00 m, how far is Amanda from the wall?

Circular Motion

Centripetal Motion on a Flat Plane

EXTENSION DERIVATION: $a_{\mathbf{c}} = \frac{v^2}{r}$

Consider a particle in circular motion.



Although there is a curvature in the path taken from P to P', note that we will take the limit as Δt approaches 0, and hence the curvature will become negligible, allowing us to approximate the two shapes as similar triangles. Also note that

$$\lim_{\Delta b \to 0} \frac{\Delta a}{\Delta b} = \frac{da}{db}$$

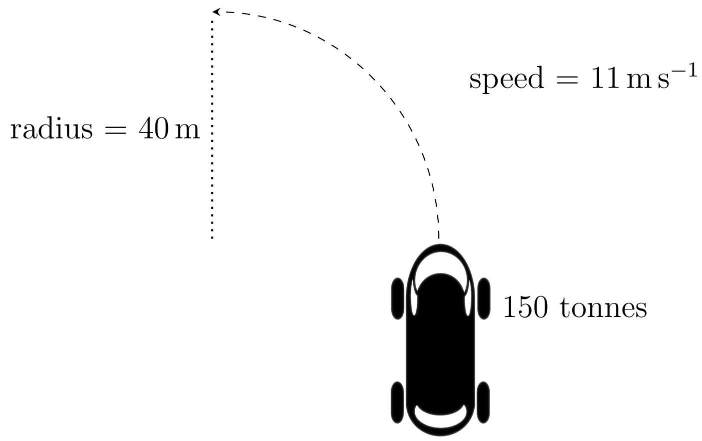
as defined in calculus.

Now using the equivalent ratios of corresponding sides in the two triangles,

$$\begin{aligned} \frac{\Delta v}{v} &\approx \frac{v\Delta t}{r} \\ \frac{\Delta v}{\Delta t} &\approx \frac{v^2}{r} \\ \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} &= \frac{v^2}{r} \\ \frac{dv}{dt} &= \frac{v^2}{r} \\ a_c &= \frac{v^2}{r} \end{aligned}$$

Question 1

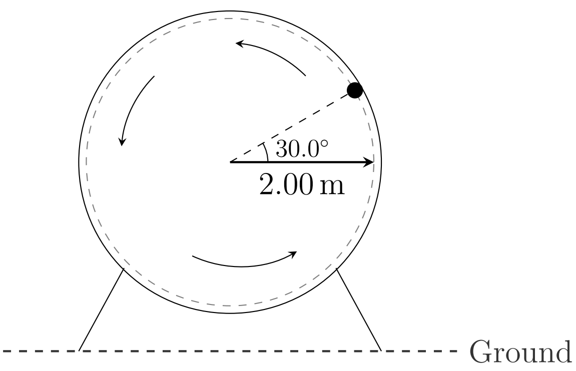
A cargo truck with a mass of 150 tonnes follows through a left turn that has a radius of 40 m at 11 m s^{-1}. To 2 significant places, give:
(a) the friction force between the truck and the road,
(b) the truck's acceleration, and
(c) the truck's angular velocity in degrees per second.



Question 2

Emmy swings a 0.80 kg rock tied to a 1.4 m long string at an angular velocity of 36 rad s^{-1} upwards, causing the string to snap and break. Give:
(a) the linear speed of the rock just before the string snaps, and
(b) the tension in the string just before it snaps.

Question 3

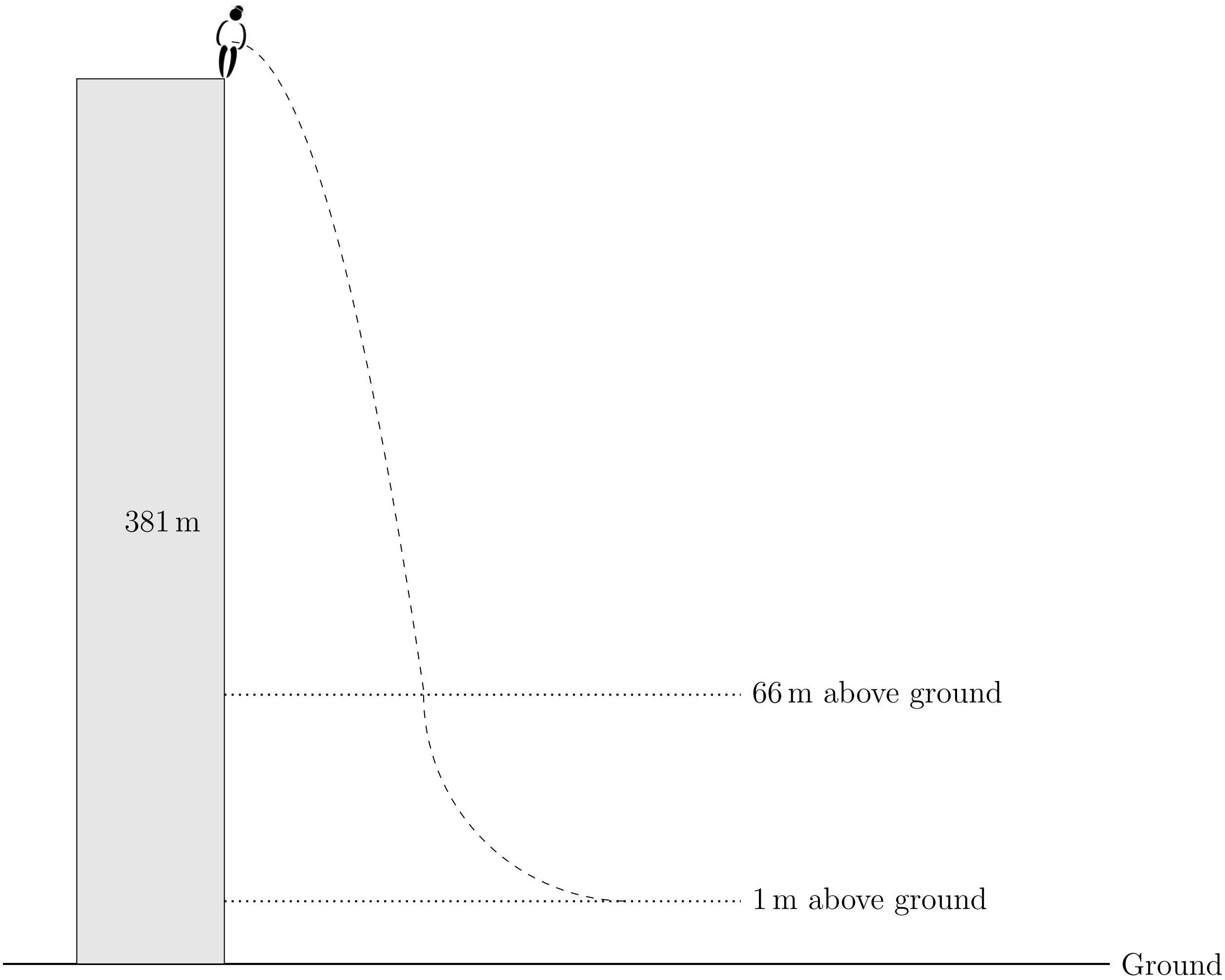


A ball is spinning around a vertical circular path of radius 2.00 m at the minimum possible speed as shown.

What is the speed of the ball in the position shown?

Question 4

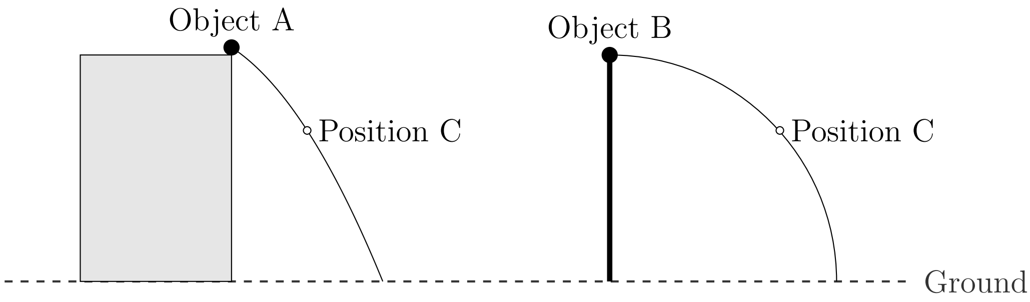
Emmy slips and falls off the Empire State Building at a height of 381 m. She falls for 315 m, but Tarzan swings in and attempts to save Emmy by swinging with him in a circular path to avoid Emmy splatting on the concrete ground. Tarzan successfully prevents Emmy from colliding with the ground, but Tarzan fails to account for the fact that if Emmy experiences g-forces of above 9, she will not survive. Does Tarzan actually save Emmy?



Hint: Use 1 $g = 9.8\,\mathrm{m\,s^{-2}}$

Question 5

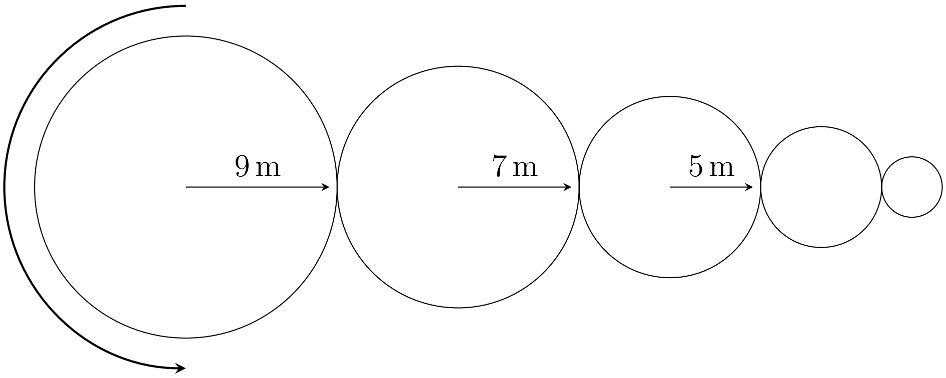
Object A is a 2.50 kg ball that rolls off a 3.00 m high ledge at 20.0 m s^{-1}, and object B is identical to object A, however, is being spun with a 3.00 m long arm in a circular path at 20.0 m s^{-1}.



At position C, both object A and object B are 2.00 m above the ground. What is the difference of the magnitudes of the net force acting on both objects at position C?

Question 6

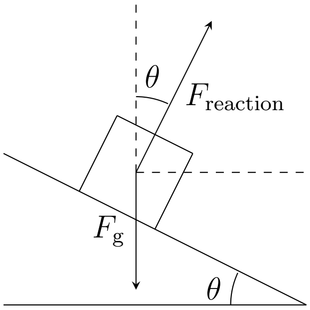
The following contraption is set:



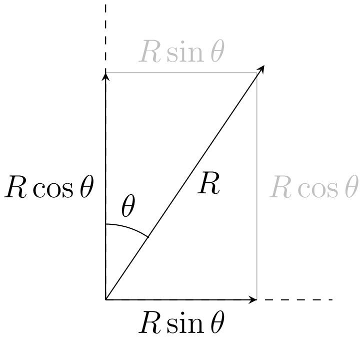
The wheel of radius 9 m is rotating at an angular velocity of 1 rad s^{-1} out of the page. If the wheels continued to decrease in radius by 2 m from left to right, and they rotated against each other without slipping, find the angular velocity of the rightmost wheel.

Circular Motion on Banked Tracks without Friction

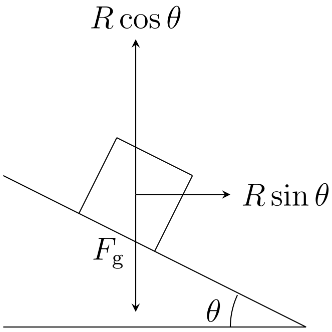
The derivations of the equations required for this topic are explained below:



Take note of this diagram. It should seem somewhat familiar. In the next diagram, we will simplify a little to clarify the individual components of the reaction force.



Now it should be clearer that the force acting towards the right, or the centre of the circular motion is $R\sin\theta$, and the force acting upwards, that is, keeping the object from vertically falling is $R\cos\theta$.



Now, if you imagine this box as the front of a car driving out of the page, we can see that $R\sin\theta$ is the centripetal force, and furthermore, that $R\cos\theta$ is the force keeping the car from falling due to its weight, meaning $R\cos\theta = mg$.

Now derive the equations for the centripetal force and maximum safe velocity.

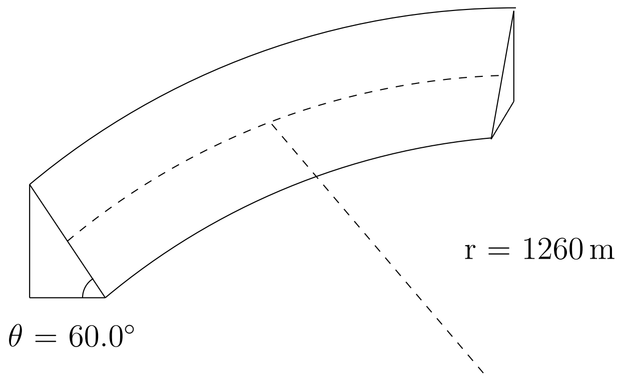
$$R\text{cos}\theta = mg$$
$$R = \frac{mg}{\text{cos}\theta}$$

$$F_\text{c} = R\text{sin}\theta$$
$$F_\text{c} = mg\text{tan}\theta$$

$$F_\text{c} = \frac{mv^2}{r}$$
$$mg\text{tan}\theta = \frac{mv^2}{r}$$
$$v^2 = rg\text{tan}\theta$$
$$v = \sqrt{rg\text{tan}\theta} \qquad \text{(Maximum safe velocity)}$$

Question 1

Acceleration due to gravity on the Moon's surface is $1.67\,\mathrm{m\,s^{-2}}$. The lunar government has installed a road called the Moon-Way, a frictionless banked curve with a radius of 1260 m, and an inclination of 60.0° as shown



If a lunar delivery truck with a mass of 2.50 tonnes drives along the Moon-Way, what will be
(a) the centripetal force on the truck,
(b) the maximum safe speed for the truck in kilometres per hour, and
(c) show why the maximum safe speed does not change for different vehicles.

Question 2

On Earth, there is a banked curve with a radius of 40.0 m. Through the sacrifices of a few brave souls for science, it has been discovered that the maximum safe velocity of this banked curve is $8.00\,\mathrm{m\,s^{-1}}$. What is the angle of inclination of the banked curve?

Question 3

If on Earth AND the moon, there exists a frictionless banked curve that is inclined at the same angle, with the curve on the Earth having a radius of 45 m and the curve on the Moon having a radius of 117 m, what is the closest ratio of the maximum safe velocity of the curve on Earth to the Moon? (Use $1.67\,\mathrm{N\,kg^{-1}}$ as the gravitational field strength of the moon)

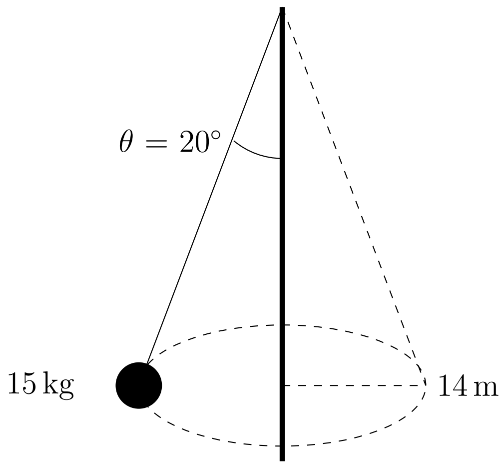
A : 2 : 1
B : 3 : 2
C : 4 : 3
D : 5 : 4

Question 4

A skateboarder is skating around a circular banked track. If the magnitude of the linear momentum of the skateboarder is 1050 N s, and the centripetal force acting on the skateboarder is 3150 N towards the centre of the circular motion, what is the magnitude of the angular velocity of the skateboarder?

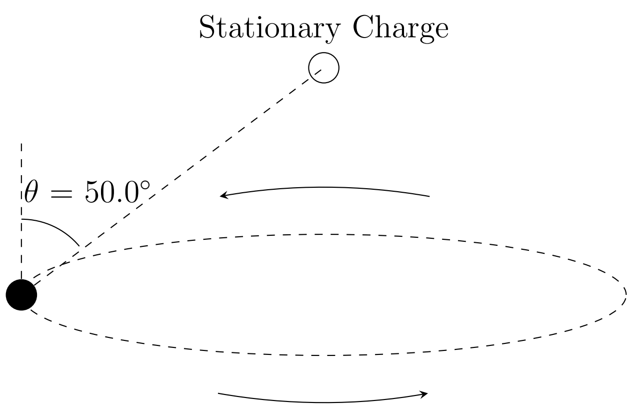
Question 5

If the tension of the string keeping the conical pendulum in motion is 800 N, what is the magnitude of the linear velocity of the mass?



Question 6

A conical pendulum is in motion; however, the mass is kept in motion by the electrostatic attraction between itself and the stationary charge.



If the linear momentum of the moving mass is 0.800 N s, and the frequency of the moving mass was 2.50 Hz, what will the magnitude of the electrostatic attraction be?

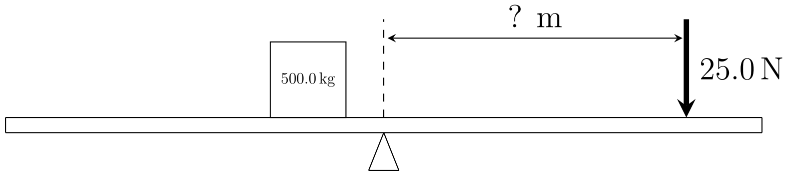
Torque

Question 1

A force of 72 N is applied directly downwards at the end of a 36 cm horizontal wrench. What is the magnitude of the torque produced?

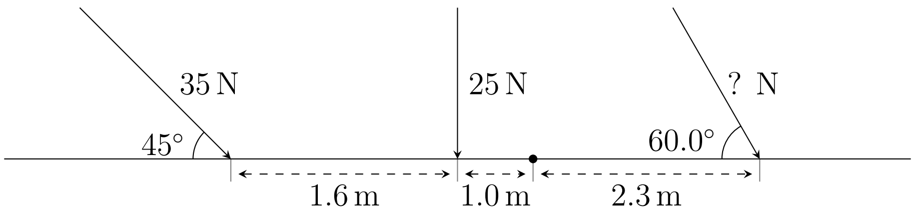
Question 2

On a beam on a fulcrum, a 500.0 kg mass is placed 1.00 m from the fulcrum point. At what distance from the fulcrum point must a force of 25.0 N be applied directly downwards to have the contraption at rest as shown?



Question 3

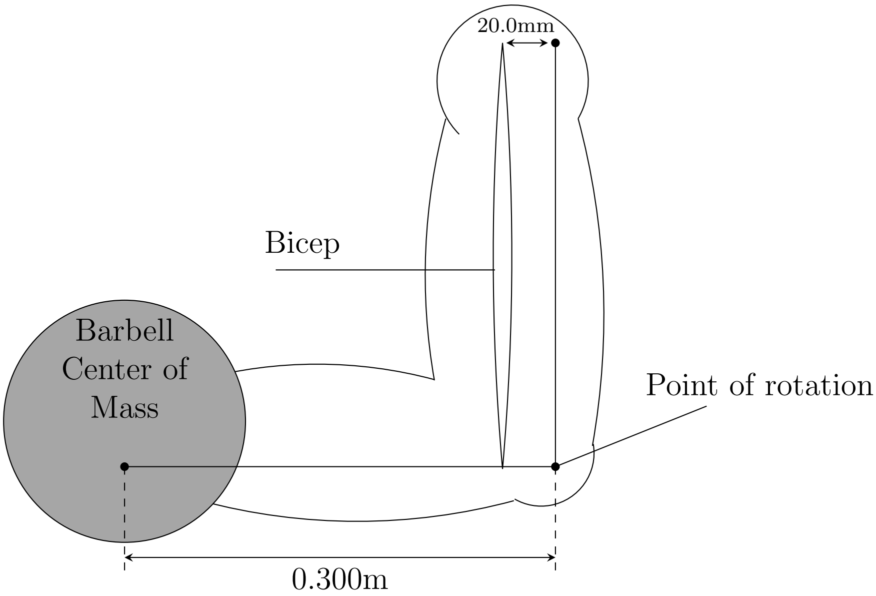
The system shown below is in equilibrium, with the black circle being the rotation point.



What must the unknown force's magnitude be?

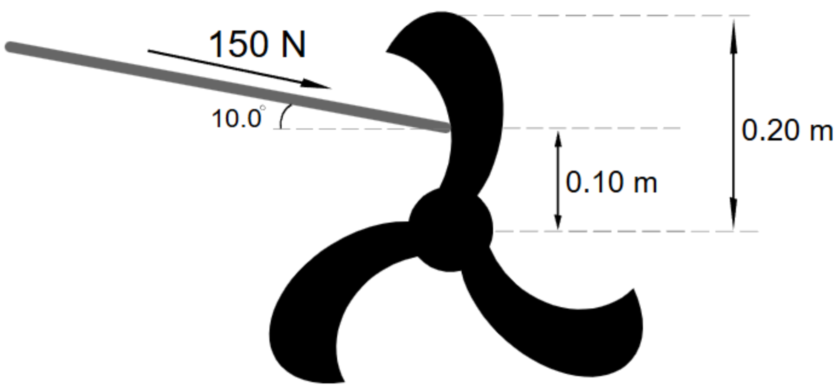
Question 4

The barbell has a mass of 145 kg. If the man is statically holding the barbell with both hands as so, what is the tension in his left bicep?



Question 5

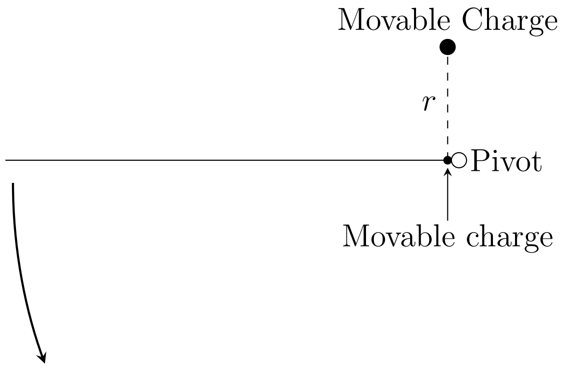
Llewellyn is probing around the fans of the outdoor component of a split system air conditioning unit. He sees that the fan is not moving, and continues to push until a force of 150 N manages to turn the fan as so:



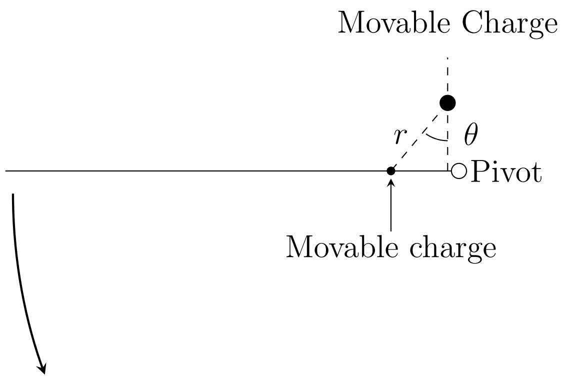
If instead of pushing at an angle of 10.0° to the horizontal 0.10 m above the pivot of the fan, Llewellyn pushed perpendicular to the vertical 0.20 m above the pivot of the fan, how much force would he need to make the fan turn?

Question 6

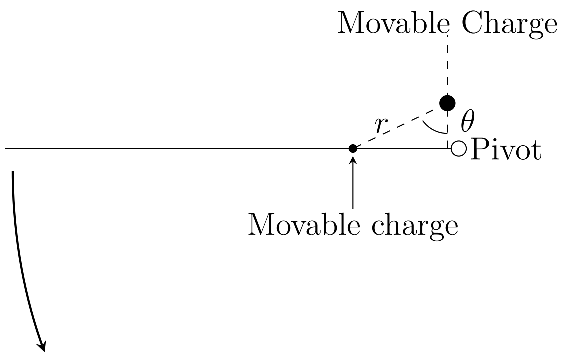
There is a lever that can rotate clockwise or anticlockwise with a positive charge attached that can move left and right across the lever. There is also another positive charge that can be moved up or down. However, the distance between the two charges is always kept constant by the charge on the lever sliding left or right, as shown:



When $\theta = 0°$



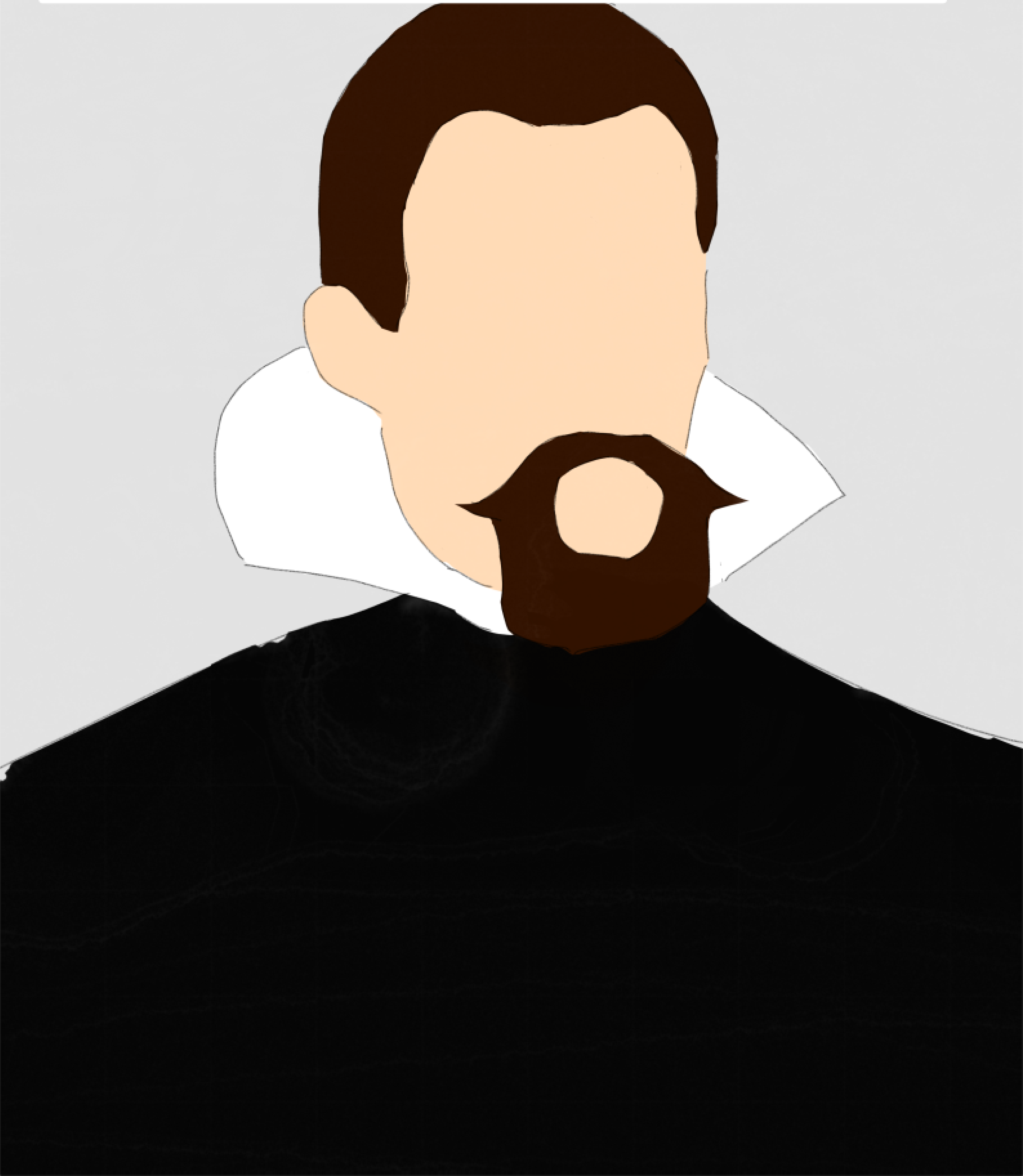
When $\theta = 30°$



When $\theta = 60°$

If it is known that the maximum value of $\sin\theta\cos\theta$ is when $\theta = 45°$, show why the lever as shown above will experience the strongest torque when $\theta = 45°$.

Motion in Gravitational Fields

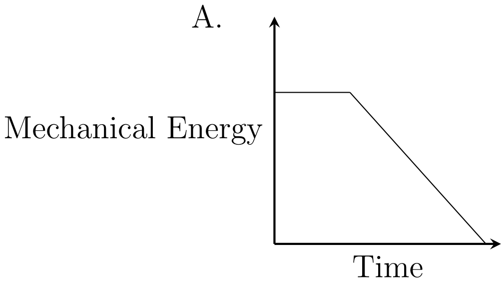


Conservation of Mechanical Energy

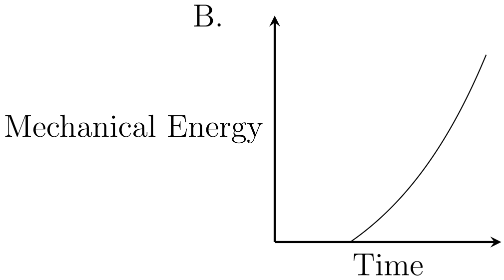
Question 1

Two kookaburras fly horizontally towards each other, and collide, where they entangle and fall directly downwards. Ignoring friction, what graph best depicts the total mechanical energy of the two kookaburras from moments before the collision to just before the kookaburras touch the ground?

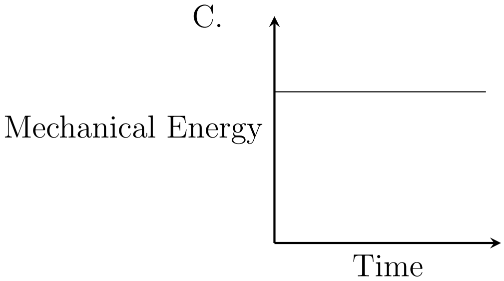
A.



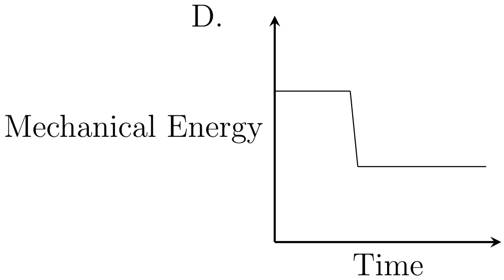
B.



C.

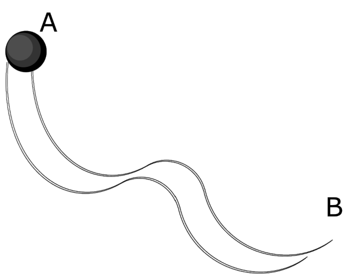


D.



Question 2

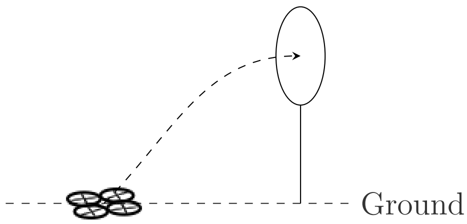
A ball is placed on a track above the ground, initially at rest at position A, and slides down the track with no friction. At position B, it has a velocity of $8.00\,\mathrm{m\,s^{-1}}$ at an angle of $50.0°$ to the horizontal as shown.



If the time it takes to reach fall to the ground after rolling off position B is $2.00\,\mathrm{s}$, what is the vertical displacement of the ball from position A to the ground?

Question 3

A 0.150 kg drone starts from the ground and flies through a target hoop as shown.



The hoop is 3.00 m above the ground, and the drone is travelling at $20.0\,\mathrm{m\,s^{-1}}$ when it goes through the hoop. If this manoeuvre was taken out at a power efficiency of 36.5%, how much energy was used during this movement?

Question 4

Suppose the average household uses 1500.0 J of energy per second. Emmy was able to create a contraption whereby taking a large boulder and placing it above the Earth's surface, the gravitational potential energy of the boulder can be converted to electrical energy with 99.9% efficiency as it falls. She manages to set a 2000.0 kg rock 310 m above the surface, and tries to power her average household through this contraption. How long can this contraption power her house for? Answer in minutes.

Question 5

The data required to solve the question is given in the table below:

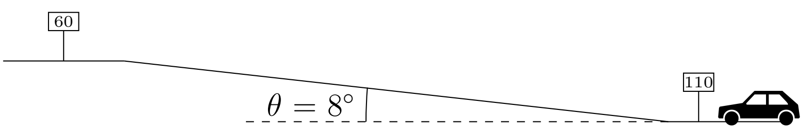
Nissan Altima Specs

Mass (No Fuel)	1451 kg
Fuel Tank Capacity	60.6 L

87 Octane Fuel Specs

Gravimetric Energy Density	$42.8\,\mathrm{MJ\,kg^{-1}}$
Density at 15 °C	$0.735\,\mathrm{kg\,L^{-1}}$

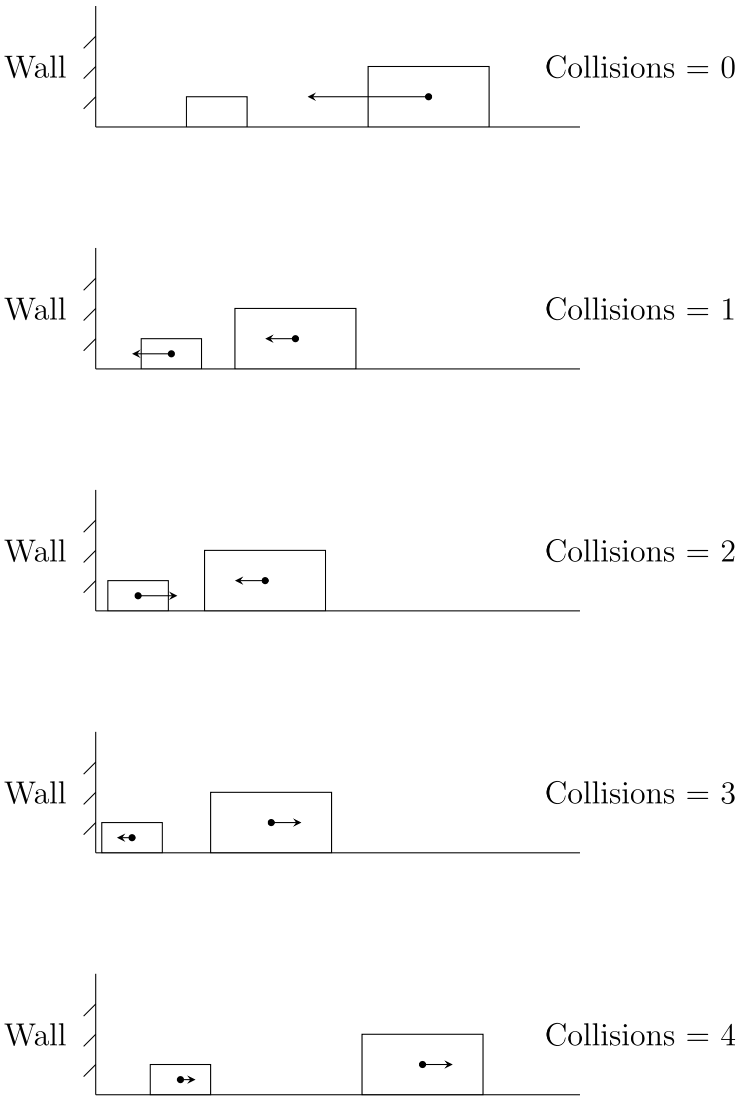
A Nissan Altima using 87 Octane fuel is riding across a $110\,\mathrm{km\,h^{-1}}$ highway on a 15 °C day. This highway is interestingly inclined at an angle of 8°. At the beginning of the highway, the fuel tank is 3 quarters full. By the end of the highway, where there is a speed limit of $60\,\mathrm{km\,h^{-1}}$, the fuel tank is only a fifth full.



The driver responsibly starts on the highway at the speed limit and ends on the highway at the speed limit. Assuming that throughout this trip, the Altima had an energy efficiency of 30%, how long is the highway? Answer in kilometres.

Question 6**

A stationary hockey puck B of mass 1 kg is placed at some arbitrary distance from a vertical wall to the right. A second hockey puck A of mass 100^n kg, $n \in \mathbb{R}$, approaches puck B from the right with some initial speed. The system evolves only by elastic collisions on a frictionless surface (diagram not to scale).



In the example above, it can be seen that there are 4 total collisions. Determine the function for the number of collisions in terms of the mass of puck A, m_A (you may assume m_A is sufficiently large), and explain why for $n \in \mathbb{Z}^+$, the number of collisions follows n digits of the mathematical constant π.

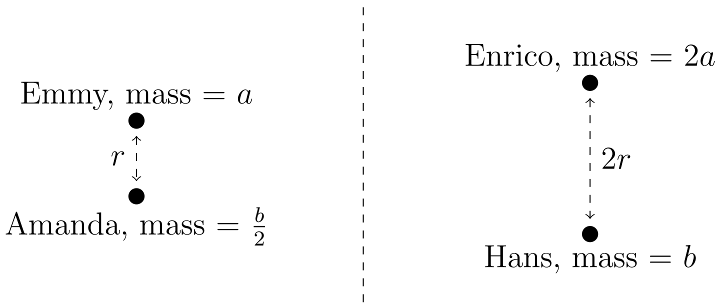
Hint: consider plotting relevant physical quantities on a Cartesian plane.

Newton's Law of Universal Gravitation

Question 1

Calculate the magnitude of the gravitational force between a person with a mass of 103 kg and another of 82 kg if they are 5.00 m apart.

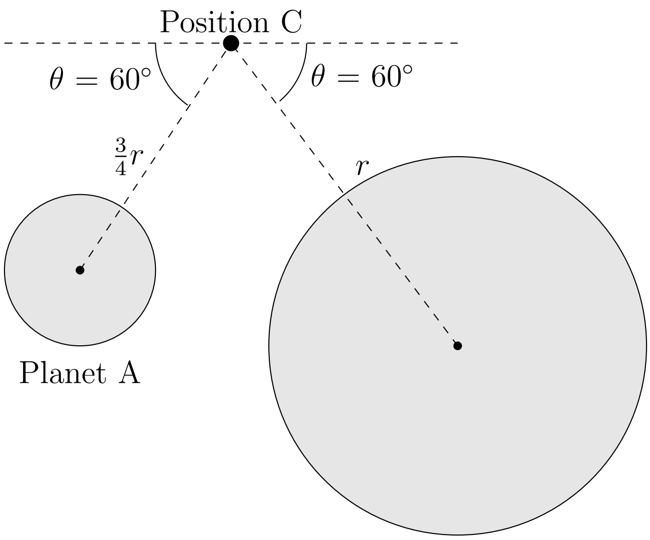
Question 2



What is the ratio of the gravitational force between Emmy and Amanda to the gravitational force between Enrico and Hans?

Question 3

There are two planets, planet A of mass a, and planet B of mass $9a$ as shown:



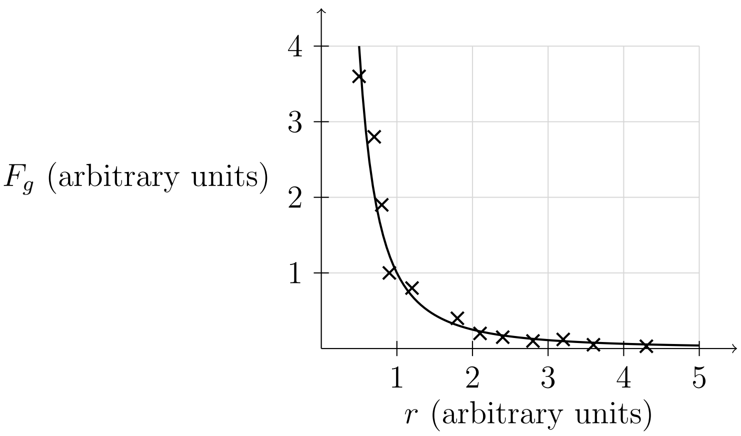
Give the direction of the resultant force on a mass at position C in degrees, to 3 significant figures.

Question 4

Given that Jupiter's mean density is $1326\,\mathrm{kg\,m^{-3}}$, and its radius is $6.991 \times 10^7\,\mathrm{m}$, what is the gravitational field strength at the surface?

Question 5

Emmy plots the magnitude of the gravitational attraction against the distance between the centres of two masses from his experiments as shown:



Which relationship is shown through this graph?

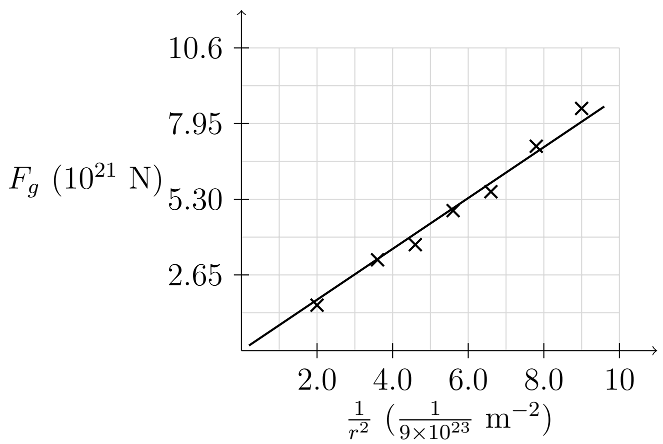
A : $F_g \,\alpha\, r$

B : $F_g \,\alpha\, r^2$

C : $F_g \,\alpha\, \frac{1}{r}$

D : $F_g \,\alpha\, \frac{1}{r^2}$

Question 6



Emmy has conducted another experiment similarly, but has linearised the graph.

From the graph, determine the product of the two masses.

Kepler's Laws

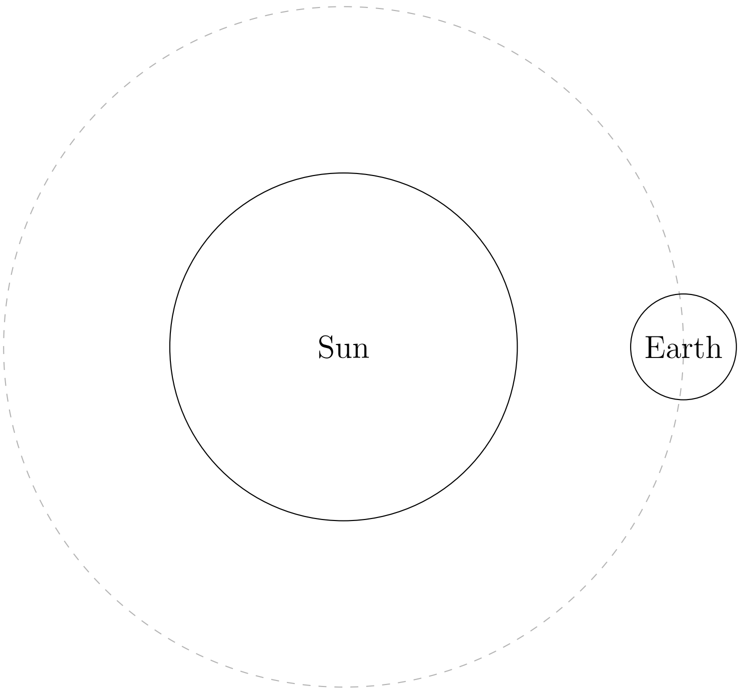
Question 1

If the mass of a star is 2.83×10^{31} kg with a diameter of 1894560 km, and has an orbiting satellite that has a diameter of 9420 km, that is at a distance of 4598390 km from surface to surface, calculate the period of the orbiting satellite.

Question 2

In an alien civilisation, the orbital radius of their home planet (Eworb) around their sun (Bvzerk) is declared as 1 zyleck, and the orbital period as 1 ewa. If the orbital radius of a satellite that orbits Bvzerk is 2.3 zylecks, what is the orbital period of this satellite in ewas to 3 significant figures?

Question 3



If the orbit of Earth around the Sun is circular and the radius is 149.6×10^9 m, taking a year as 365.25 days, what is the mass of the Sun?

Question 4

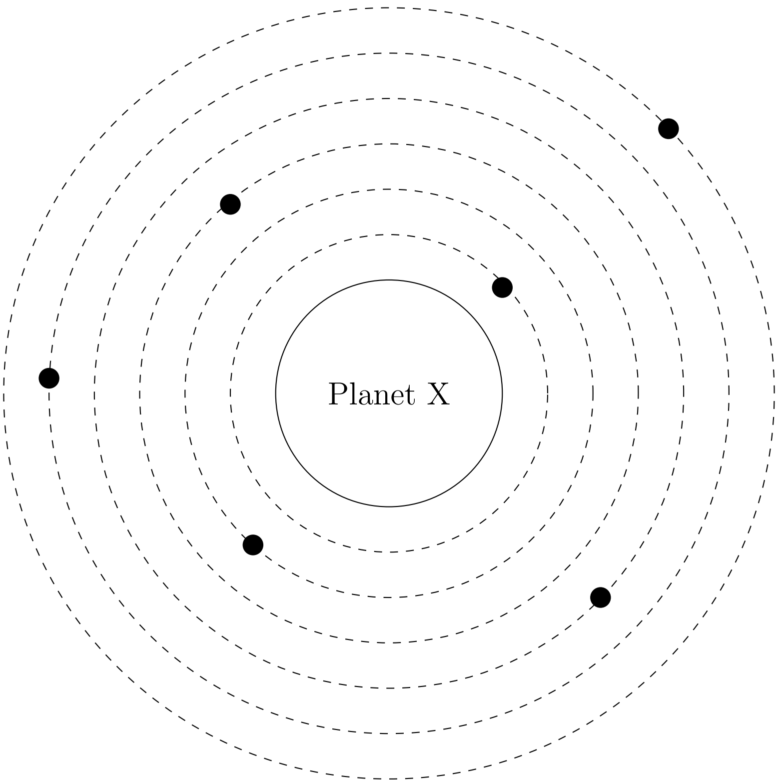
The International Space Station orbits Earth at an altitude of 408 km. Calculate its orbital velocity.

Question 5

A satellite is orbiting a large mass. If the ratio of the orbital velocity to the orbital period of the satellite was 2:1, determine the gravitational field strength the satellite is experiencing due to the large mass in exact values.

Question 6*

Planet X has 23 moons, a diagram that is not to scale is shows Planet X and its 6 closest moons:



Let the closest moon of Planet X be $moon_1$, the second closest be $moon_2$, and so on. If $moon_2$ has double the orbital period of $moon_1$, and $moon_3$ has triple the orbital period of $moon_2$, and $moon_4$ has quadruple the orbital period of $moon_3$ and so on, what is the orbital radius of $moon_{23}$ in terms of the orbital radius of $moon_1$?

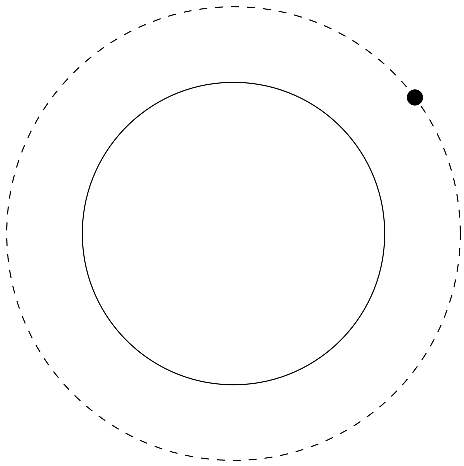
Energy & Orbits

Question 1

What is the escape velocity of a 1.0 kg mass if it were being launched from a 100.0 km altitude on Earth?

Question 2

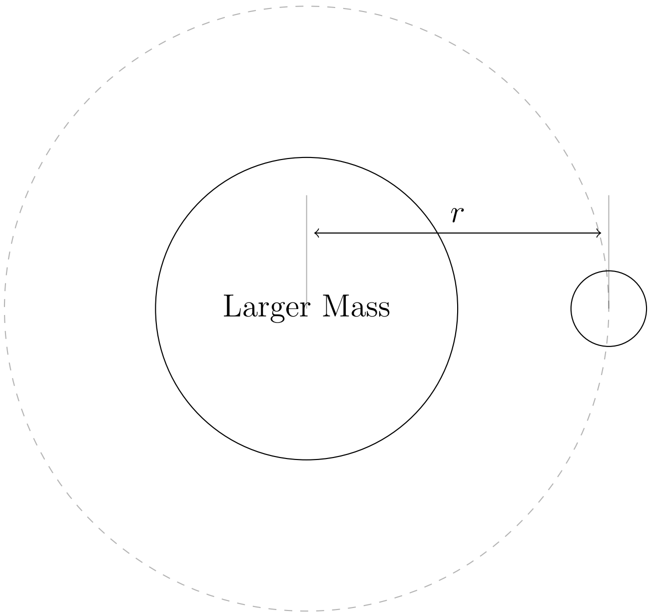
A satellite is in orbit as shown.



If the gravitational potential energy of the satellite was to double, what would occur to the kinetic energy of the satellite?

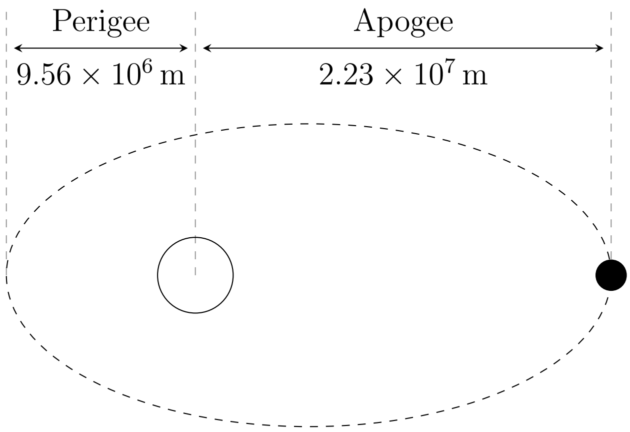
Question 3

If the orbital velocity is x, find an expression for the escape velocity in terms of x.



Question 4

A not-to-scale diagram of an elliptical orbit is shown below:



If the orbital speed of the satellite at its apogee is $3.28 \times 10^3\,\mathrm{m\,s^{-1}}$, find the magnitude of the orbital velocity of the satellite at its perigee.

Question 5

Planet A and Planet B are in orbit around each other. They have equal mass a, and are separated by distance x from centre to centre. If their speeds are constant throughout their orbit, find an expression for the magnitude of the velocity for either of the planets.

Question 6

For the following question you may use the given information.

The semi-major axis of an elliptical orbit a is given by

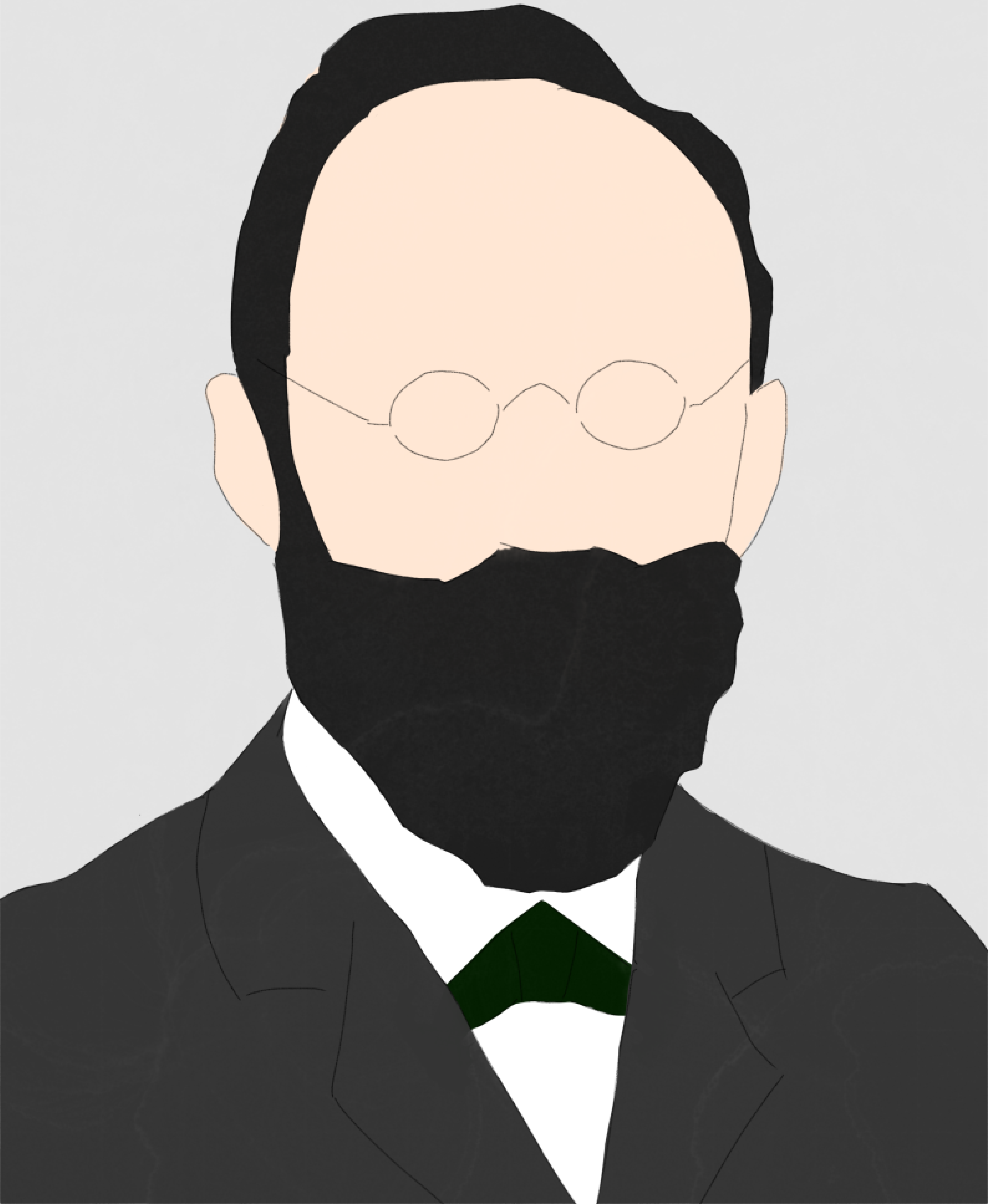
$$a = \frac{r_\mathrm{P} + r_\mathrm{A}}{2}$$

Where r_P is the orbital radius at the periapsis and r_A is the orbital radius at the apoapsis. When finding the period of an elliptical orbit, the semi-major axis is substituted for the orbital radius.

Two geosynchronous satellites, X and Y, orbit the Earth around the equator in opposite directions. Their masses are 1000 kg and 500 kg. When the satellites collide, they coalesce. After the collision, the maximum speed they achieve is 7.41×10^3 meters per second. Find the orbital period T (in hours) of the orbit after the collision (assume the coalesced mass is in an elliptical orbit).

Electromagnetism

Charged Particles in Electric and Magnetic Fields



Charged Particles in Electric Fields

Question 1

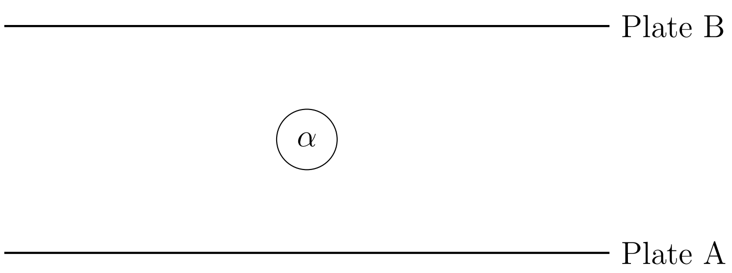
A charge of $-1.6\,\mu C$ experiences a force of 2.72×10^{-2} N East at a certain point. Find the electric field at the certain point.

Question 2

Find the magnitude of the acceleration of an electron in an electric field of strength $1.76 \times 10^{-6}\,N\,C^{-1}$.

Question 3

Emmy sees the following apparatus on the surface of the moon. The alpha particle is stationary.



Which plate is positively charged?

Question 4

A proton is accelerated through a uniform electric field of magnitude $2500\,V\,m^{-1}$ for a distance of 4.0 cm. Find the speed of the proton once fully accelerated.

Question 5

Which relationship of velocity and mass of an electron accelerated through a voltage is correct?

A. $v \propto m$

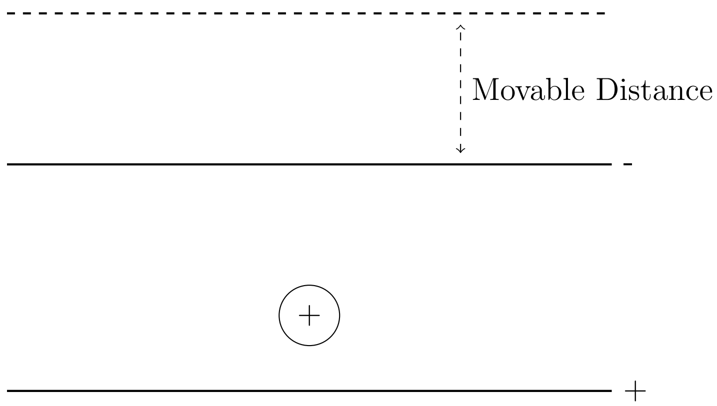
B. $v^2 \propto m$

C. $v \propto \frac{1}{m}$

D. $v^2 \propto \frac{1}{m}$

Question 6

Emmy has the following apparatus, where the top negative plate can be moved upwards or downwards, however, the position of the positive charge relative to the positive bottom charge and the voltage between the plates are kept constant.



Let the magnitude of the charge be q, and the potential difference between the plates to be V. Mathematically show that as Emmy moves the top plates upwards the work done on the charge will increase towards qV, but will never reach it.

Trajectories in Electric Fields

Question 1

Two scenarios are shown below. Scenario A is a baseball that is thrown horizontally. Scenario B is an electron that is shot horizontally, immediately entering an electric field upwards.

Scenario A

Scenario B

What can be said about the two scenarios' paths?

A. Both scenarios will have parabolic paths

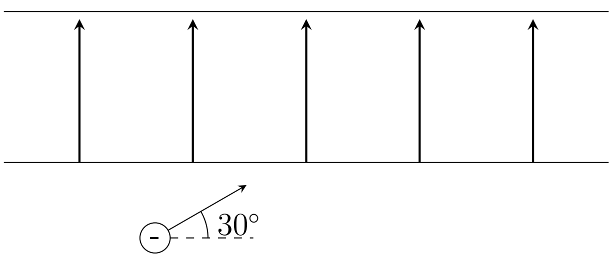
B. Both scenarios will have circular paths

C. Scenario A will have a parabolic path, while Scenario B will have a circular path

D. Scenario A will have a circular path, while Scenario B will have a parabolic path

Question 2

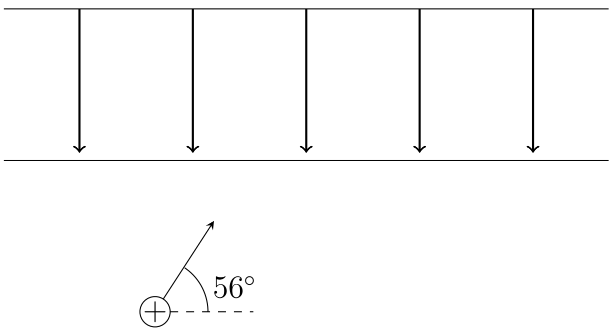
An electron is shot out of an electron gun through a voltage of 2000 V and enters an electric field as shown.



The time difference between the electron entering and exiting the electric field is 5.027×10^{-8} s. Find the magnitude of the electric field strength to 2 significant figures.

Question 3

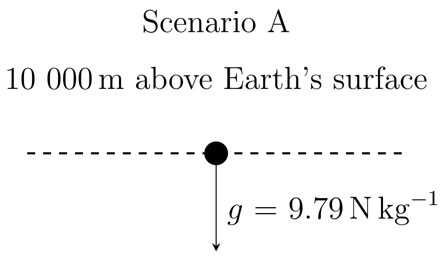
A proton is shot out of a proton gun through a voltage of an unknown voltage and enters an electric field of strength 4200 $\mathrm{N\,C^{-1}}$ as shown.

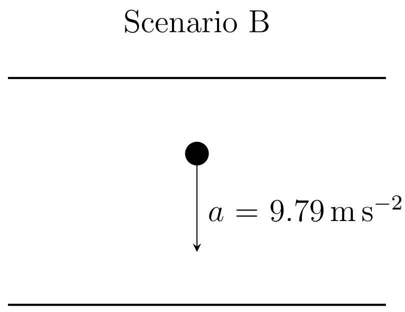


The time difference between the proton entering and exiting the electric field is 6.988×10^{-6} s. Find the voltage of the proton gun.

Question 4

Take the following two scenarios. Scenario A takes place as Emmy skydives 10 000 m above the Earth's surface, where the gravitational field strength is 9.79 $\mathrm{N\,kg^{-1}}$ (ignore air resistance). Scenario B takes place between two plates at an extremely large distance apart from each other (so that an object can fall without colliding with the bottom plate), where an object experiences an acceleration of 9.79 $\mathrm{m\,s^{-2}}$ downwards. This is shown in the following diagram.

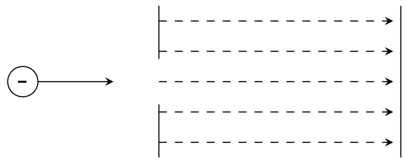




After some time, will Emmy from scenario A fall further, or the object from scenario B fall further?

Question 5

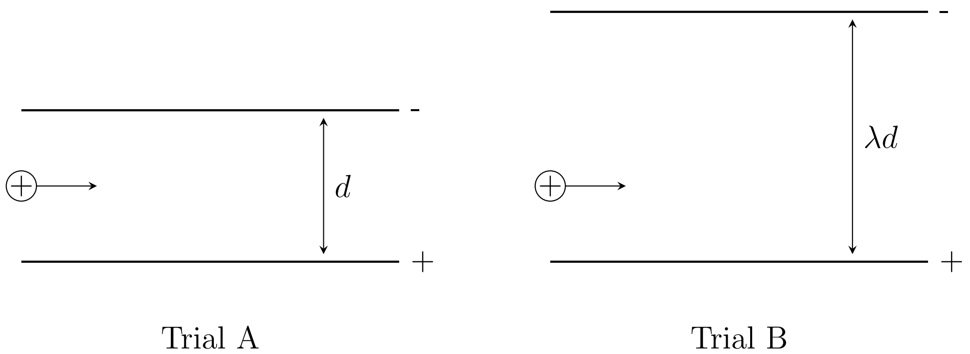
A voltage gun can be used to shoot an electron into an electric field as shown.



The polarity of the gun and the electric field can be swapped whilst keeping the magnitude of the voltage and the electric field strength constant to shoot a proton similarly. Assume that the plates supplying the electric field are far apart enough so that neither the proton nor the electron can reach the farther plate. Therefore, both the proton and the electron will travel a certain distance past the closest plate before changing direction. For that certain distance, which of the proton and the electron will travel further?

Question 6

Emmy continues to use her apparatus from **Question 6** of **Charged Particles and Electric Fields**. He now conducts two experiments. In one trial, the positively charged particle enters from the left of the plates while having equal vertical distance from both plates. In the other, the particle enters from the left from the same vertical distance relative to the bottom plate, but Emmy has elevated the top plate so that the distance between the plates is now a certain multiple of the original distance, as shown.



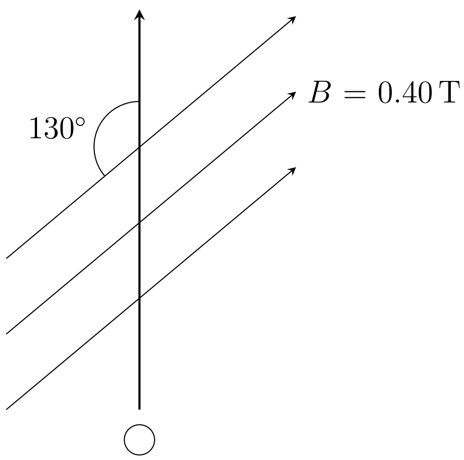
Both particles are travelling at the same horizontal velocity. If the particle in Trial B travels horizontally 2 times further than the particle in Trial A before colliding with the plate above, prove that $\lambda = \frac{1+\sqrt{33}}{4}$.

Hint: When proving something, do NOT begin with the statement that you are proving. Instead, you must finish with the statement that you are proving.

Charged Particles in Magnetic Fields

Question 1

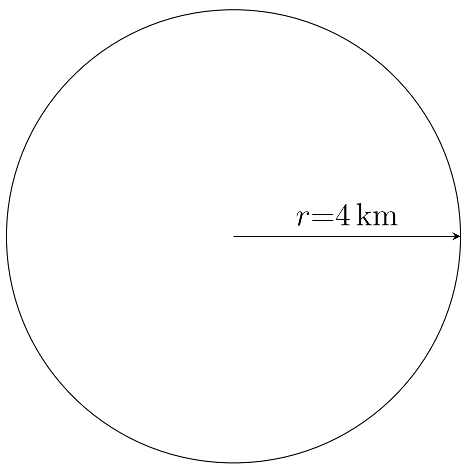
A charged particle moving through a magnetic field of strength 0.40 T, as shown experiences a force of 2.7×10^{-2} N out of the page.



If the particle's velocity is $6.3 \times 10^5\,\mathrm{m\,s^{-1}}$, find the charge of the particle.

Question 2

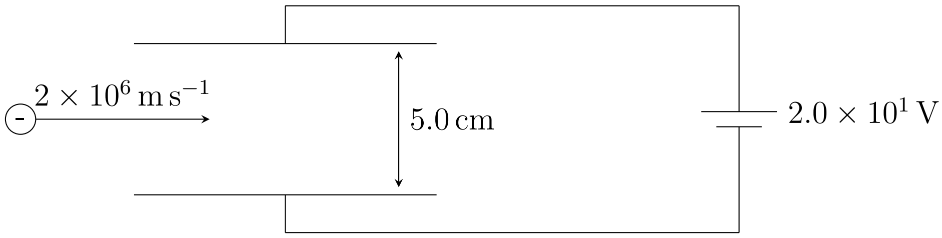
In the Large Hadron Collider, assume that a proton is accelerated to 99.9% of the speed of light. A top-down view of the LHC is given below.



Ignoring relativistic effects, find the magnitude of the magnetic field required to keep such a proton in circular motion.

Question 3

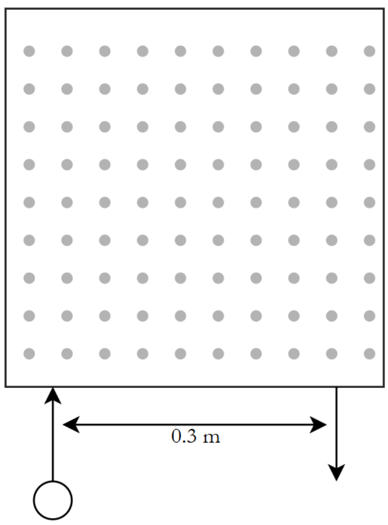
An electron is shot through an electric field as shown.



Determine the magnetic field required to allow the electron to continue through the electric field undeflected.

Question 4

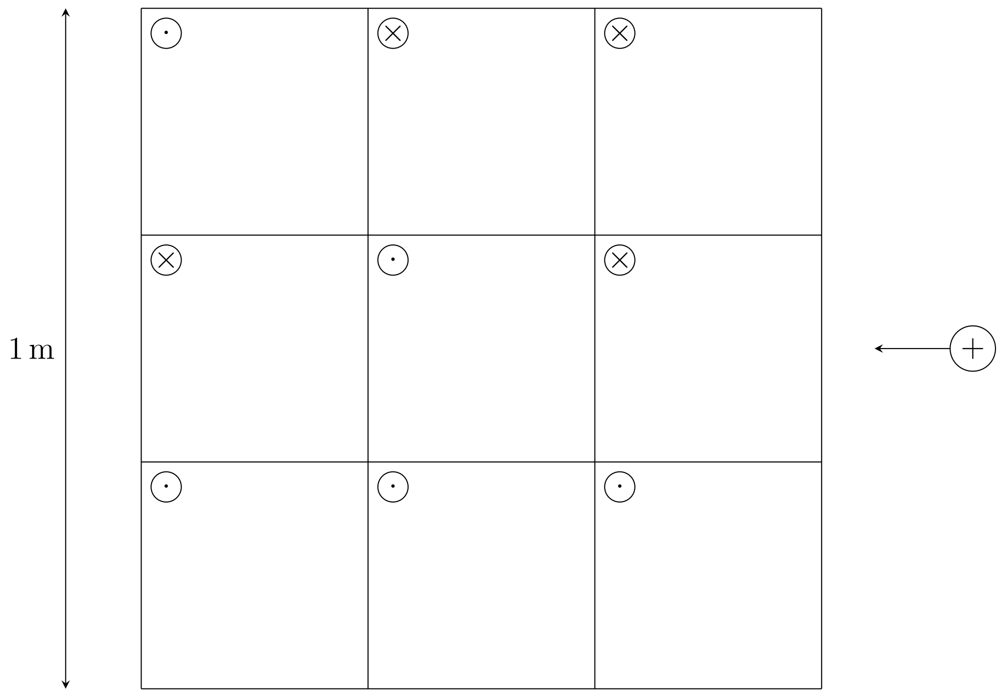
A charged particle enters and exits a magnetic field as shown below.



If the magnetic field strength is 0.1 T, and the velocity of the particle is $7.2 \times 10^5\,\mathrm{m\,s^{-1}}$, find the charge-to-mass ratio of the particle.

Question 5

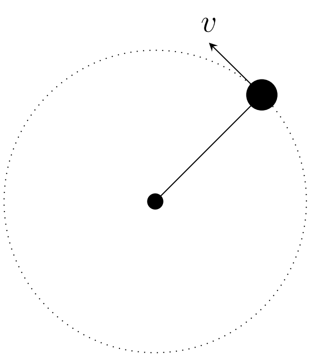
See the following apparatus. The dot or cross shows the direction of the magnetic field of strength 6 T in that square region.



The charged particle depicted enters this region at a velocity of $1\,\mathrm{m\,s^{-1}}$. If this hypothetical charged particle's charge-to-mass ratio is $1\,\mathrm{C\,kg^{-1}}$, show that the particle will be in the magnetic field for $\frac{5\pi}{6}$ seconds.

Question 6

Imagine that an object of mass 3 kg and charge -5×10^{-1} C is being swung by a rope of length 2 m in a circular path as so. This apparatus is not in a gravitational field.



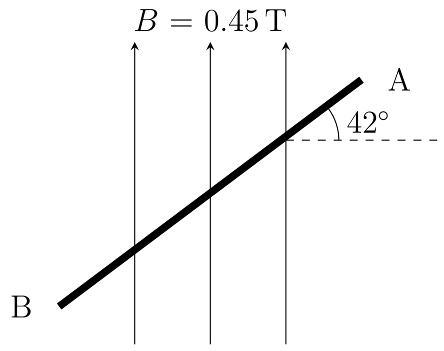
Without a magnetic field, the object can be spun at a frequency of 6 Hz before breaking. Determine the magnetic field that will cause the rope to break when the object is spun at a frequency of 5.9 Hz.

The Motor Effect

Currents in Magnetic Fields

Question 1

The wire shown experiences a force per length of $7.6 \times 10^{-2}\,\mathrm{N\,m^{-1}}$ into the page.



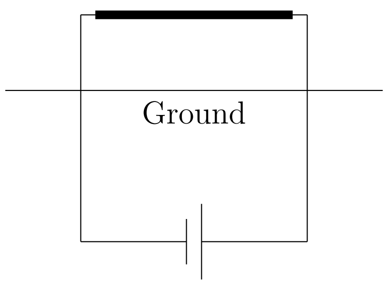
Find the direction and magnitude of the current.

Question 2

Mathematically derive the formula $F = lIB$ from the formula $F = qvB$.

Question 3

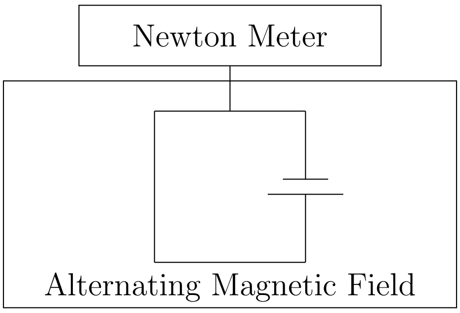
A metre length wire carrying 400 A has a mass of 2.5 kg. The metre-long section is darkened, and the other sections of the circuit have negligible mass.



Determine the magnetic field required to suspend the wire in midair.

Question 4

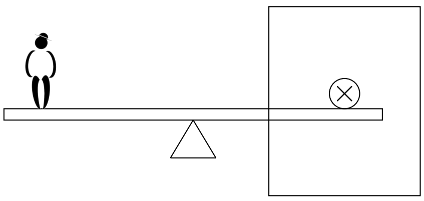
A circuit is suspended as depicted. Its weight is measured by a Newton meter.



The circuit hangs in a rapidly alternating magnetic field between in and out of the page. Describe the value of the measured weight of the coil as the magnetic field alternates in direction for 2 marks.

Question 5

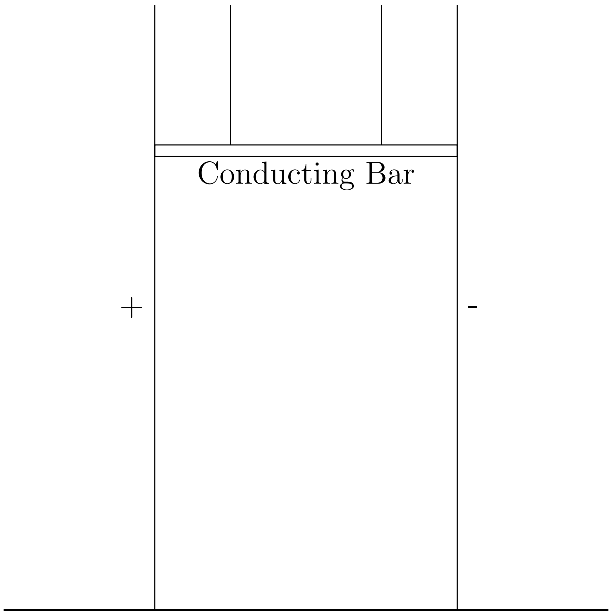
Emmy, who is 78 kg, stands on one side of a lever while a 50.0 cm length wire with a current is on the other end of the lever as shown. The wire is 300.0 g



It is known that the distance between Emmy and the fulcrum is the same as the distance between the wire and the fulcrum. If the current running in the wire is 200.0 A, find the magnetic field the wire resides in for the lever to be stationary.

Question 6

See the following apparatus. The conducting bar is 5.0 m, and is suspended 20.0 m above the ground.

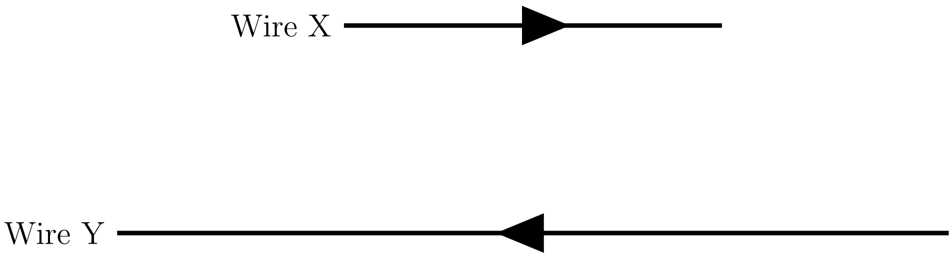


The bar in the middle of the two plates carries a current of 300.0 A. This bar is held in place with two ropes. The entirety of the apparatus is in the presence of a magnetic field of strength 4.2 T out of the page. At an instant, the rope holding the bar snaps, causing the bar to drop downwards whilst maintaining the current through constant contact with the plates. Determine the time it takes for the bar to drop to the floor if the bar's weight is 500.0 N.

Parallel Wires

Question 1

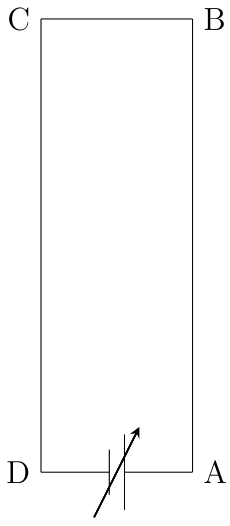
Two parallel wires are separated by a distance of 1.25 m. Wire X is 2.5 m long and carries a current of 4.2 A. Wire Y can be considered to be infinitely long and carries a current of 1.0 A. The currents run in opposite directions.



Determine the force between the two wires, and identify what will happen if Wire X started to increase in length for 2 marks.

Question 2

Emmy has the following circuit with a variable voltage supply.



Which of the following relationships is correct for the voltage of the supply and the force experienced by the circuit sections AB and CD?

A. $F \propto V$

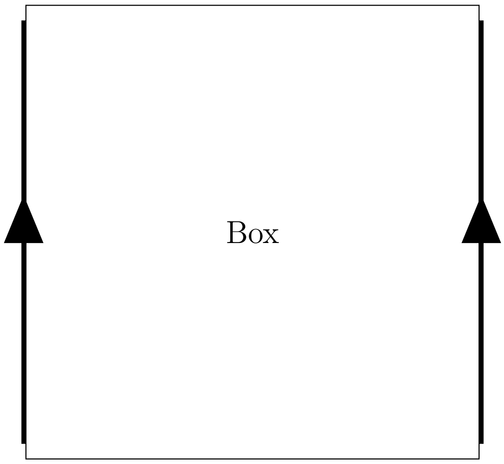
B. $F \propto V^2$

C. $F \propto \frac{1}{V}$

D. $F \propto \frac{1}{V^2}$

Question 3

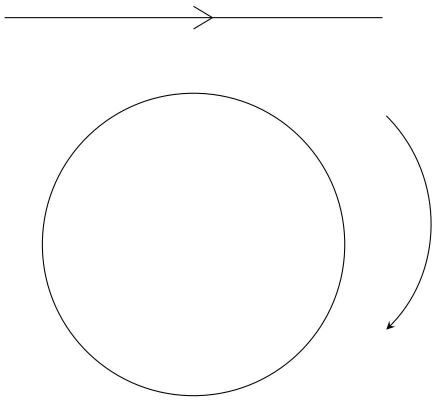
Two wires are found to be in contact with a cube box of side length 3.0 cm as so. Each 2.5 cm wire has a mass of 7.0 g and carries the same current.



Each wire is held onto the wall through friction. If the static coefficient of friction between the wire and the box is 2.3, with the information given in the question, find the magnitude of the current in each wire.

Question 4

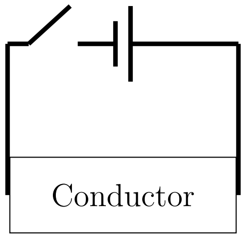
A circular circuit is placed near a straight current-carrying wire as shown.



Explain how the circular circuit is affected by the straight wire for 3 marks.

Question 5

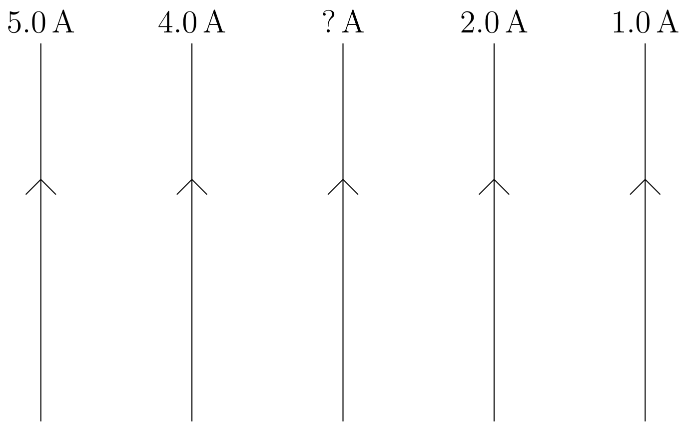
Emmy constructs the following circuit. The wires on the left and right hang freely so that they are in contact with the conductor to complete the circuit.



Emmy then attaches a globe to the circuit and turns the switch on. However the globe continuously flickers on and off. Explain why this occurs for 4 marks.

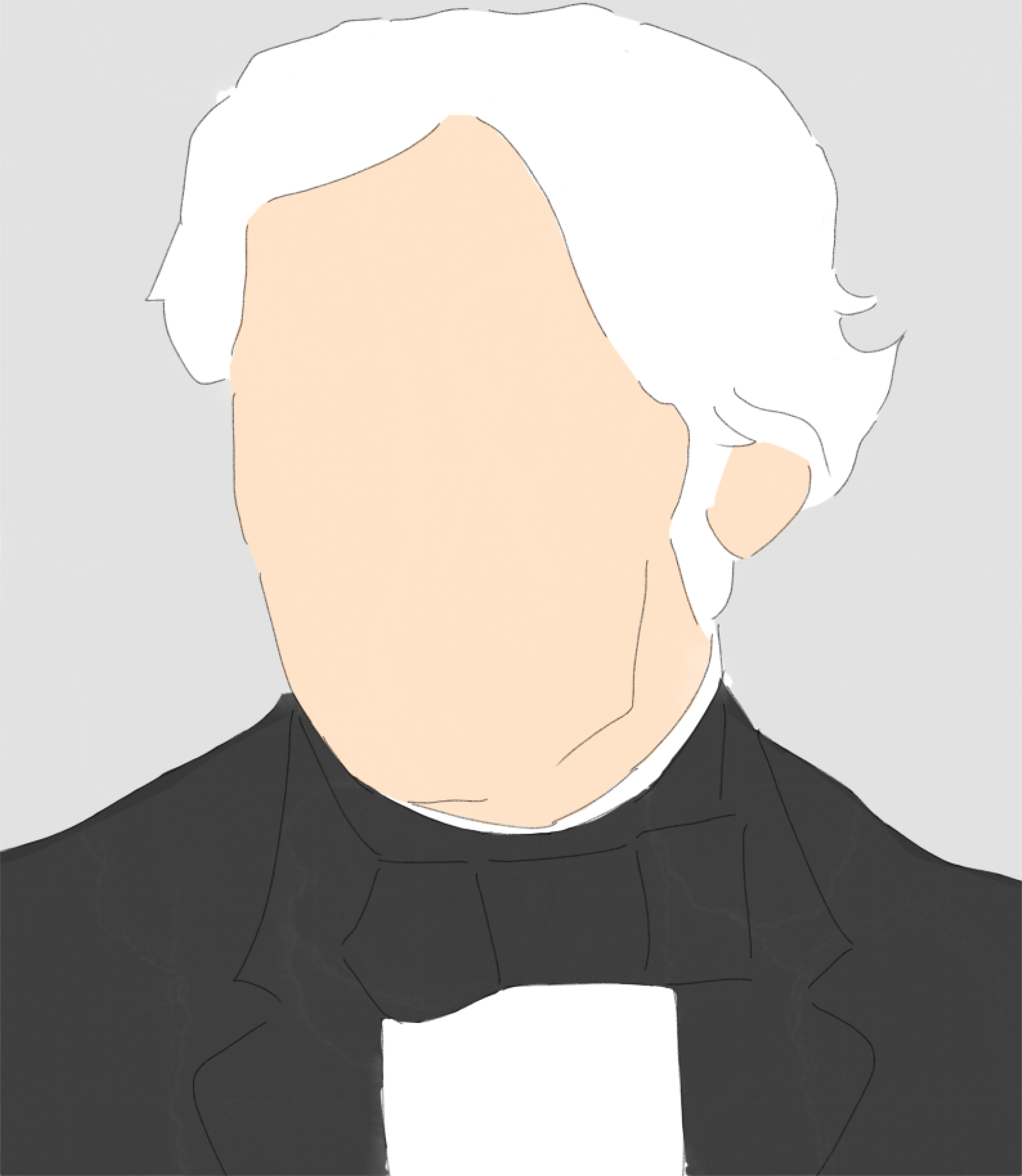
Question 6

5 wires are positioned as shown. Each wire is the same length, and each wire is separated by 3.0 cm from its neighbours.



Emmy removes the wire that carries the current of 4.0 A. Determine the resultant force the middle wire (i.e. the wire with the unknown current) experiences after the removal.

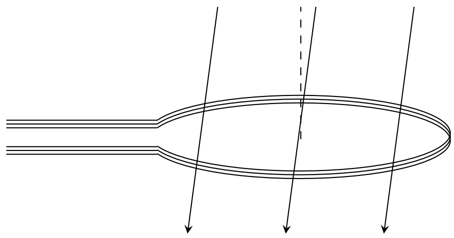
Electromagnetic Induction



Magnetic Flux

Question 1

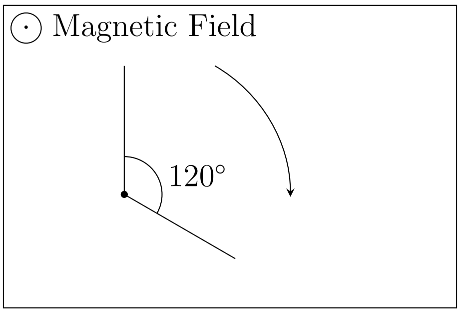
A flat circular coil of 300 turns is placed in a uniform magnetic field of strength 0.15 T so that the magnetic field lines make an angle of 10.0° to the normal.



If the flux linkage of the coil is 8.7×10^{-2} Wb, find the radius of the circular coil to 2 significant figures.

Question 2

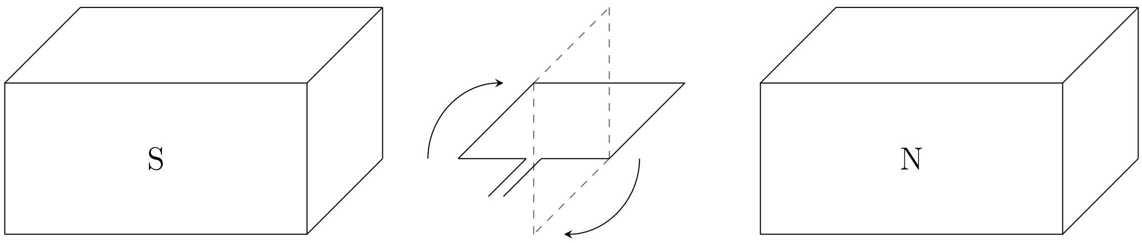
A straight and rigid wire of length 2 m is swung in a circular path within a magnetic field of strength 2 T as shown.



Find the change in magnetic flux for the wire by substituting the area as the area swept by the wire to 2 significant figures.

Question 3 Solution

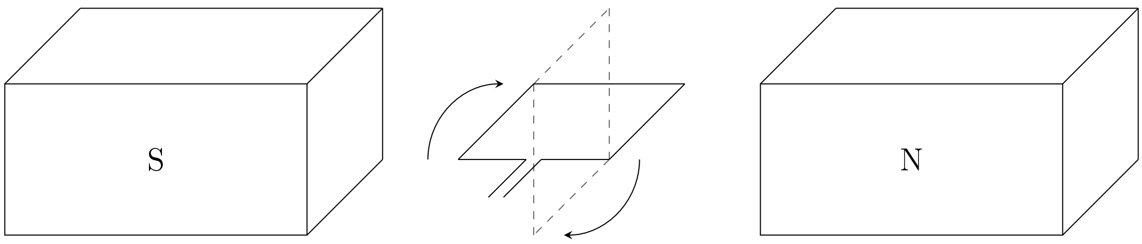
A coil rotates on an axis in a magnetic field as shown.



Draw the graph of the magnetic flux through the coil as it rotates through one full revolution starting from the position depicted above.

Question 4

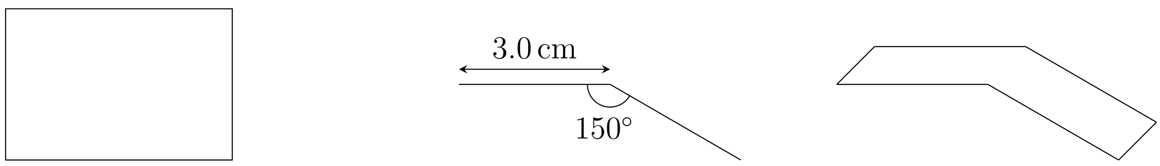
A coil rotates on an axis in a magnetic field as shown.



The coil rotates at 3400 rpm. How many times will the coil's flux be at a maximum as the coil rotated for 3.5 seconds starting from the position depicted above?

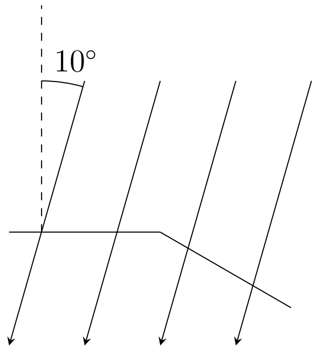
Question 5

A particular coil has a shape of two squares at an angle, as shown.



TOP VIEW SIDE VIEW 3D VIEW

The coil is placed in a uniform magnetic field of strength 0.55 T as shown.

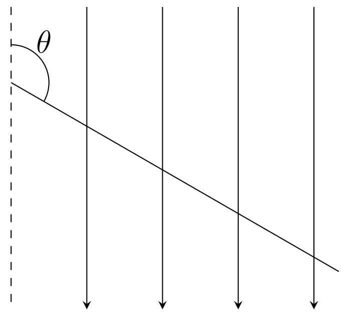


SIDE VIEW

Find the flux linkage of the coil to 2 significant figures.

Question 6

A square coil of side length 4.0 cm is placed in a uniform magnetic field of strength 0.4 T as shown.



If the magnetic flux linkage is 3.2×10^{-4} Wb, find θ.

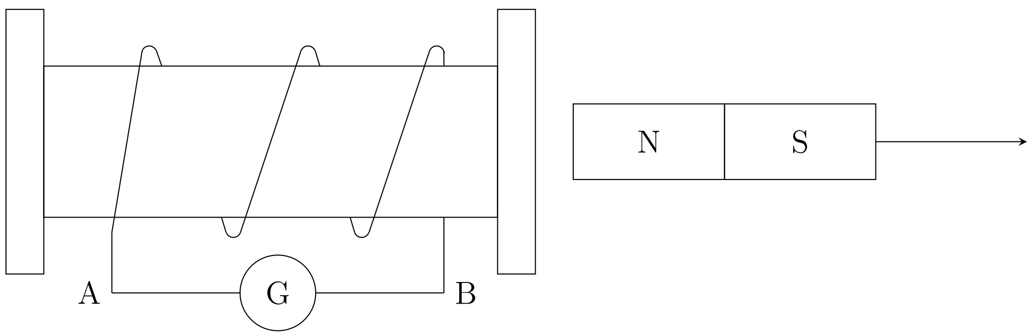
Faraday's Law and Lenz's Law

Faraday's law and Lenz's law are both fundamental principles in electromagnetic induction, however, they address different aspects of electromagnetic induction.

Faraday's law simply states that a changing magnetic flux through a closed loop induces an electromotive force (emf). Lenz's law however, provides the direction of the induced current resulting from the said emf. Lenz's law states:

> The direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field.

However, the statement may be slightly difficult to understand. A possibly easier way to understand Lenz's law is through an example application: if a magnet is moved near a closed loop, the loop will have an induced current that will "try to make the movement of the magnet harder". A diagram and explanation are given below.



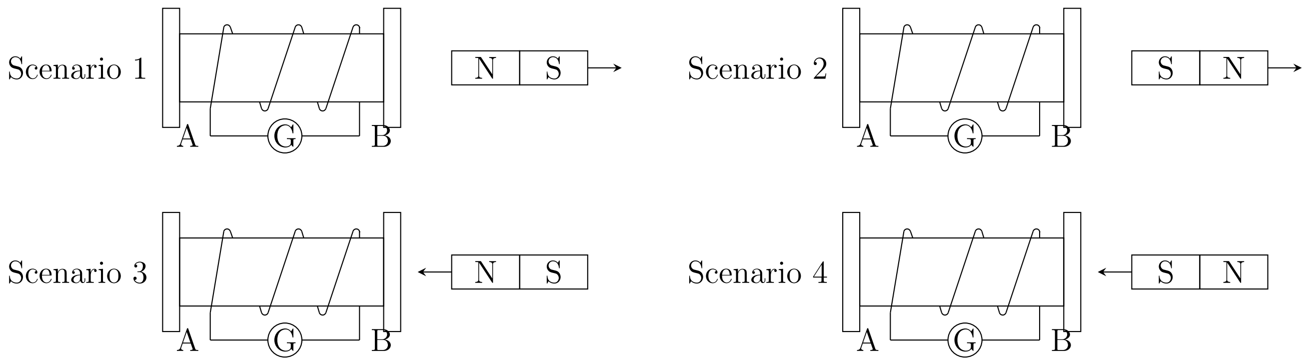
In the scenario above, imagine that the solenoid will have an induced polarity to make the movement of the magnet more "difficult". Therefore, the solenoid will have an induced South pole on the right to make the magnet moving away more difficult through magnetic attraction. Thus, by the right hand rule, the current must move from B to A through the galvanometer (denoted by the G). This is the essence of Lenz's law.

Thus, in whatever change of magnetic flux, a closed loop will have an induced current that results in a magnetic field that opposes the initial change in magnetic flux.

Questions are given on the following page.

Question 1

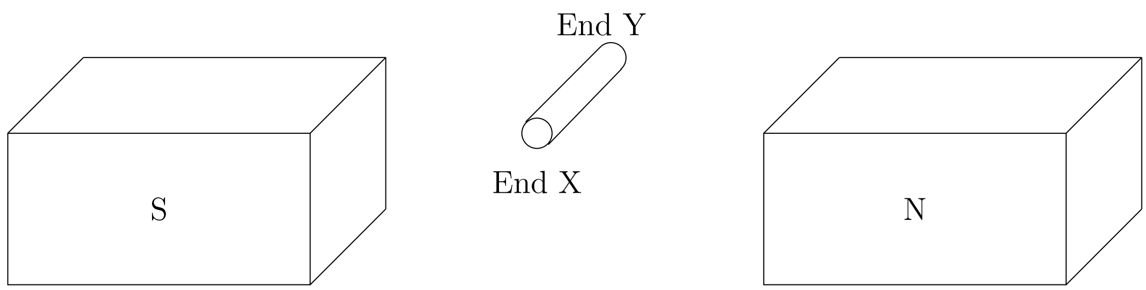
Four scenarios are shown below.



Identify in which direction the current will flow through the galvanometer in each scenario.

Question 2

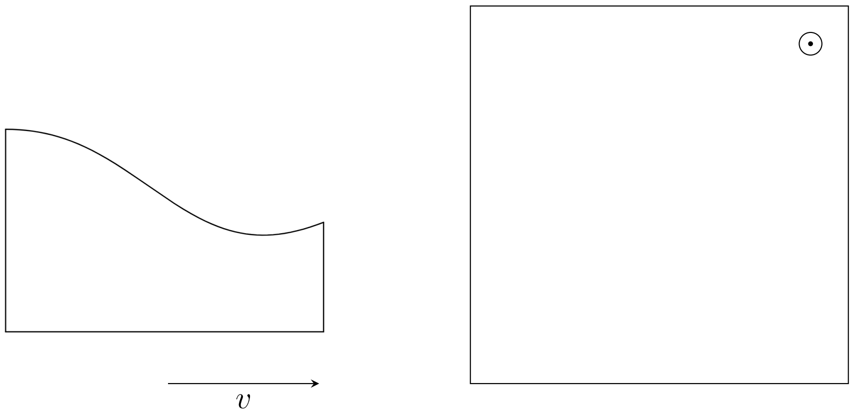
A small section of a wire that is capable of conducting a current is dropped from the position shown.



Identify the direction of the induced conventional current during the fall and explain why this occurs for 3 marks.

Question 3*

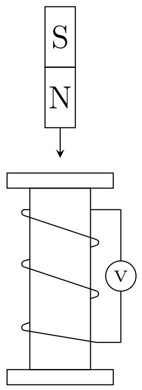
A conductor that is capable of carrying a current is constructed so that it exactly resembles the area bounded by the curve $y = \cos(x^{\cos x})$, the y-axis, the x-axis and the line $x = \frac{\pi}{2}$. This loop then enters a constant magnetic field at a constant velocity as shown below.



Express the magnitude of the emf produced in the loop as it enters the field ε_{i} in terms of the magnitude of the emf produced in the loop as it is completely immersed in the field ε_{f}.

Question 4

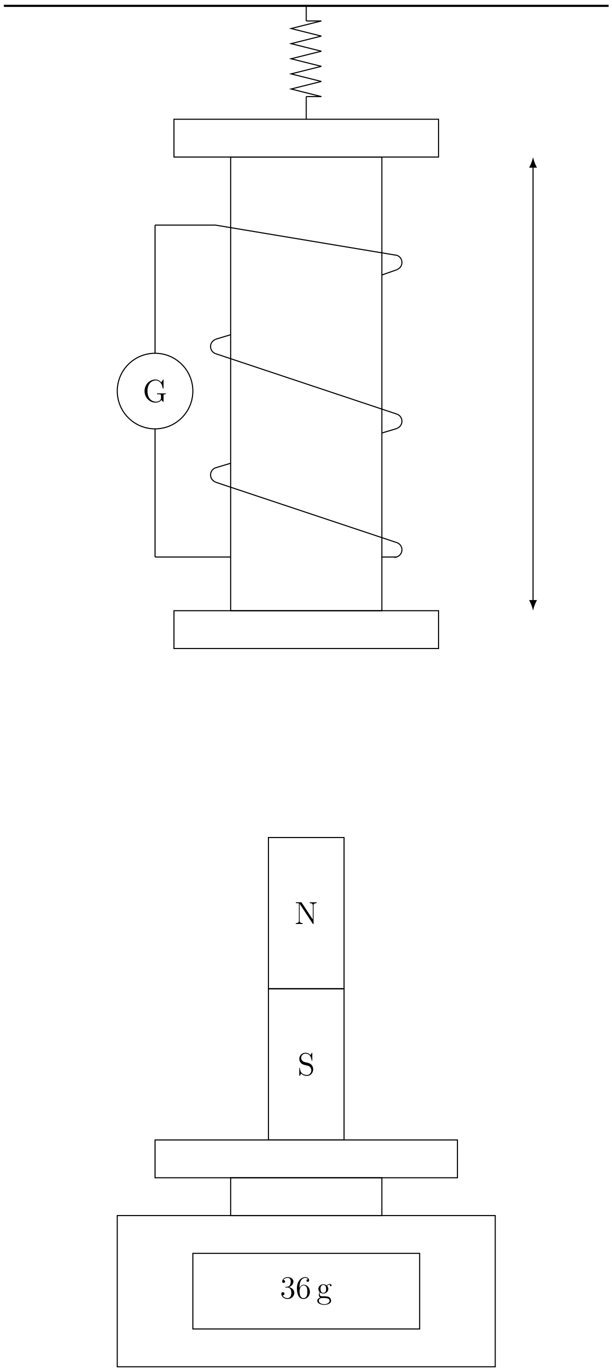
A bar magnet is dropped through the centre of a hollow solenoid, and the voltage is recorded as shown.



Assuming that the magnet enters the solenoid at a velocity lower in magnitude than the terminal velocity inside the solenoid, draw a graph of the potential difference against time after the bar magnet is dropped for 3 marks.

Question 5

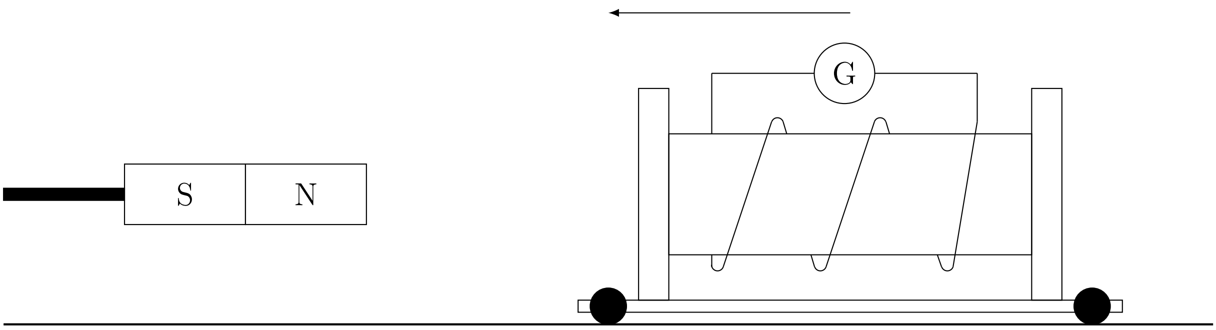
A bar magnet is placed on a sensitive electronic balance, and a hollow solenoid is hung via a spring so that the solenoid oscillated downwards and upwards as depicted.



By referring to the induced polarity of the solenoid throughout the oscillation, explain how the measured weight of the bar magnet changes throughout the oscillation for 4 marks.

Question 6

A cart with a mounted solenoid travels towards a bar magnet placed so that it would pass through the hollow solenoid without collision, as shown.



Assuming that no energy is lost by friction, explain what will occur to the cart's motion and energy before passing completely through the bar magnet for 3 marks.

Transformers

Question 1

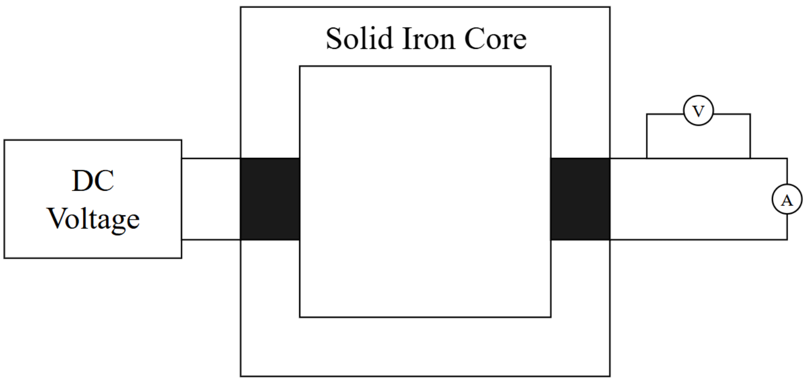
Emmy states that step up transformers do not follow the conservation of energy as the voltage increases in the secondary coil. Including mathematical reasoning, show that energy is conserved in ideal transformers for 2 marks.

Question 2

From the formula $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$, mathematically derive the formula $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for an ideal transformer.

Question 3

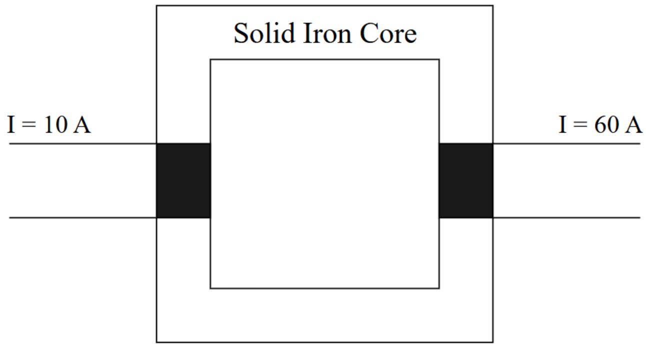
Emmy sets up the following ideal transformer.



If the DC voltage is turned on only once and kept on for an extended period of time, explain how this transformer will result in a current and voltage in the secondary circuit, and why the transformer cannot produce a constant non-zero voltage and current in the secondary circuit for 3 marks.

Question 4

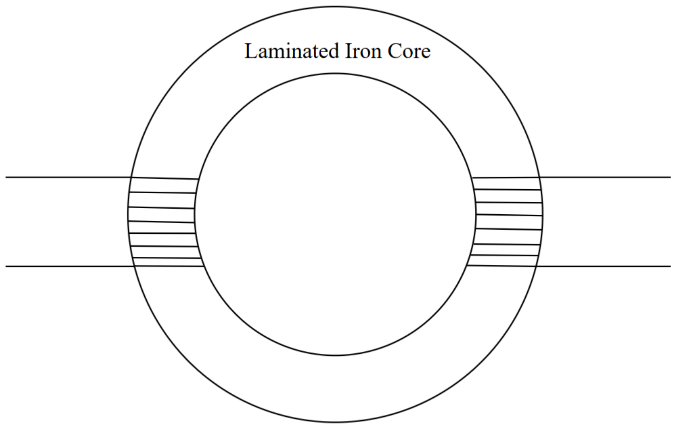
An ideal transformer is shown below.



Determine if the transformer is step down or step up, and find the ratio of primary coils to secondary coils.

Question 5

A transformer is shown below.



Explain how 2 modifications have improved its efficiency for 4 marks.

Question 6

Explain the necessity of transformers in the distribution of power in our modern society. Refer to how a transformer works and power loss during transmission in your response for 5 marks.

Applications of the Motor Effect

Back emf

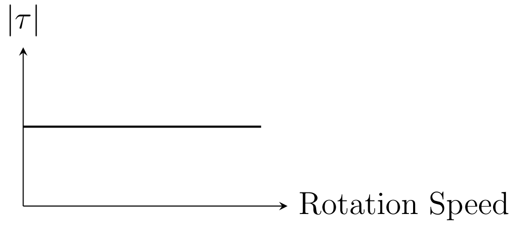
Question 1

The car window of a 1997 Ford Taurus can be rolled up or down by an electric motor. However, it is known that if you attempt to roll up the car window while it is blocked by some debris, the motor will overheat, and the rolling of the window will no longer work. Explain why the motor overheats for 1 mark.

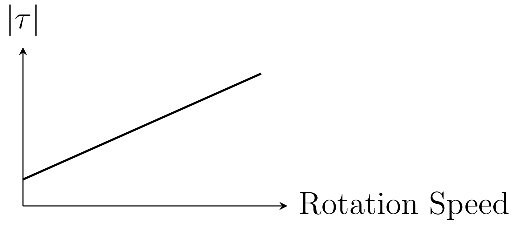
Question 2

Which graph is most accurate in representing the magnitude of torque $|\tau|$ of a DC motor from startup to rotation speed?

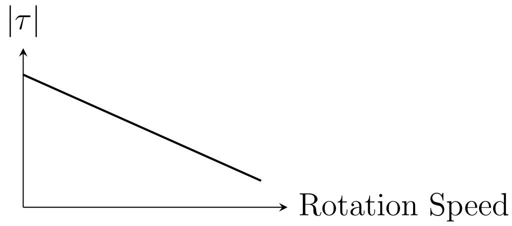
A.



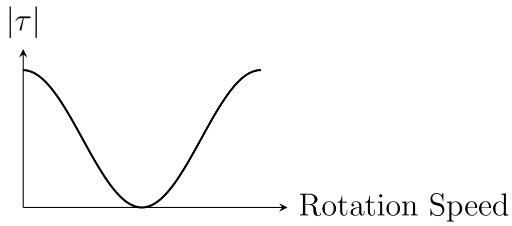
B.



C.

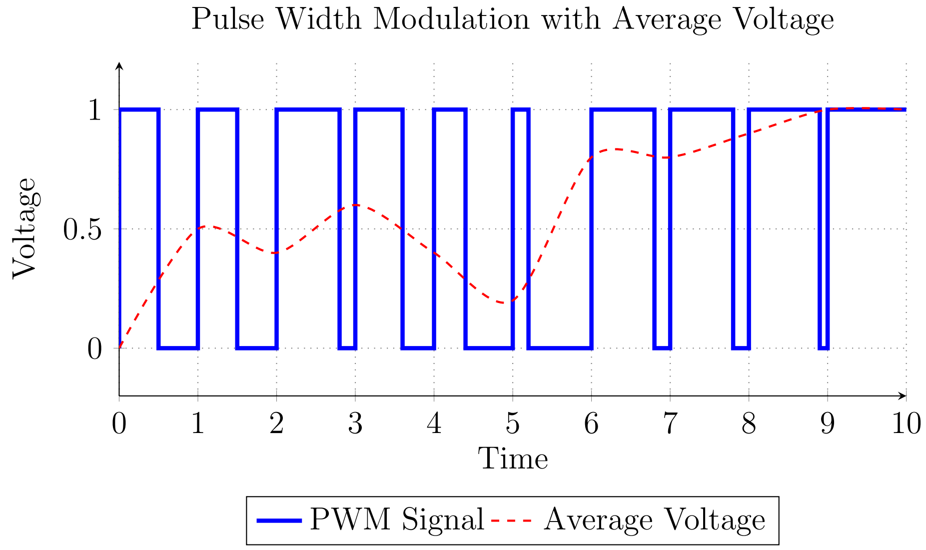


D.



Question 3

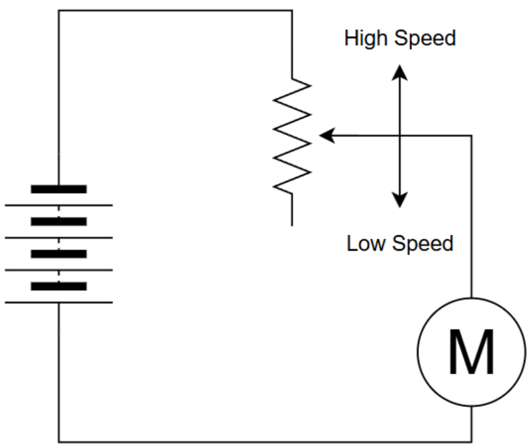
Sometimes, having a variable voltage supply or a variable resistor (rheostat) to control the voltage of a motor as it winds up to operating speed can be inefficient. Modern motors often use Pulse-Width Modulation (PWM) to gradually increase voltage as the motor rotates to operating speed, limiting initial current. This is shown below.



Explain how PWM protects motors for 3 marks.

Question 4

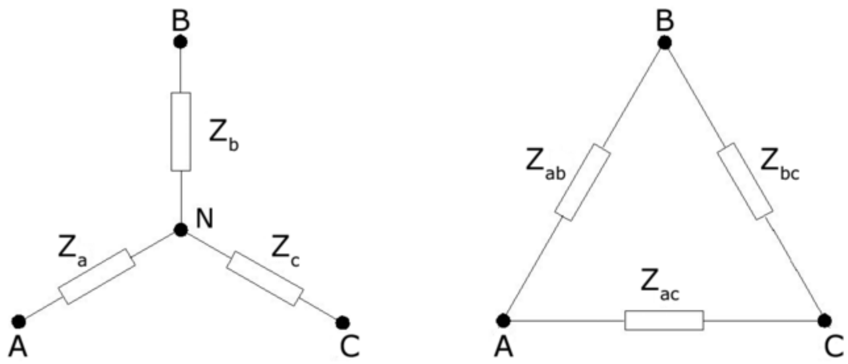
Suppose an apparatus where, as the rotation speed of a motor increases, the centrifugal force produced through the rotation results in the connection point of the circuit to a resistor move upwards, decreasing the resistance of the circuit, and vice versa, as shown below.



Explain how this apparatus protects the motor for 3 marks.

Question 5

AC motors limit the initial voltage by starting the motor in a high-impedance ”star” configuration, then eventually switching to the ”delta” configuration for full power. The delta connection, shown on the right in the diagram below, has a lower resistance than the star connection shown on the left.



Explain how the star-delta starter configuration protects motors for 3 marks.

Question 6

An ideal electrical motor is connected to a DC voltage supply, so it rotates at a constant 200 rpm. No load is placed on the motor. Which of the following statements is most accurate?

A. The DC voltage is greater than the back emf, as the back emf itself is capable of supplying the voltage for the motor to run.

B. The DC voltage is equal to the back emf, as no work is being done on the motor.

C. The DC voltage is less than the back emf, as the back emf itself is capable of supplying the voltage for the motor to run.

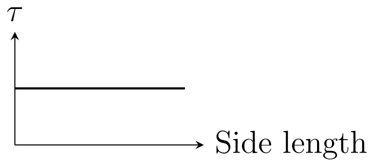
D. The DC voltage is greater than the back emf, as a current must be present in the rotor to continue rotation through the motor effect.

DC Motors

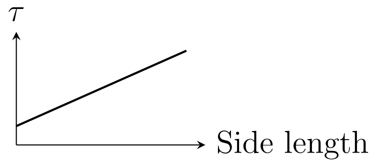
Question 1

If the side length of the square coil of a DC motor were to be increased, how would the average torque of the DC motor appear to increase?

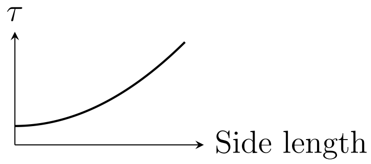
A.



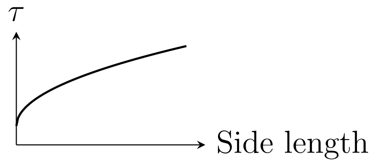
B.



C.

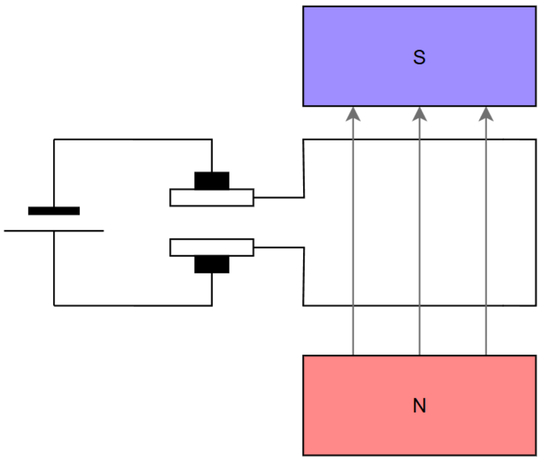


D.



Question 2

The diagram below represents a DC motor.



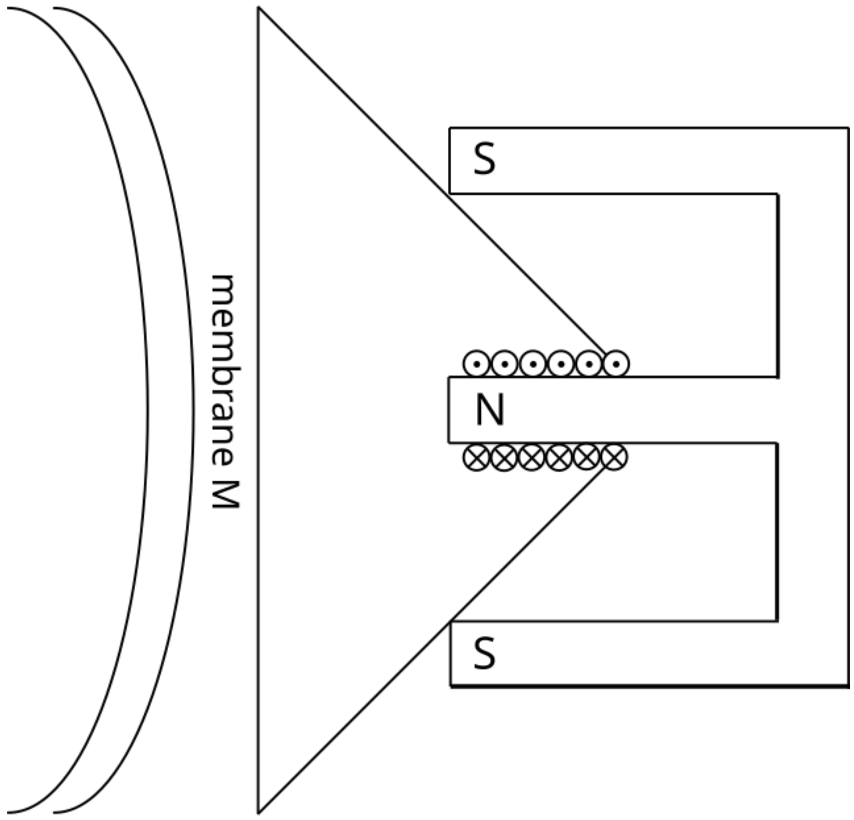
Explain how the direction of the torque is maintained as the loop rotates for 2 marks.

Question 3

Emmy has a DC motor. The DC motor is capable of producing a torque of 0.10 N m on a voltage supply of 6.0 V. Emmy then plugs the DC motor into a voltage supply of 9.0 V. If the motor is capable of handling such a voltage, what would its torque be capable of producing now?

Question 4

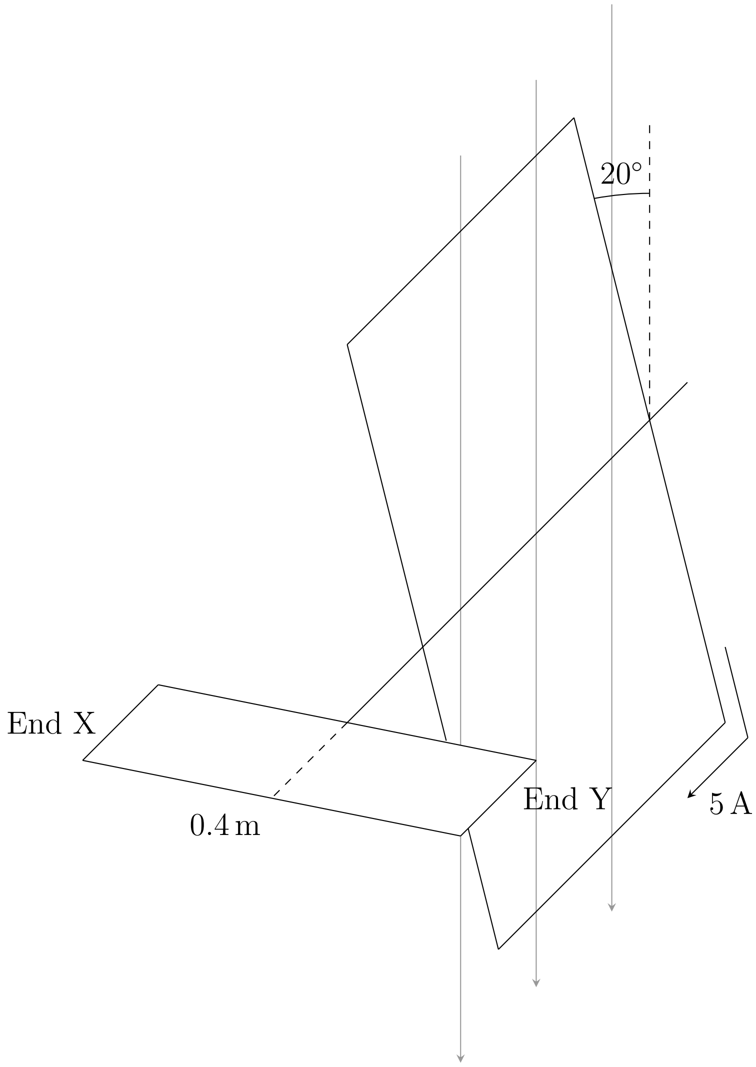
Loudspeakers work by the same principle as DC motors. In the diagram shown below, the coil will move to the left, resulting in the membrane moving to the left. This causes a longitudinal pressure wave in the atmosphere that is transmitted as sound.



Explain how loudspeakers convert electrical signals to sound waves in the air for 3 marks.

Question 5

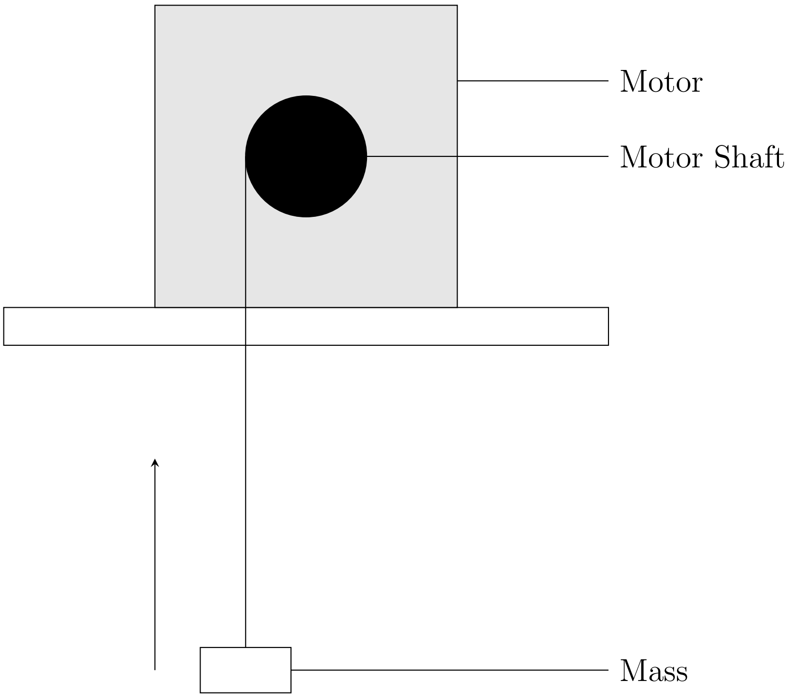
A simple DC motor of a flat rectangular coil with 500 turns in a magnetic field of strength 0.400 T directly downwards as shown below.



The coil has an area of $0.240\,\text{m}^2$ and carries a current of 5.00 A. The motor is connected to a lever of length 0.400 m at a rotation axis in the middle of the lever. Determine what mass at which end will prevent the motor from turning in the position shown (assume the mass is placed 0.200 m from the rotation axis).

Question 6

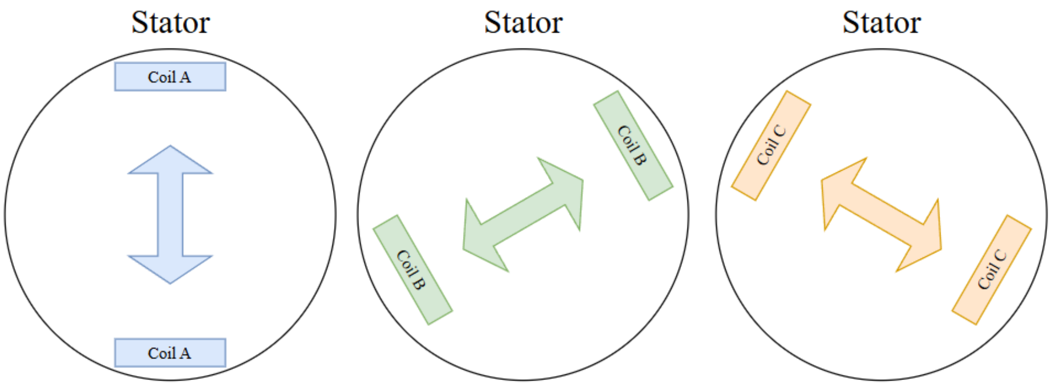
A 0.50 kg mass is lifted at a constant speed by a DC motor on top of a table as shown.

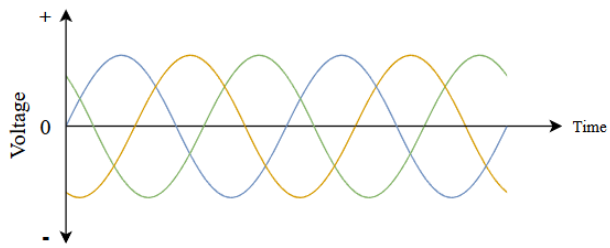


The motor has a coil of 50 turns in a 0.10 T magnetic field. The area of the coil is $2.0 \times 10^3\,\text{mm}^2$. The motor shaft has a diameter of 0.010 m. If the DC voltage of the motor is 12 V, find the minimum instantaneous power output of the motor.

AC Motors

The typical AC motor is referred to as the three phase AC induction motor. The "AC" refers to how the motor naturally runs on AC, and the "induction" refers to how the motor primarily works through electromagnetic induction. However, the "three phase" refers to how the stator (the stationary component of the motor) is constructed. The AC motor is called three phase as there are essentially three coils that have each three different phases of voltages through them, as shown.



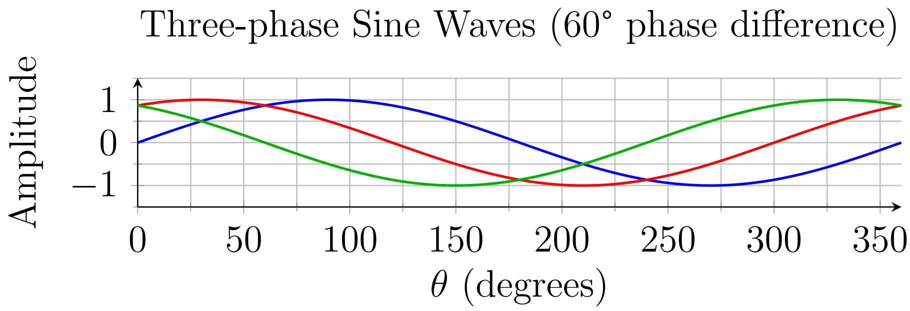


Each coil will have a magnetic field (shown by the arrows) that continually reverses in direction as the voltages reverse due to the nature of AC voltages. Now, each phase is 120° out of phase with each other. This is extremely important, as this will produce between all three coils a NET rotating magnetic field. This can be intuitively understood by the analogy of Tusi motion (it is recommended to search for videos of this illusion on the web). In this illusion, focus on one ball and the illusion will disappear as you realise each ball is simply travelling back and forth in a straight line, similar to how each of the three coils in an AC induction motor only reverse in direction. However look at the general picture of the balls, and because all the balls are out of sync in a particular timing, it creates the illusion that all of the balls are rotating, similar to how if you take the sum of all of the magnetic fields of the coils, due to their specific phases, they create a net rotating magnetic field. Now the rotor itself does not produce any current by itself, but due to the squirrel cage structure, each of the bars experience a change in magnetic field (due to the rotating magnetic field of the stator) resulting in Lenz's law to induce a current that results in the motor effect, leading to each bar essentially "chasing" the rotating magnetic field (but it lags, known as "slip"), and thus the rotor rotates.

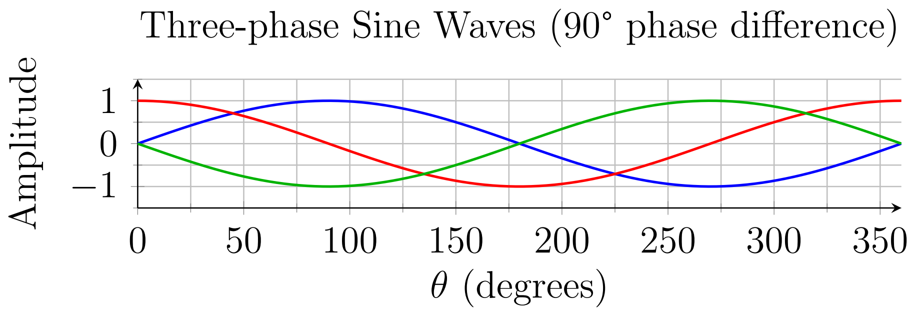
Question 1

Which of the following most accurately represents the voltages of the three phase induction motor?

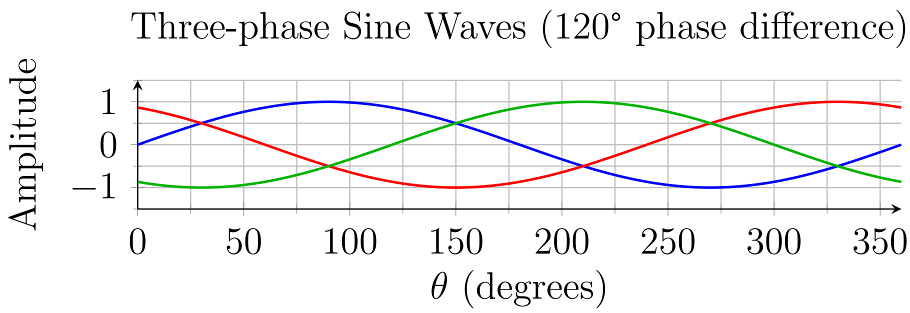
A.



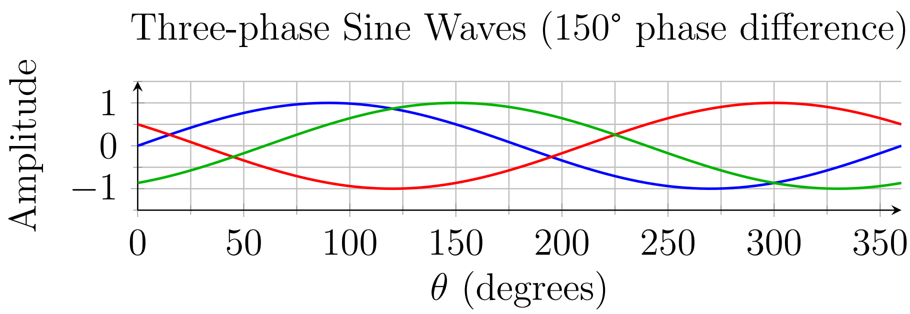
B.



C.



D.



Question 2

Explain why the rotor rotates in the same direction as the magnetic field's rotation in an AC induction motor for 2 marks.

Question 3

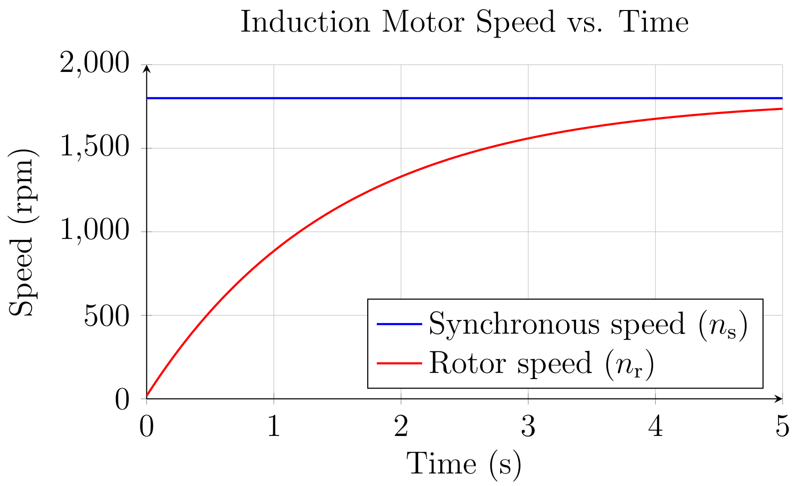
Explain how a rotating magnetic field is created in a three phase AC induction motor for 3 marks.

Question 4

Describe the role of the rotor and stator in a three phase AC induction motor for 4 marks.

Question 5

The rotational speed of the magnetic field produced by the stator in an AC induction motor is called the synchronous speed, while the rotational speed of the rotor is called the motor speed. The graph below shows how the two values increase with respect to time after the motor is started.



Explain why the rotor speed does not reach the synchronous speed for 4 marks.

Question 6

Explain how a three phase AC induction motor converts electrical energy in the form of AC into mechanical energy for 5 marks.

DC and AC Generators

Question 1

List 4 different changes to an electrical generator that will increase its output for 2 marks.

Question 2

Describe the role of Faraday and Lenz's law in the operation of a generator for 3 marks.

Question 3

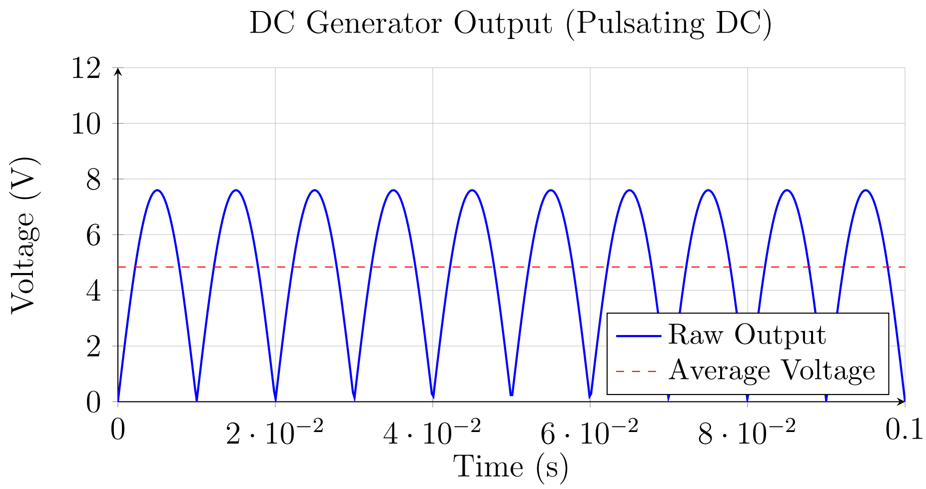
Explain how an AC generator converts mechanical energy to electrical energy in the form of AC for 3 marks.

Question 4

Compare the function of the split ring commutator and the slip ring commutator in DC and AC generators, and sketch the respective currents each would draw from the generator on the same axes for 3 marks.

Question 5

The electrical output of generators can include unwanted fluctuations known as noise, as shown below.

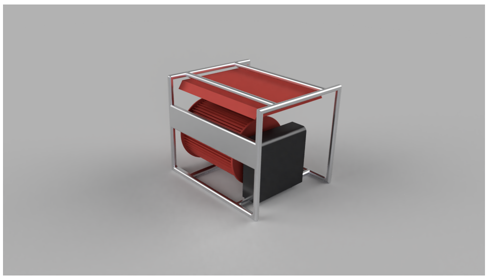


Noise can be particularly undesirable in some circumstances where a very constant electrical output is required. Identify and justify a change to a DC generator that will reduce noise, and sketch the DC voltage drawn after the change for 3 marks.

Question 6

AC generators are more common at larger scales, while DC generators can be found typically at a smaller scale. An example of a large system of AC generators is the Grand Coulee Dam, consisting of 33 hydroelectric AC generators that produce an estimated staggering 4% of all of the United States' hydroelectric power generation. An example of a DC generator is a small gas-driven multi-use generator, as shown below.

Grand Coulee Dam



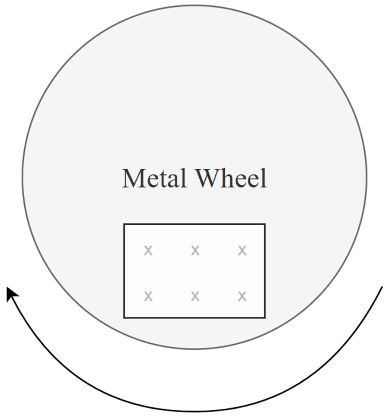
DC generator

With reference to 3 unique advantages, explain why AC generators are much more common for larger scales for 4 marks.

Applications of the Conservation of Energy in Electromagnetism

Question 1

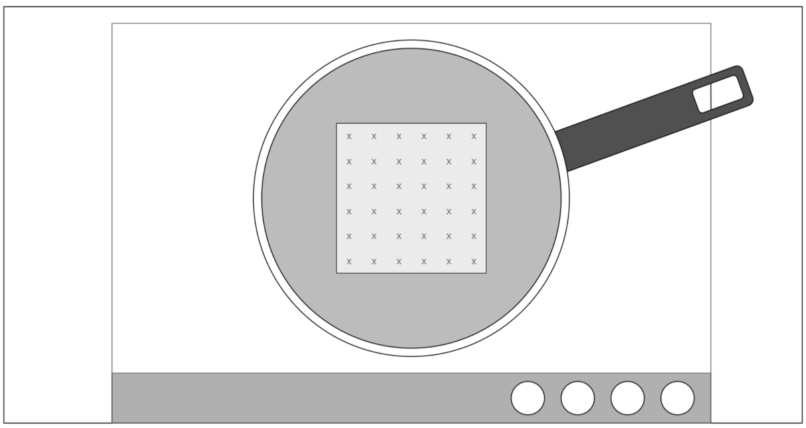
A rotating metal wheel that is in a magnetic field is shown below.



Draw the eddy currents that arise due to this motion for 1 mark.

Question 2

An induction cooktop works by inducing eddy currents in cookware, resulting in heat. A snapshot of a frying pan on an induction cooktop with the magnetic field is shown below.



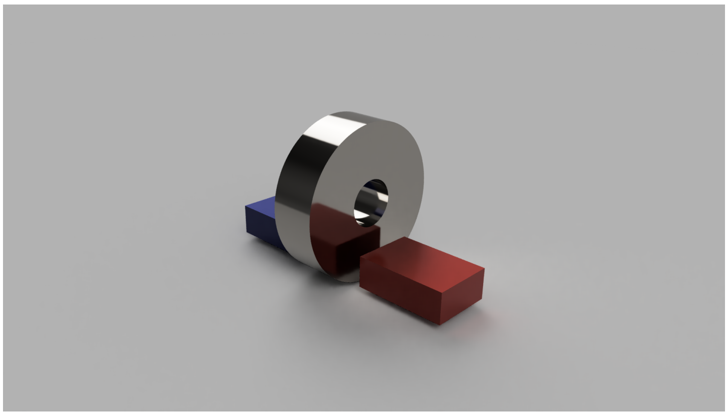
Is the magnetic field of the induction cooktop constant or alternating? Explain why for 2 marks.

Question 3

Explain how lamination in a metal core improves efficiency for 2 marks.

Question 4

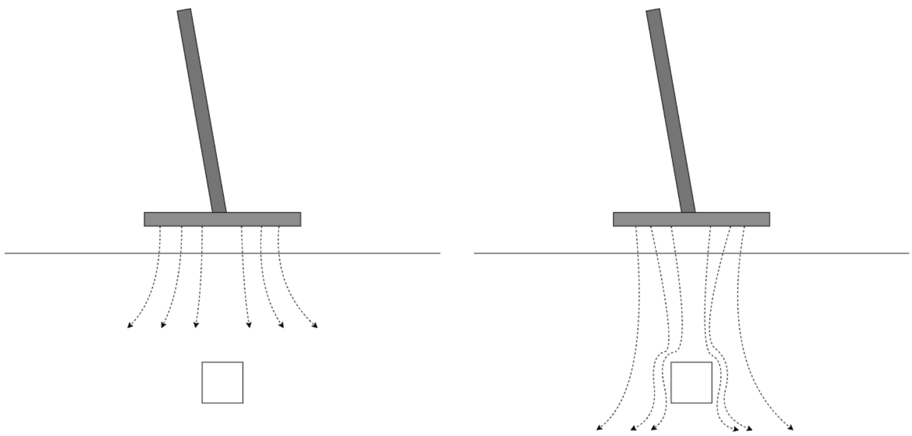
A metal wheel rotates in a magnetic field as shown.



A phenomenon known as electromagnetic braking occurs. Explain how the kinetic energy of the rotating metal wheel is converted to other forms for 2 marks.

Question 5

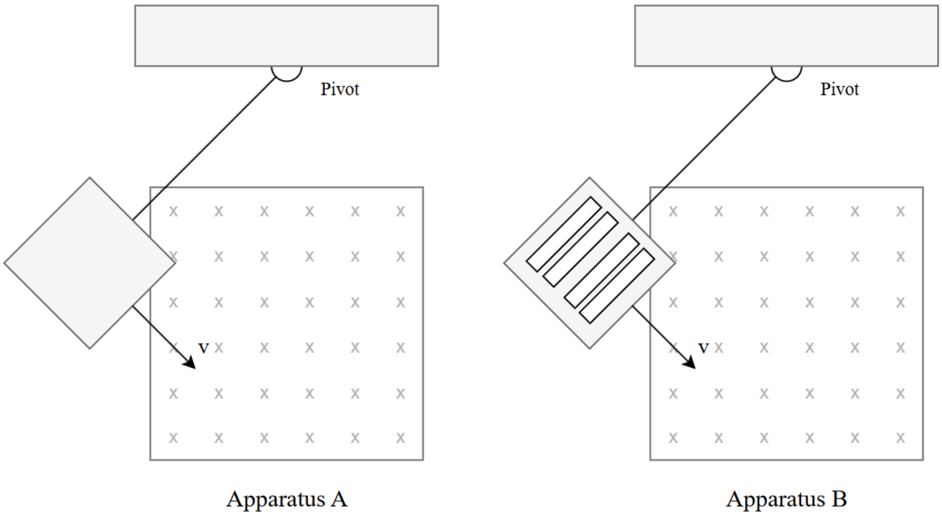
A metal detector works by producing an alternating magnetic field. The metal detector then detects a disturbance in its own magnetic field in the presence of a metal as shown.



Explain how the metal object disturbs the alternating magnetic field of the metal detector for 3 marks.

Question 6

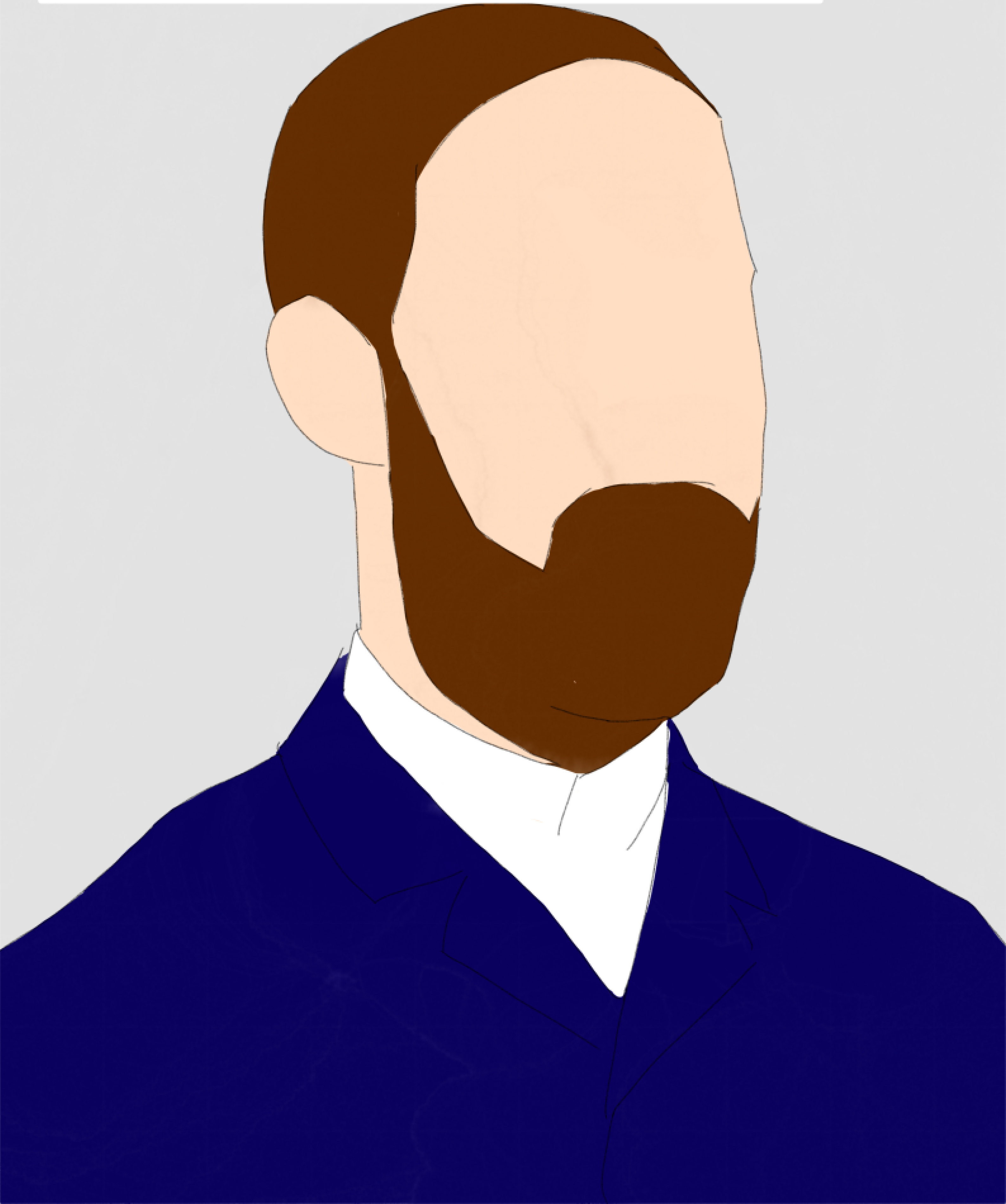
A conducting plate is hung from a pivot point to swing freely. The conducting plate moves through a magnetic field. The experiment is carried out again, however, the conducting plate now has slits cut out of the plate as shown.



Explain why apparatus A comes to rest faster with reference to the conservation of energy for 3 marks.

The Nature of Light

The Electromagnetic Spectrum



Maxwell's Theory of Electromagnetism

Maxwell's 4 equations are a set of (coupled partial differential) equations that form the foundation of classical electromagnetism. Although the equations are not explicitly studied in the HSC Physics course, it is beneficial to have a brief understanding of his work.

Partial Differential Form	Integral Form
$\nabla \cdot \vec{E} = \frac{q}{\varepsilon_0}$	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot d\vec{A} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt}$

There's no need to memorise or mathematically understand these equations, but a brief description of each equation is given below.

1. Gauss's Law for Electricity:
States that the electric flux through a closed surface is proportional to the charge enclosed by the surface.

2. Gauss's Law for Magnetism:
States that the magnetic flux through a closed surface is a net 0, implying magnetic monopoles do not exist.

3. Faraday's Law of Induction:
This should be familiar, as it's studied in module 6, electromagnetism.

4. Ampère-Maxwell Law:
States that a changing electric field or current produces a magnetic field. This is especially important, as this allows for the self-propagation of EM waves.

Some other notable impacts of Maxwell's theory of electromagnetism include:

- The unification of electricity and magnetism into a single theory.
- The prediction of electromagnetic waves as self-propagating electric and magnetic fields.
- The prediction of the speed of light as $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$.
- The implication that light has a momentum $\frac{E}{c}$.

Question 1

What is the expression for the speed of light derived from Maxwell's equations?

Question 2

What led Maxwell to discover that light is an electromagnetic wave? Identify for 1 mark.

Question 3

Identify the most widely accepted model of light prior to Maxwell's theory of electromagnetism for 1 mark.

Question 4*

Given below are Maxwell's equations.

Partial Differential Form	Integral Form
$\nabla \cdot \vec{E} = \frac{q}{\varepsilon_0}$	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot d\vec{A} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt}$

Provide a brief description of each of Maxwell's equations for 2 marks.

Question 5

Explain the significance of Maxwell's theory of electromagnetism for 3 marks.

Question 6

Compare the understanding of electricity & magnetism, and fields before and after Maxwell's theory of electromagnetism for 3 marks.

Production and Propagation of Electromagnetic Waves

Question 1

What are electromagnetic waves?

A. Transverse waves made of 90° out-of-phase oscillating electric and magnetic fields that are perpendicular to each other.

B. Transverse waves made of in-phase oscillating electric and magnetic fields that are perpendicular to each other.

C. Longitudinal waves made of 90° out-of-phase oscillating electric and magnetic fields that are parallel to each other.

D. Longitudinal waves made of in-phase oscillating electric and magnetic fields that are perpendicular to each other.

Question 2*

Which pair of Maxwell's equations is most relevant in the explanation of self-propagation of electromagnetic waves?

A. Gauss's Law for Electricity and Gauss's Law for Magnetism

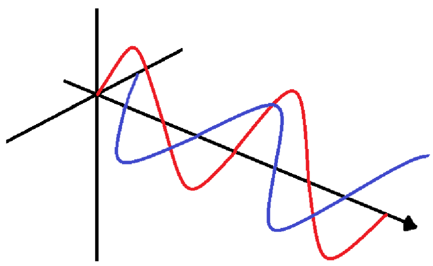
B. Gauss's Law for Electricity and Faraday's Law

C. Gauss's Law for Magnetism and the Ampère-Maxwell Law

D. Faraday's Law and the Ampère-Maxwell Law

Question 3

Emmy was suddenly asked by her professor to draw an electromagnetic wave. She quickly sketched the diagram shown below.



Is Emmy's diagram correct? Explain for 1 mark.

Question 4

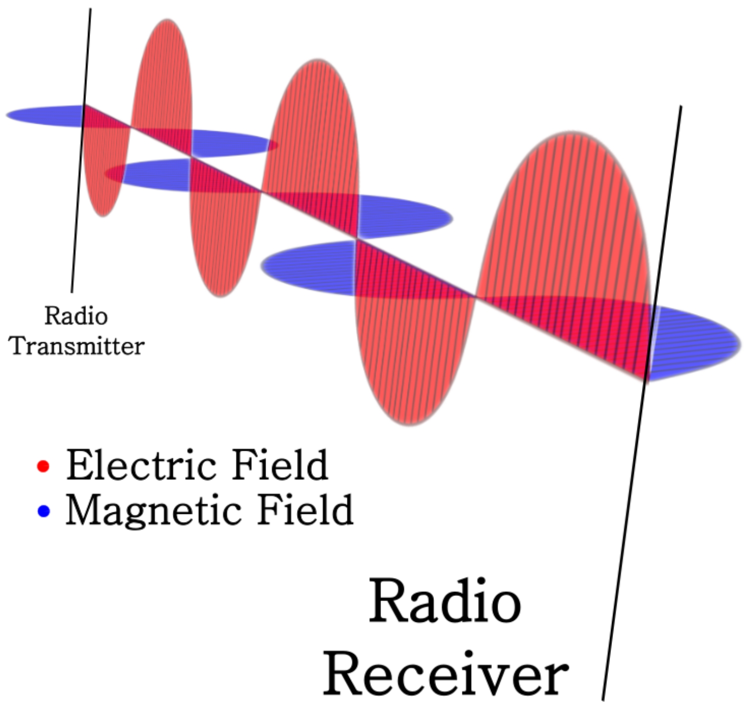
Explain how electromagnetic waves propagate without a medium for 2 marks.

Question 5

Accelerating charges will produce electromagnetic radiation, while stationary or uniformly moving charges will not. Can a wire of AC produce continuous electromagnetic radiation? Can a wire of DC? Explain your answer for 4 marks.

Question 6

Radio waves are created by AC oscillating in a radio transmitter antenna. This is shown below.



This is because the oscillating electrons of the AC produce an oscillating electric field. Explain how the radio receiver antenna's electrons begin to oscillate with the same frequency as the radio transmitter antenna's electrons for 4 marks.

Investigations of the Speed of Light

Question 1

An investigation of the speed of light was carried out by observing the variations of the orbital period of Io, a moon of Jupiter. Who was responsible for taking out the measurements of the orbital periods?

A. Ole Rømer

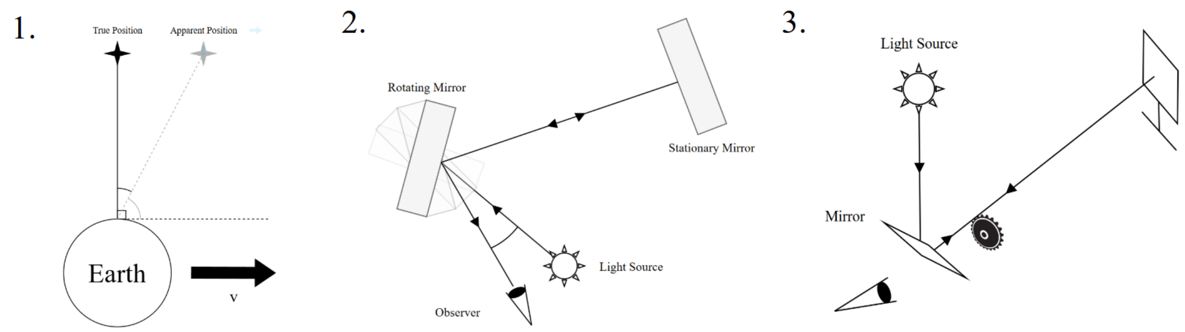
B. James Bradley

C. Hippolyte Fizeau

D. Léon Foucault

Question 2

Three historical investigations of the speed of light are shown below.



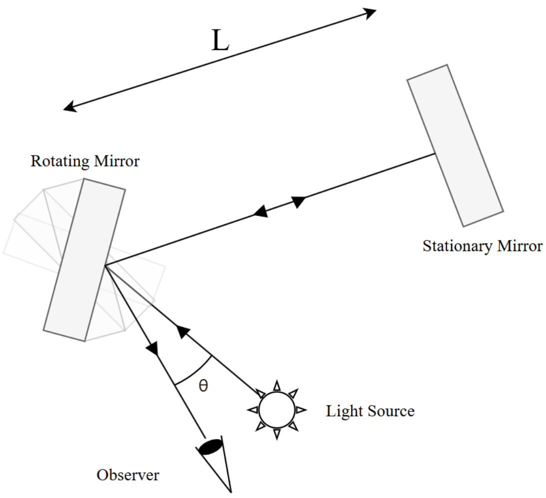
Link each investigation to the historical figure that was responsible for the measurement for 2 marks.

Question 3

Rømer's "measurement" of the speed of light of approximately $2.3 \times 10^8\,\mathrm{m\,s^{-1}}$ is significantly lower than the value for c accepted today. Identify why Rømer's determination of the speed of light was still historically important for 1 mark.

Question 4

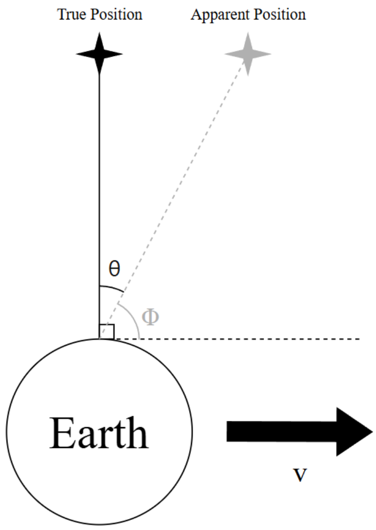
Foucault's apparatus for measuring the speed of light is shown below.



Given the angular velocity of the rotating mirror is ω, find the expression for the speed of light in terms of L, ω and θ.

Question 5

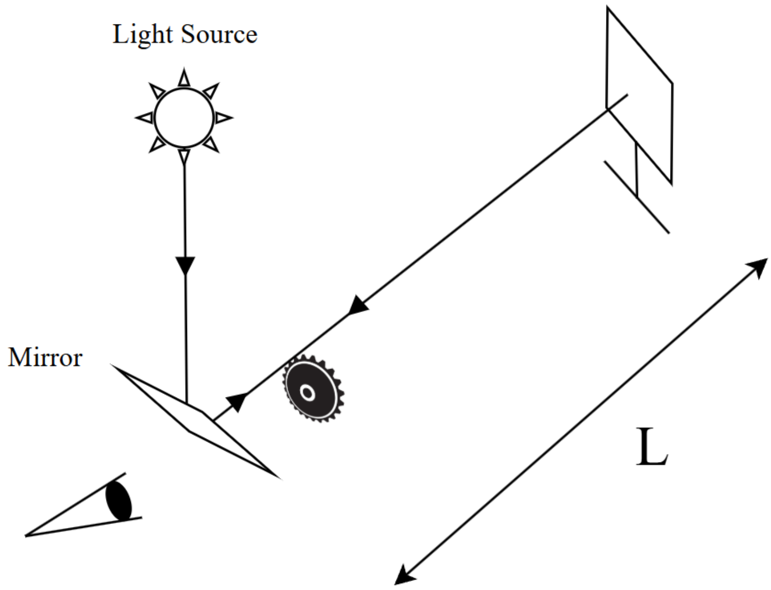
Emmy records the position of a star when its true position is supposed to be directly above her. She finds that there is a small discrepancy in the recorded angle of elevation required to see the star and the true angle of elevation, 90 degrees. This is shown below.



Find the angle difference θ to the nearest second, given that the Earth's orbital radius is 1.496×10^{11} m and the mass of the sun is 1.989×10^{30} kg.

Question 6

Fizeau's apparatus for measuring the speed of light is shown below.



Let the number of teeth of the cog be n and the frequency of its circular motion be f. Assume that the width of each tooth of the cog is to be equal to the gap between each tooth. If the light ray travels through the middle of the opening to travel to the far mirror, then as it travels back is blocked by the middle of the subsequent tooth after the opening, determine the expression for the speed of light in terms of L, n and f.

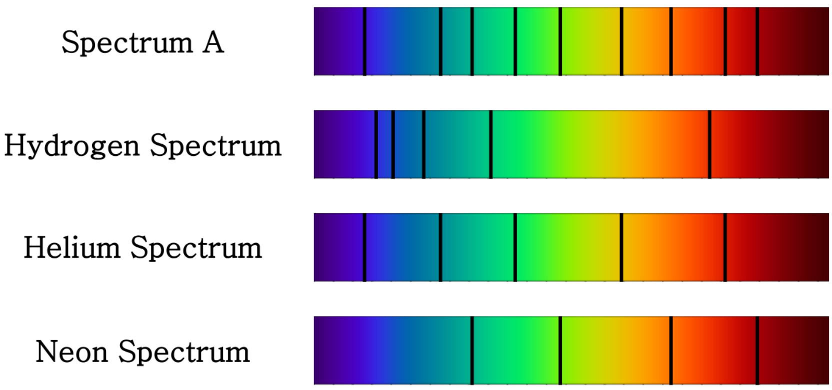
Spectra of Stars

Question 1

Among the spectra of the Sun, a gas discharge tube and an incandescent filament, which two are most similar? Explain why the two are similar and how the spectrum of the other object is different for 2 marks.

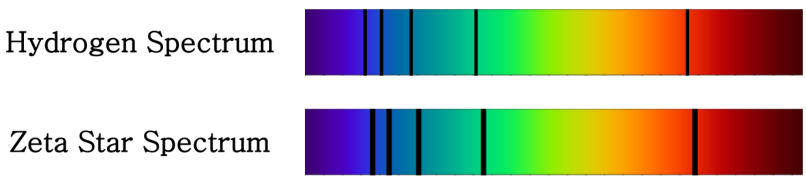
Question 2

Identify which 2 elements are in the discharge tube that produce the emission spectrum A for 2 marks.



Question 3

Below is the spectrum of hydrogen in a laboratory setting, and the stellar spectra of a fictitious star Zeta.



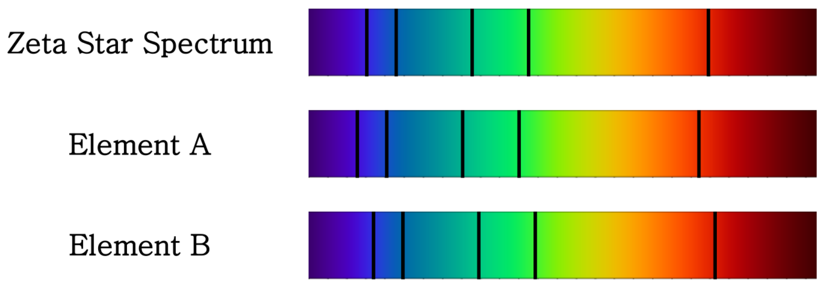
Describe the motion of Zeta and justify for 3 marks.

Question 4

Describe the importance of the electromagnetic spectrum in the identification of elements with an example for 2 marks.

Question 5

The fictitious star Zeta is composed of only one of two also fictitious elements, and is approaching Earth. The left of the spectrum signifies the smaller frequency. Identify which element Zeta is composed of by examining the spectra below.



Question 6

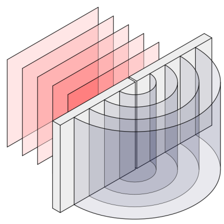
Describe how the spectrum of a star can allow for the determination of its various features for 7 marks.

Light: Wave Model

Diffraction and Interference of Light

Question 1

A wave phenomenon is shown below.



The phenomenon shown is

A. Diffraction

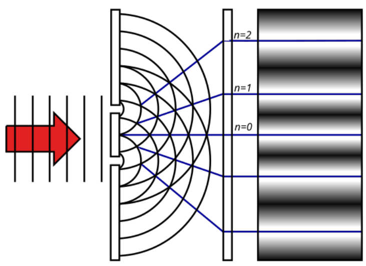
B. Refraction

C. Dispersion

D. Reflection

Question 2

An experiment that displays wave features of light is shown below.



The phenomenon that produces the pattern of bands is

A. Polarisation

B. Harmonisation

C. Resonance

D. Interference

Question 3

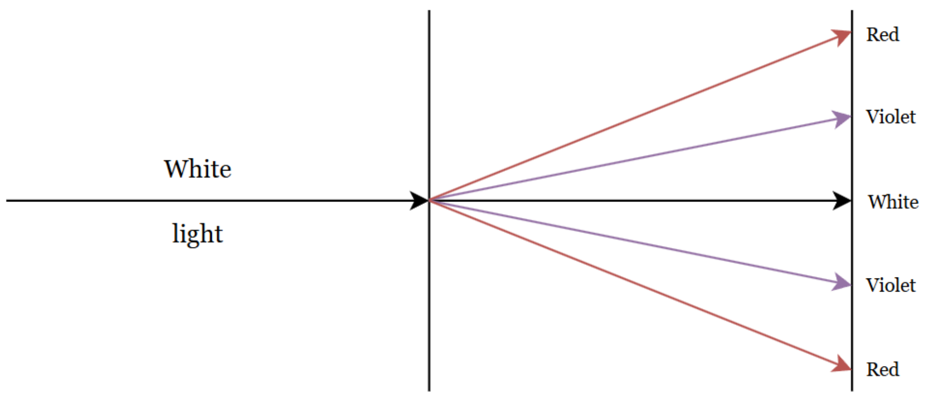
Explain why Young's double slit experiment supports Huygens' model of light for 2 marks.

Question 4

Identify the two aspects of a laser that allow it to produce an interference pattern in Young's double slit experiment for 1 mark.

Question 5

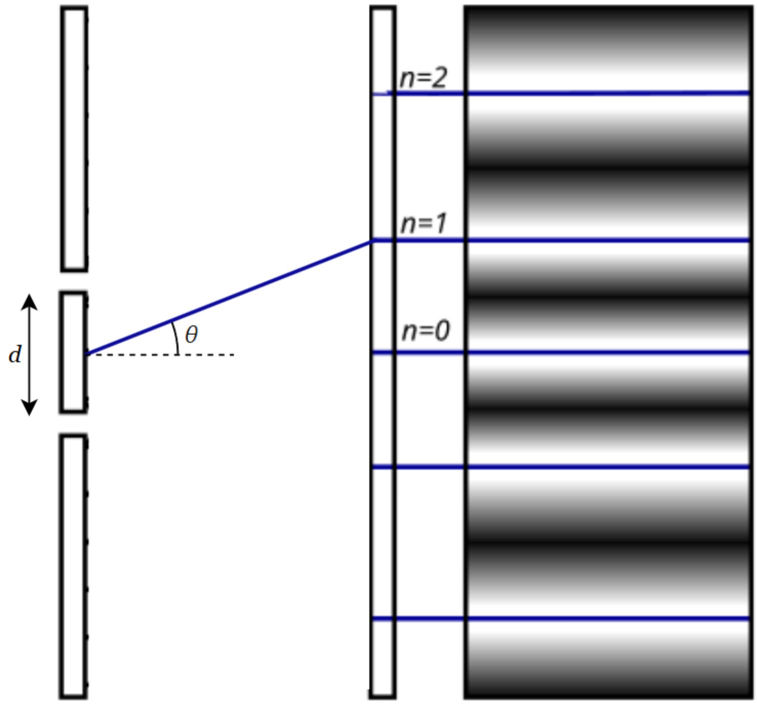
The diagram below demonstrates white light incident upon a diffraction grating.



Explain why red light is deviated more than violet light for 2 marks.

Question 6

Young's double slit experiment is shown below.



Show the derivation of the equation $n\lambda = d \sin\theta$, where λ is the wavelength of light.

Newton and Huygens' Model of Light

Question 1

Which of the following correctly states Huygens' principle?

A. Light particles called corpuscles travel in straight lines.

B. The diffraction of light can be explained by light's electromagnetic wave nature.

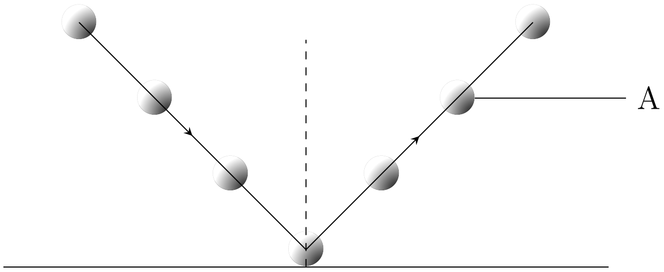
C. Each point of a wavefront may be considered as a source of secondary wavelets.

D. Every point of a wavefront is interconnected, so that the propagation of light can occur.

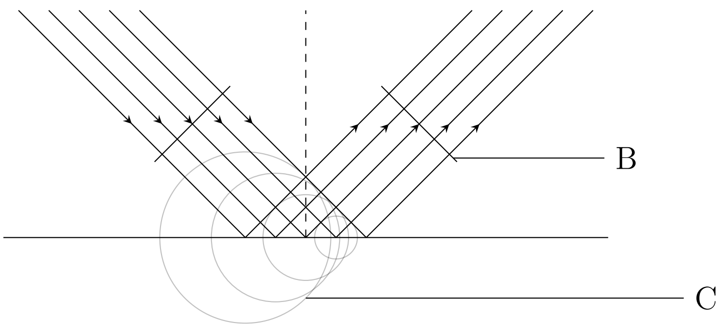
Question 2

A diagram reflection according to Newton and Huygens' model of light is shown below.

Newton's Model



Huygens' Model



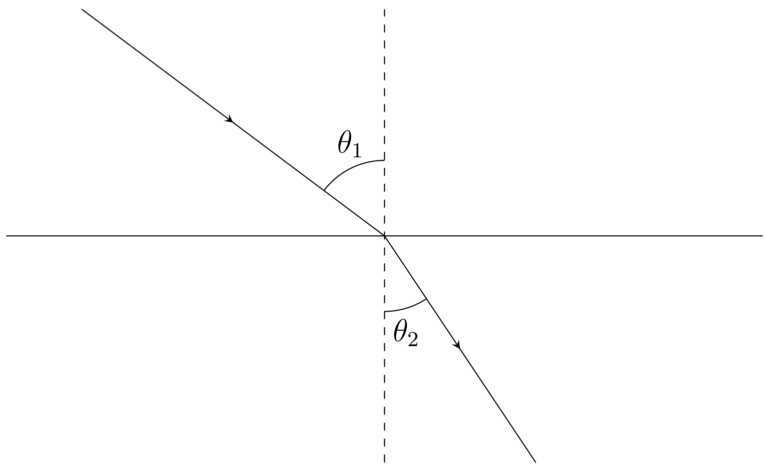
Identify the features labelled A, B and C.

Question 3

Explain how Foucault's investigation of the speed of light in various mediums supports Huygens' model of light in favour of Newton's model for 2 marks.

Question 4

Below is the refraction of light from air to water.



By Newton's model of light, derive Snell's law in the form $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

Question 5

Explain how refraction occurs from both perspectives of Newton's model and Huygens' model of light for 3 marks. Utilise a diagram in your response.

Question 6

Considering refraction according to Huygen's model of light, derive the provided form of Snell's law: $v_1 \sin\theta_1 = v_2 \sin\theta_2$ where v_1 and θ_1 is the velocity of light and angle of incidence in the first medium, and v_2 and θ_2 are the velocity of light and angle of incidence in the second medium.

Malus' Law

Question 1

Explain how the polarisation of light supports the idea that light is a transverse wave rather than a longitudinal wave or a stream of particles for 1 mark.

Question 2

Explain why a polarising filter reduces the intensity of unpolarised light by 50% regardless of its orientation for 2 marks.

Question 3

A beam of unpolarised light passes through two polarising filters. After emerging from the second filter, its intensity is measured to be 46.65% of the original. Determine the angle between the two filters.

Question 4*

A vertically polarised ray of light of intensity 100 cd is passed through a rapidly rotating polarising filter, creating a strobe light effect. If the polarising filter rotates at an angular velocity of $4\pi \,\text{rad}\,\text{s}^{-1}$, sketch a graph of the intensity of the pulsating light from $t = 0\,\text{s}$ to $t = 1\,\text{s}$, assuming the polarising filter is vertical at $t = 0\,\text{s}$.

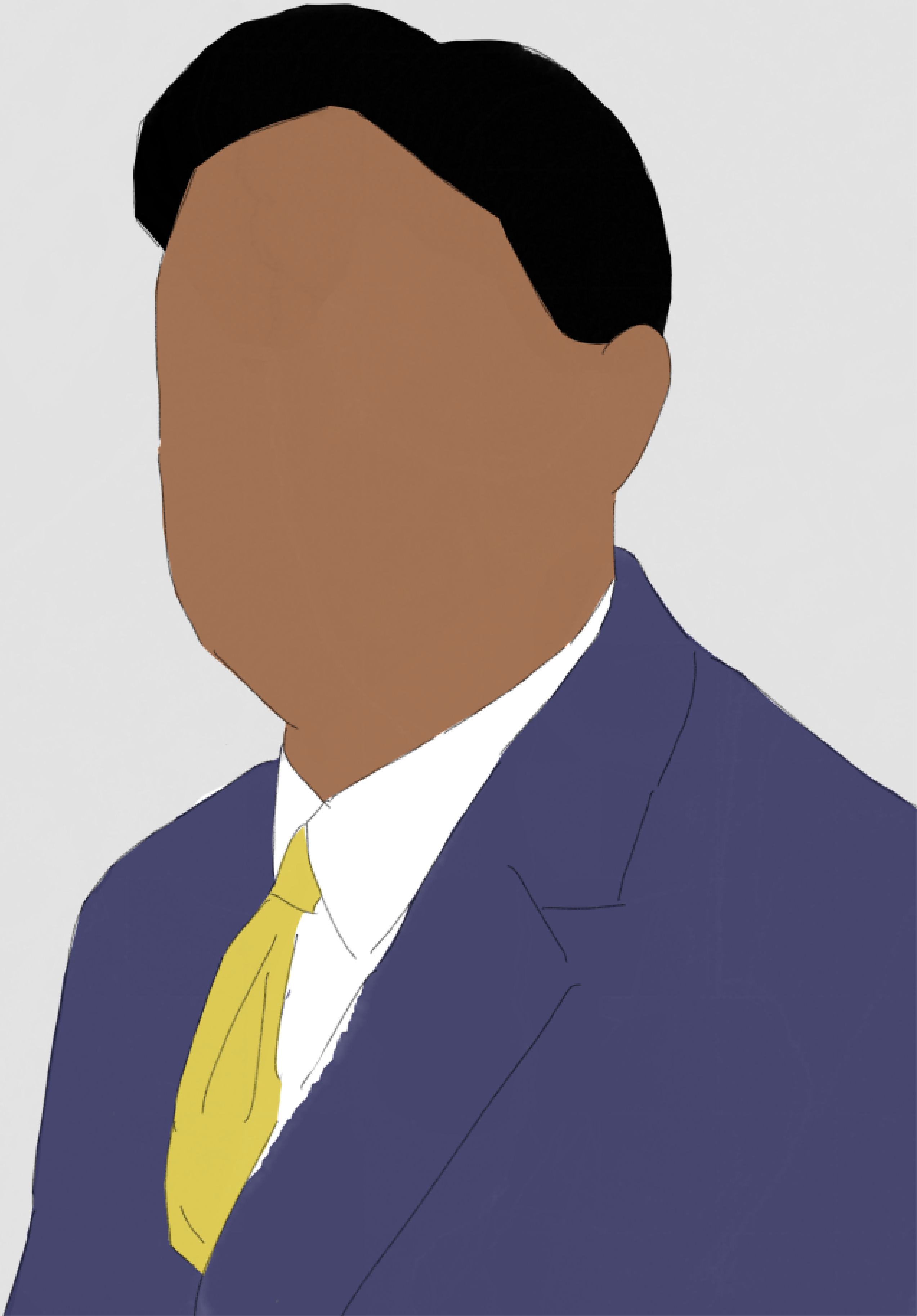
Question 5

Emmy observes the sunlight reflecting off the lake's surface on a bright afternoon. Since water can act as a dielectric surface, the reflected sunlight is partially polarised. Explain the observations Emmy would make as she rotates the polarising filter through 1 revolution for 2 marks.

Question 6

Light is composed of an electric field and a magnetic field. The electric field is also able to be polarised. Given that the intensity of light is proportional to the square of the electric field's amplitude, derive Malus' law $I = I_0 \cos^2 \theta$.

Light: Quantum Model



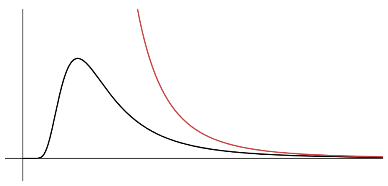
Black Body Radiation

EXTENSION DERIVATION: $E = hf$

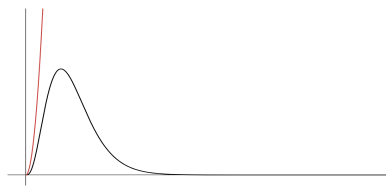
Before Planck's quantisation of light as $E = h\nu$ (ν is used instead of f for frequency for notational purposes), the Rayleigh-Jeans approximation was used to predict the electromagnetic radiation from a black body. Two forms of the Rayleigh-Jeans approximation are provided (energy density per unit wavelength and energy density per unit frequency from left to right. You may think of these quantities as intensity to suit the HSC level).

$$\frac{du}{d\lambda} = \frac{8\pi kT}{\lambda^4} \qquad\qquad \frac{du}{d\nu} = \frac{8\pi kT\nu^2}{c^3}$$

where k is the Boltzmann constant, T is the temperature and c is the speed of light. The Rayleigh-Jeans approximation is accurate at very long wavelengths or very short frequencies, and is extremely inaccurate at small wavelengths and large frequencies (remember this, as this will be important later). This can be seen by the graph below, where the black line is the experimental data gathered and the red line is the Rayleigh-Jeans' prediction.

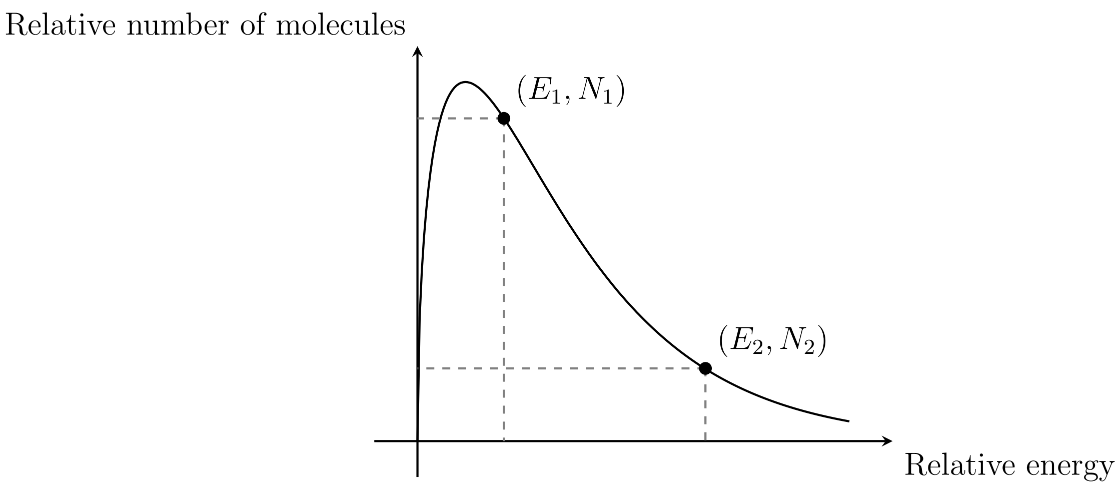


Energy density per unit wavelength ($\frac{du}{d\lambda}$)



Energy density per unit frequency ($\frac{du}{d\nu}$)

Planck then aimed to resolve this catastrophe, aptly called the UV catastrophe (due to the catastrophic discrepancies when the electromagnetic radiation passes the ultraviolet range). To begin, we should consider a beaker of water in thermal equilibrium and the Maxwell-Boltzmann curve for such a beaker (HSC Chemistry may help for familiarisation).



As we can see, we have chosen two energy levels where $E_2 > E_1$. The probability of a transition from E_1 to E_2 and a transition from E_2 to E_1 is assumed to be proportional to the energy density per unit frequency (i.e. proportional to $\frac{du}{d\nu}$). We will take the constants of proportionality to be B_{12} and B_{21} respectively. Furthermore, as $E_2 > E_1$, we consider the probability of spontaneous transition from E_2 to E_1 as A_{21}. As the beaker is in thermal equilibrium, the number of transitions from E_2 to E_1 must be equal to the transitions from E_1 to E_2.

$$\begin{aligned}
\text{Transitions}_{12} &= \text{Transitions}_{21} \\
(\text{N. molecules}_1)[\text{Prob. Transition}_{12}] &= (\text{N. molecules}_2)[(\text{Prob. Transition}_{21}) + A_{21}] \\
N_1\left[B_{12}\left(\frac{du}{d\nu}\right)\right] &= N_2\left[B_{21}\left(\frac{du}{d\nu}\right) + A_{21}\right]
\end{aligned}$$

By Boltzmann's distribution,

$$\begin{aligned}
N_1 &= N_0 e^{-\frac{E_1}{kT}} \\
N_2 &= N_0 e^{-\frac{E_2}{kT}} \\
\therefore N_0 e^{-\frac{E_1}{kT}}\left[B_{12}\left(\frac{du}{d\nu}\right)\right] &= N_0 e^{-\frac{E_2}{kT}}\left[B_{21}\left(\frac{du}{d\nu}\right) + A_{21}\right] \\
\frac{N_0 e^{-\frac{E_2}{kT}}}{N_0 e^{-\frac{E_1}{kT}}} &= \frac{B_{12}\left(\frac{du}{d\nu}\right)}{B_{21}\left(\frac{du}{d\nu}\right) + A_{21}} \\
e^{-\frac{E_2 - E_1}{kT}} &= \frac{B_{12}\left(\frac{du}{d\nu}\right)}{B_{21}\left(\frac{du}{d\nu}\right) + A_{21}}
\end{aligned}$$

Now solve for $\frac{du}{d\nu}$.

$$\begin{aligned}
B_{12}\left(\frac{du}{d\nu}\right) &= e^{-\frac{E_2 - E_1}{kT}}\left[B_{21}\left(\frac{du}{d\nu}\right) + A_{21}\right] \\
B_{12}\left(\frac{du}{d\nu}\right) - B_{21}\left(\frac{du}{d\nu}\right)e^{-\frac{E_2 - E_1}{kT}} &= A_{21}e^{-\frac{E_2 - E_1}{kT}} \\
\frac{du}{d\nu} &= \frac{A_{21}e^{-\frac{E_2 - E_1}{kT}}}{B_{12} - B_{21}e^{-\frac{E_2 - E_1}{kT}}} \\
\frac{du}{d\nu} &= \frac{A_{21}}{B_{12}e^{\frac{E_2 - E_1}{kT}} - B_{21}}
\end{aligned}$$

Now consider the case where $T \to \infty$,

$$\lim_{T \to \infty} \frac{du}{d\nu} = \frac{A_{21}}{B_{12} - B_{21}}$$

The derivation continues over the page.

Einstein maintained that as $T \to \infty$, $\frac{du}{d\nu} \to \infty$.

$$\begin{aligned} \therefore \frac{A_{21}}{B_{12} - B_{21}} &\to \infty \\ B_{12} - B_{21} &\to 0 \\ B_{12} &\to B_{21} \\ \therefore \text{Let } B_{12} &= B_{21} \\ \therefore \frac{du}{d\nu} &= \frac{A_{21}}{B_{21} e^{\frac{E_2 - E_1}{kT}} - B_{21}} \\ \frac{du}{d\nu} &= \frac{A_{21}/B_{21}}{e^{\frac{E_2 - E_1}{kT}} - 1} \end{aligned}$$

Now we introduce Planck's hypothesis.

$$\begin{aligned} E &= h\nu \\ \therefore E_2 - E_1 &= h\nu \\ \therefore \frac{du}{d\nu} &= \frac{A_{21}/B_{21}}{e^{\frac{h\nu}{kT}} - 1} \end{aligned}$$

Then we consider the case as $\nu \to 0$. Firstly, we can approximate $e^{\frac{h\nu}{kT}}$ to a more useful value using a Maclaurin series (a technique above the scope of the HSC).

$$\begin{aligned} f(x) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + ... + \frac{f^{(n)}(0)}{n!}x^n + ... \\ \text{Let } f(x) &= e^x \\ \therefore e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ... \\ \text{Let } x &= \frac{h\nu}{kT} \\ \therefore e^{\frac{h\nu}{kT}} &= 1 + \frac{h\nu}{kT} + \frac{1}{2!}\left(\frac{h\nu}{kT}\right)^2 + \frac{1}{3!}\left(\frac{h\nu}{kT}\right)^3 + ... + \frac{1}{n!}\left(\frac{h\nu}{kT}\right)^n + ... \\ \because \nu \to 0, &\frac{h\nu}{kT} \ll 1, \frac{1}{n!}\left(\frac{h\nu}{kT}\right)^n \approx 0 \text{ for } n > 1 \\ \therefore e^{\frac{h\nu}{kT}} &\approx 1 + \frac{h\nu}{kT} \end{aligned}$$

Now substitute back into our original expression for the energy density per unit wavelength,

$$\begin{aligned} \frac{du}{d\nu} &= \frac{A_{21}/B_{21}}{e^{\frac{h\nu}{kT}} - 1} \\ \frac{du}{d\nu} &= \left(\frac{A_{21}}{B_{21}}\right)\left(\frac{kT}{h\nu}\right) \end{aligned}$$

The derivation continues over the page.

Now recall that in the case where $\nu \to 0$ or where the frequency is extremely small, the Rayleigh-Jeans approximation is accurate. Hence we can let our previous expression to be equal to the Rayleigh-Jeans approximation.

$$\left(\frac{A_{21}}{B_{21}}\right)\left(\frac{kT}{h\nu}\right) = \frac{8\pi kT\nu^2}{c^3}$$
$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3}$$

Now we subsitute this known value to our previous expression.

$$\frac{du}{d\nu} = \frac{A_{21}/B_{21}}{e^{\frac{h\nu}{kT}} - 1}$$
$$\frac{du}{d\nu} = \frac{8\pi h v^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

This is Planck's radiation law in terms of frequency, but using this we can also derive his law in terms of wavelength.

$$\lambda\nu = c$$
$$\nu = \frac{c}{\lambda}$$
$$\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$\frac{du}{d\lambda} = \frac{du}{d\nu} \times \left|\frac{d\nu}{d\lambda}\right| \qquad \text{(Absolute as energy density is a scalar)}$$
$$\frac{du}{d\lambda} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1} \times \frac{c}{\lambda^2}$$
$$\frac{du}{d\lambda} = \frac{8\pi h\left(\frac{c}{\lambda}\right)^3}{c^3}\frac{1}{e^{\frac{h\left(\frac{c}{\lambda}\right)}{kT}} - 1} \times \frac{c}{\lambda^2}$$
$$\frac{du}{d\lambda} = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

This is Planck's radiation law in terms of wavelength, and correctly predicts the electromagnetic radiation from a black body. Since his hypothesis allowed him to arrive at the correct mathematical expression for black body radiation, resolving the UV catastrophe, his hypothesis is accepted as

$$E = h\nu.$$

Question 1

What was Planck's contribution to the development of quantum physics?

A. He unified the wave theory and the particle theory of light.

B. He carried out the photoelectric experiment to describe light as quantised energy packets.

C. He unified the wave model and the quantum model of light.

D. He hypothesised that the radiation absorbed and emitted by a black body radiator is quantised.

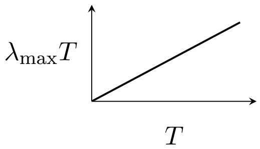
Question 2

Describing the features of a black body, evaluate if the Earth can be described as a black body for 3 marks.

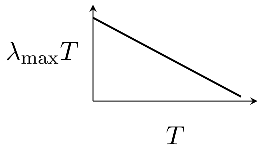
Question 3

If the product of the wavelength of maximum intensity λ_{max} and the surface temperature T for stars with various surface temperatures were plotted, which of the following graphs would best represent the relationship?

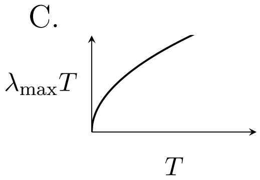
A.



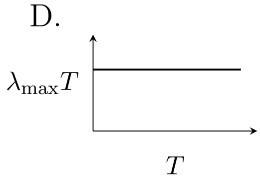
B.



C.

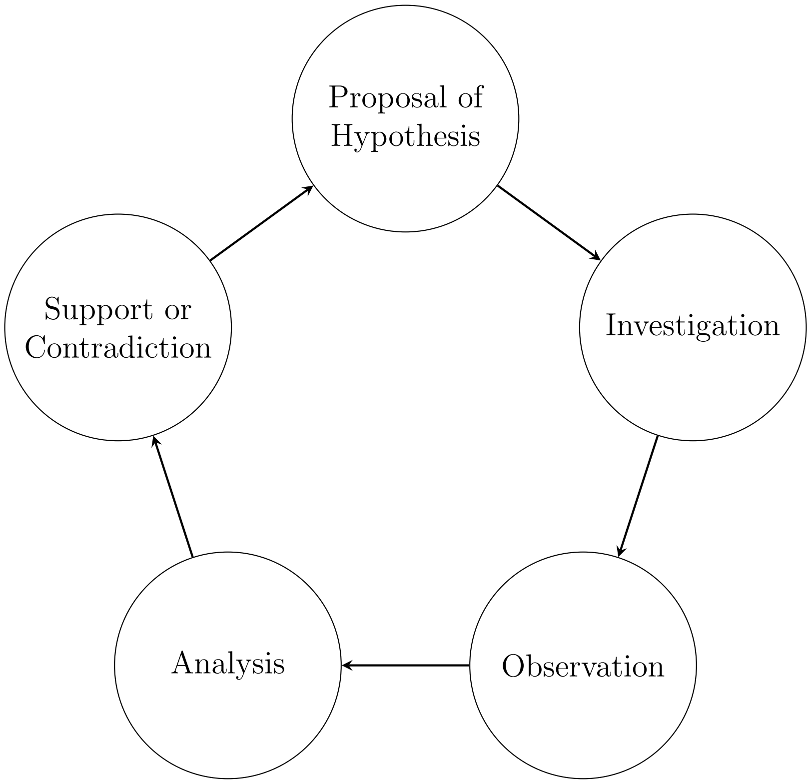


D.



Question 4

The diagram below abstracts the scientific method.



Link each component to Planck's contribution to quantum physics for 3 marks.

Question 5

Describe the UV catastrophe and explain how Planck resolved the catastrophe for 3 marks.

Question 6

> If observations are made that cannot be explained by the current model, the model must be changed or replaced by one that can. Throughout history, there have been moments when not just scientific laws were changed, but our core beliefs on how the universe works.

Support this statement using Planck's contribution to quantum physics for 3 marks.

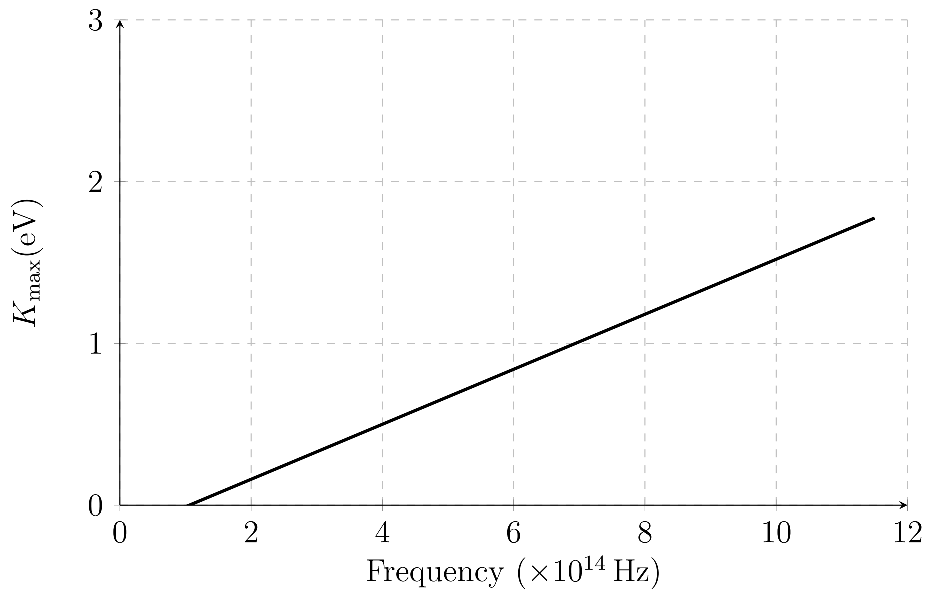
Photoelectric Effect

Question 1

Emmy conducts an experiment investigating the photoelectric effect using a particular metal. She plots the maximum kinetic energy of the emitted photoelectrons K_{max}, against the energy of the incident photons, E_{photon}. Identify how Emmy can determine the work function of the metal from this graph.

Question 2

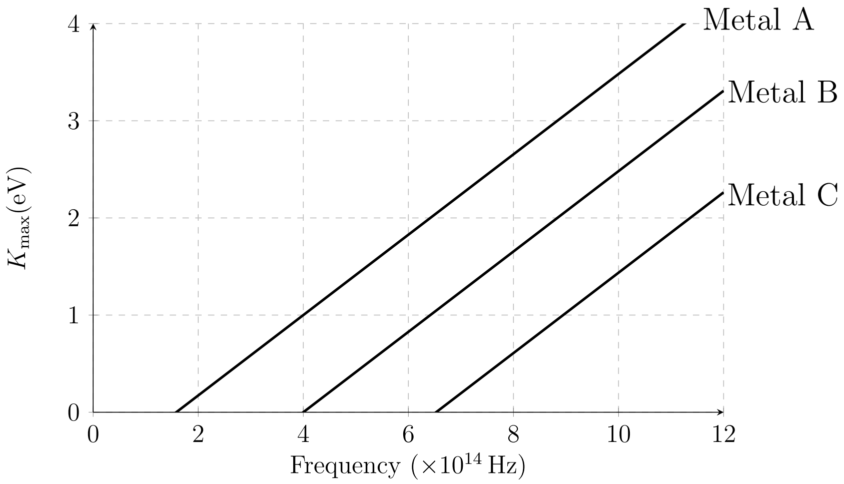
Emmy conducts an experiment investigating the photoelectric effect using a particular metal. She records the data and plots the maximum kinetic energy of the emitted photoelectrons K_{max}, against the frequency of the light source f as shown.



Identify whether Emmy's data is accurate or not, mathematically supporting your answer.

Question 3

Emmy conducts an experiment investigating the photoelectric effect using various metals. She records the data and plots the maximum kinetic energy of the emitted photoelectrons K_{max}, against the frequency of the light source f as shown.



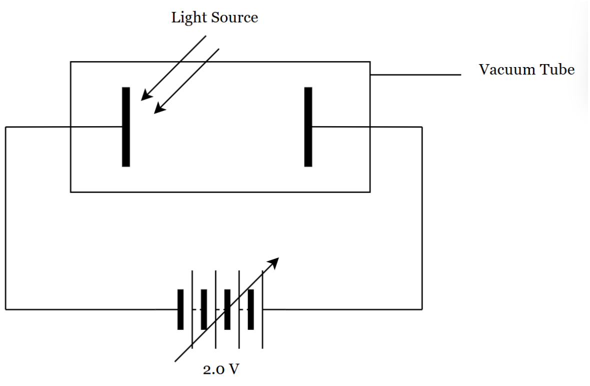
A light source of wavelength 750 nm is used for all metals in the experiment. Explain the effect of doubling the intensity of the light source on the photoelectric emission for all metals for 3 marks.

Question 4

Explain why the results of the photoelectric effect led to the development of the quantum model of light instead of the classical wave theory for 3 marks.

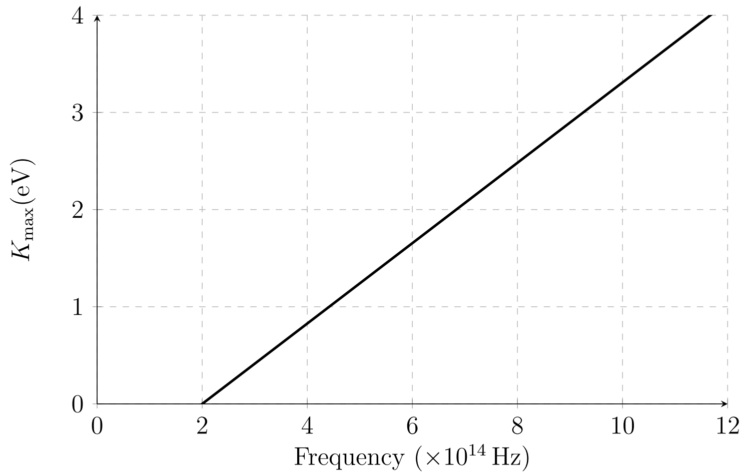
Question 5

Emmy conducts an experiment using a photocell as shown.



The question continues over the page

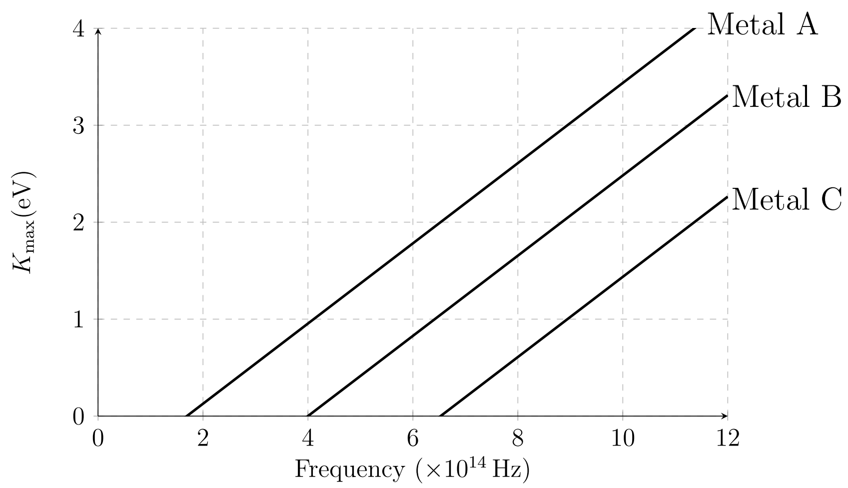
She then records the data and plots the maximum kinetic energy of the emitted photoelectrons K_{max}, against the frequency of the light source f as shown.



The experiment is repeated without a voltage. Draw the resulting graph on the same axes.

Question 6

Emmy conducts an experiment investigating the photoelectric effect using various metals. She records the data and plots the maximum kinetic energy of the emitted photoelectrons K_{max}, against the frequency of the light source f as shown.



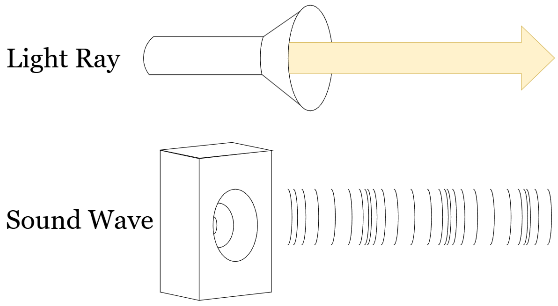
There are three settings for the light source; the 770 nm wavelength light setting, 520 nm wavelength light setting, and the 350 nm wavelength light setting. All settings of the light source have a power of 1 W. Assuming all photons result in a photoelectron if possible, determine the maximum photocurrent for each metal.

Light and Special Relativity

Einstein's Postulates

Question 1

Emmy can see a sound wave and a light ray travelling to the right in the air as shown.



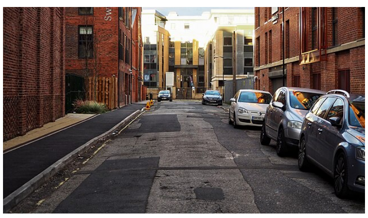
Emmy then continues to observe the propagation of these two waves, while also moving to the right at $340\,\mathrm{m\,s^{-1}}$. Explain how the light ray and sound wave will appear relative to Emmy for 2 marks.

Question 2

Amanda intends to perform the Davisson-Germer experiment in a spaceship travelling at an extremely fast uniform velocity. Emmy dissuades Amanda, stating that the speed of the spaceship may interfere with obtaining accurate results. Explain why Emmy's prediction is incorrect for 2 marks.

Question 3

Emmy and Amanda are parked on a street in Mansfield, New York.



Emmy says, "I can make all the cars move backwards." She then starts driving forward and says, "See? All the cars are moving backwards relative to me." Amanda scoffs, but then Emmy adds, "I can change the relative speed of anything, except one thing." Identify what Emmy cannot change the relative speed of, and explain why for 2 marks.

Question 4

The following is a paraphrased extract from Galileo Galilei's *Dialogue Concerning the Two Chief World Systems*, translated by Stillman Drake, University of California Press, 1953.

> Shut yourself up in a cabin with no window, and have with you some flies, bees, and other flying insects. Now, have the ship travelling at any uniform velocity you would like. You will discover there is not the least change in how the insects fly about the room, nor could you tell from any of them whether the ship was moving or standing still.

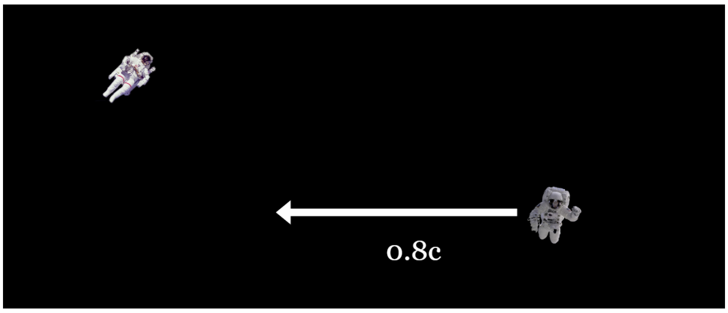
Identify which of Einstein's postulates is supported by this thought experiment, and explain how for 2 marks.

Question 5

Describe an observation that supports Einstein's postulate of the absolute velocity of light, and explain how the results support Einstein's postulate for 3 marks.

Question 6

Amanda is floating in free space. In the abyss, she sees Emmy emerging and passing by her at $0.8c$. She waves, and Emmy, being the gentlewoman she is, waves back.



As according to Special Relativity, Amanda describes that Emmy's wave appears to occur in slow motion. State how Emmy would describe Amanda's wave for 1 mark.

Time Dilation and Length Contraction

To avoid confusion, it is important to clarify the meanings of the variables t, t_0 (also commonly written as τ), l and l_0 represent. The symbol t_0 refers to the proper time; the time interval of an event measured in the frame of reference in which the event occurs. Conversely, t is the dilated time, measured from a frame not at rest with respect to the event. Similarly, l_0 denotes the proper length; the length measured in the frame at rest relative to the object. The contracted length l is measured from a frame in motion relative to the object being measured.

Question 1

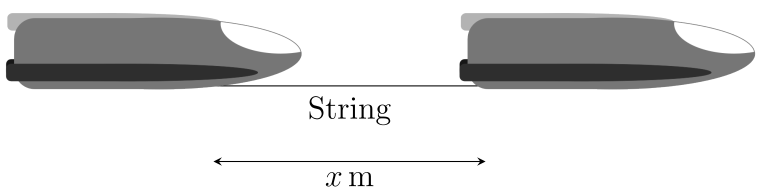
A pole vaulter holding an 11 m long pole horizontally passes through a barn that is 10 m long. How fast would the pole vaulter have to travel for the pole to fit in the barn from the barn's frame of reference? Give your answer in terms of c.

Question 2

Cosmic muons have an average lifetime of 2.2 µs, created at around 15 km above the surface. If on average they travel a distance of 5 994 m before decaying, what is their velocity in terms of c? (Use $c = 299\ 792\ 458\,\mathrm{m\,s^{-1}}$)

Question 3

A string of exactly length x m is attached between two spaceships at rest. They are remotely controlled by Emmy from a stationary lab.



The ships start to accelerate, and to prevent the string from snapping, Emmy expertly maintains exactly x m between the spaceships from his frame of reference. However, when the ships' speed approaches relativistic speeds, the string snaps despite Emmy's unfaltering precision. Explain why the string snaps for 1 mark.

Question 4

Derive the formula

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}.$$

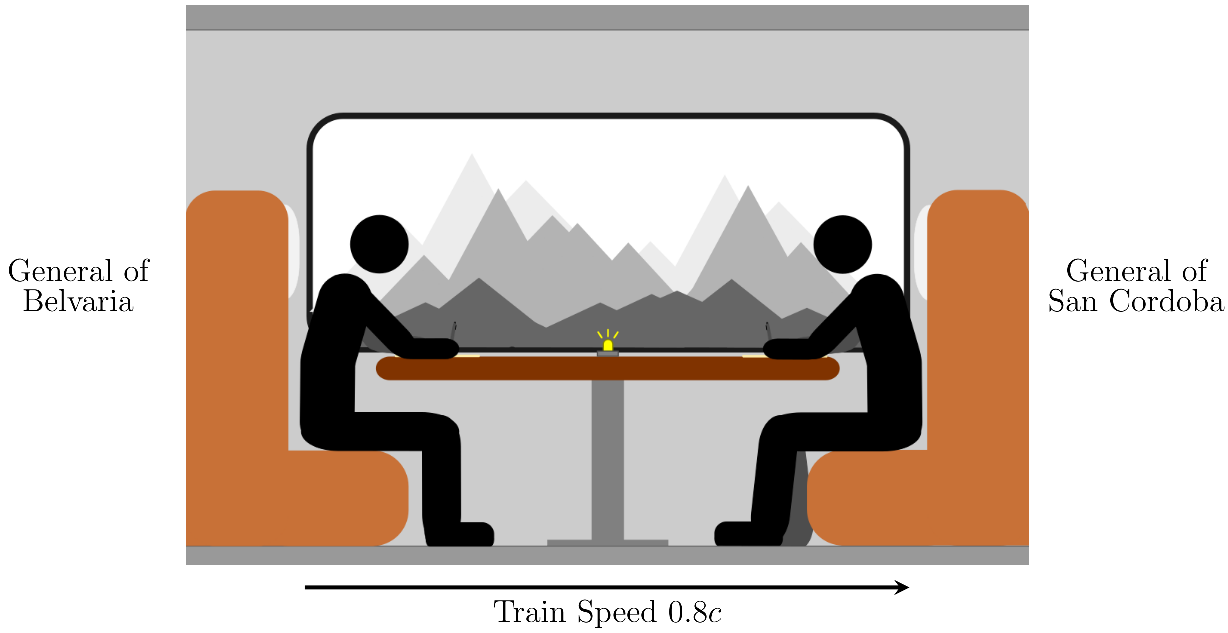
Question 5

Derive the formula

$$l = l_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}.$$

Question 6

Two generals of fictional countries, the general of Belvaria and the general of San Cordoba, are travelling in the same carriage on a train travelling at $0.8c$. They receive the information that a war is about to break out between the two countries, and they must sign a peace treaty exactly at the same time to prevent conflict. To ensure they sign the treaty simultaneously, they sit at opposite ends of a table and agree to sign the treaty as soon as they see a light placed precisely in the middle of the table turn on.



The generals see they've succeeded and celebrate their prevention of war. However, one of the generals received a phone call explaining that a spy outside the train witnessed the other general signing the peace treaty after her, and thus, war has been declared. Identify the general who received the phone call, and explain why this has happened for 2 marks.

Relativistic Momentum

To ensure clarity, the textbook will now use γ in the context

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}.$$

This is a common practice, and is referred to as the Lorentz factor. Do not use this frivolously in the HSC as it is not explicitly mentioned in the NESA-supplied syllabus.

AN IMPORTANT DISCUSSION: What is mass?

Relative mass is not an accurate concept, and the model

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

should not be used. For the more ambitious students that would like to know why, continue below.

As a quantity becomes more fundamental, it becomes continually more difficult to provide it with a non-trivial explanation. A good example of this is mass. Mass still lacks a universally accepted definition, and this textbook does not aim to propose a solution. However, this textbook will aim to remove one of its definitions from the discussion as it now does not work in modern models, and as Professor Lev Okun describes it as the "pedagogical virus of relativistic mass".

Relativistic mass is a natural result of the definition of mass as the measure of how an object resists acceleration. And it seems to make sense that the relativistic mass increases as an object travels faster, as it gains relativistic momentum, resisting acceleration. With equations such as,

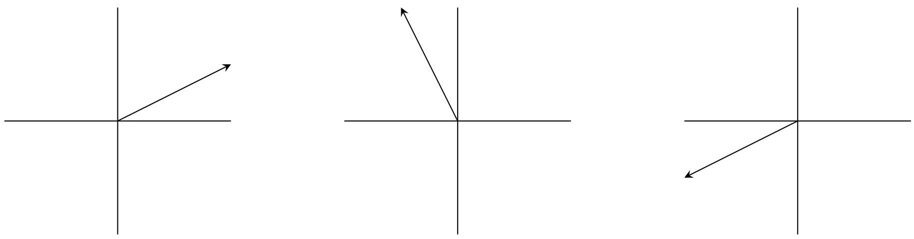
$$F_{\mathrm{c}} = \frac{\gamma m v^2}{r}, \text{ and}$$
$$p = \gamma m v$$

the definition of relativistic mass as

$$m = \gamma m_0$$

seems to integrate nicely. However, we must not rely on intuitive patterns to state physical laws, as mass is most definitely a constant quantity that does not change in the theory of special relativity.

The explanation of why mass is a constant quantity in special relativity is a long one, which is far beyond the scope of this textbook. However, to attempt to explain it in familiar (and necessarily slightly inaccurate) terms, first imagine a 2-dimensional vector.



No rotation Rotation of 90 degrees Rotation of 180 degrees

You may notice that in all of these rotations of the vector, certain characteristics are constant (invariant) and some change (variant). We will focus on the invariant characteristic: length. Now, rotating a vector around a 2-dimensional space will conserve its length, and rotating a vector in a 3-dimensional space will still conserve its length. Rotation conserves the length of a vector. Now, place this knowledge aside, as we will examine a concept called the Minkowski space, a 4-dimensional space (1 time dimension, 3 spatial) that is used to explain special relativity. Now the position vector in this Minkowski space (appropriately named the four-position) is given by,

$$\mathbf{r} = (ct, x, y, z)$$

Now the velocity is the derivative of displacement (position) with respect to time.

$$\mathbf{v} = \frac{d}{dt}(ct, x, y, z)$$
$$\mathbf{v} = (c, \dot{x}, \dot{y}, \dot{z})$$

Now multiply through by mass to find the momentum vector (four-momentum).

$$\therefore \mathbf{P} = m\mathbf{v}$$
$$\mathbf{P} = (mc, m\dot{x}, m\dot{y}, m\dot{z})$$
$$\mathbf{P} = \left(\frac{E}{c}, p_{\mathrm{x}}, p_{\mathrm{y}}, p_{\mathrm{z}}\right)$$
$$\mathbf{P} = \left(\frac{E}{c}, \mathbf{p}\right) \qquad \text{where } \mathbf{p} = (p_{\mathrm{x}}, p_{\mathrm{y}}, p_{\mathrm{z}})$$
$$\text{Now } |\mathbf{P}|^2 = \left(\frac{E}{c}\right)^2 - |\mathbf{p}|^2$$

If you're confused as to why the above statement is true, think of the 2-dimensional vector $\mathbf{u} = (3, 4)$. The length of this vector can be found by the Pythagorean theorem as $|\mathbf{u}|^2 = 3^2 + 4^2$. However, in Minkowski space, the length is found by subtracting the square of the "second side". This is because the Minkowski space is hyperbolic, not a Euclidean space where "typical" geometry applies. Unfortunately this textbook cannot provide a visualisation, as the author struggles to comprehend 4 spatial dimensions.

$$
\begin{aligned}
|\mathbf{P}|^2 &= \left(\frac{E}{c}\right)^2 - |\mathbf{p}|^2 \\
|\mathbf{P}|^2 &= \left(\frac{\gamma mc^2}{c}\right)^2 - (\gamma m\mathbf{v})^2 \\
|\mathbf{P}|^2 &= \gamma^2 m^2 c^2 - \gamma^2 m^2 v^2 \qquad \text{where } v = |\mathbf{v}| \\
|\mathbf{P}|^2 &= \gamma^2 m^2 (c^2 - v^2) \\
|\mathbf{P}|^2 &= \frac{m^2(c^2 - v^2)}{1 - \frac{v^2}{c^2}} \\
|\mathbf{P}|^2 &= (mc)^2
\end{aligned}
$$

Now this 4-momentum vector can be **rotated** (in hyperbolic space) to represent it from a different inertial frame of reference by an action called a Lorentz transformation. However, rotations in Minkowski space also conserves the lengths of vectors, much like how we examined before in Euclidian space. Thus, in all frames of references (as Lorentz transformations change the frame of reference), the length of the 4-momentum $|\mathbf{P}| = mc$ is invariant. Since c is constant, m must be also constant across all frames of references. Hence, m is a constant value, and does not change within special relativity.

For further reading into the definition of mass, this article is recommended.

Question 1

Calculate the velocity of a proton with momentum $2 \times 10^{-19}\,\mathrm{kg\,m\,s^{-1}}$ in terms of c.

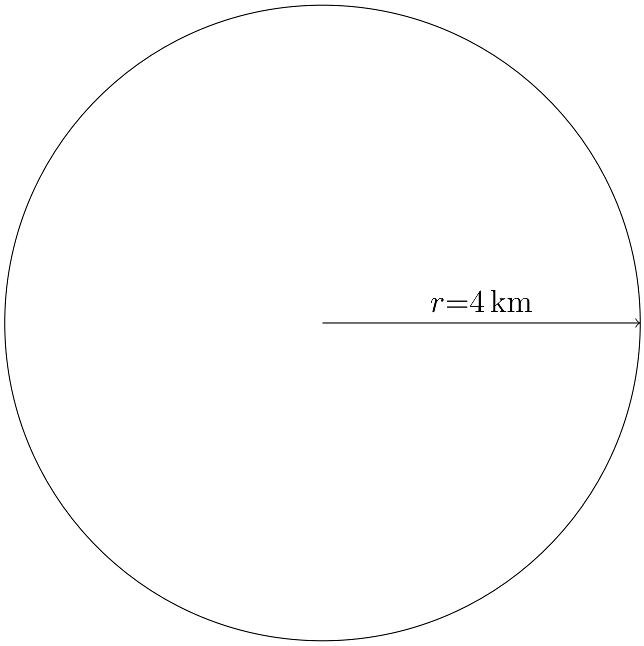
Question 2

For this question, you may assume

$$F_\mathrm{c} = \frac{\gamma m v^2}{r}$$

In **Electromagnetism, Charged Particles in Magnetic Fields, Question 2**, you have answered the following question, ignoring relativistic effects.

In the Large Hadron Collider, assume that a proton is accelerated to 99.9% of the speed of light. A top down view of the LHC is given below.

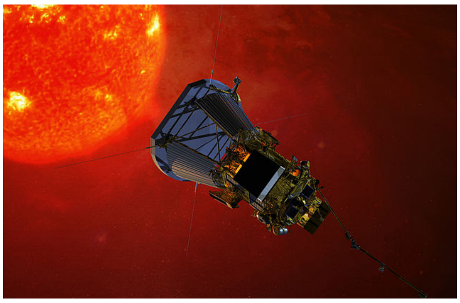


Now accounting for relativistic effects, find the magnitude of the magnetic field required to keep such a proton in circular motion.

For reference, the answer ignoring relativistic effects was $7.8 \times 10^{-4}\,\mathrm{T}$

Question 3

The fastest man-made object is NASA's Parker Solar Probe, travelling at speeds up to 700 000 km h^{-1} relative to the sun.



Using this example, explain why the classical formula of momentum, $p = mv$, so accurately models momentum in everyday situations for 2 marks.

Question 4

Outline how special relativity imposes a maximum velocity on objects with mass, contrasting this with classical expectations. Include a labelled diagram to support your explanation for 4 marks.

Question 5*

Calculate the velocity of an electron with a kinetic energy of 2×10^{-14} J in terms of c.

Question 6*

Derive the formula

$$p = \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Hint: Consider velocity as $\frac{dx}{dt_0}$

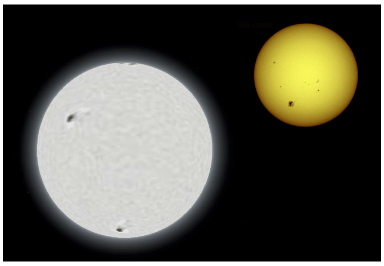
Einstein's Mass-Energy Equivalence

Question 1

If a Higgs boson's mass is denoted as 125 GeV/c^2, find its mass in kg.

Question 2

The brightest star in the night sky is Sirius.



A comparison between Sirius and the Sun in scale.

If its power output is 9.77×10^{27} W, determine the mass Sirius loses every second.

Question 3

An electron and a positron collide, annihilating each other in the following process.

$$^{0}_{+1}\mathrm{e} + ^{0}_{-1}\mathrm{e} \longrightarrow 2\,\gamma$$

If both particles were travelling at a speed of $2 \times 10^8\,\mathrm{m\,s^{-1}}$ before annihilation, what is the energy produced in this reaction?

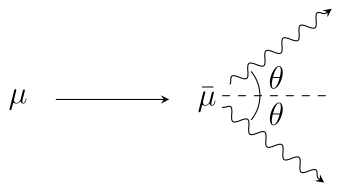
Question 4

Assuming that the sun is 1.989×10^{30} kg, and given that a $1\,\text{m}^2$ area at the equator receives a maximum solar power of 1000 W after accounting for the 25% atmospheric solar energy loss, determine the mass the Sun loses every second.

Hint: Consider Kepler's laws.

Question 5

A muon travelling to the right at $0.8c$ collides with a stationary antimuon. This produces two photons of equal energy as shown below.



Find θ.

Question 6

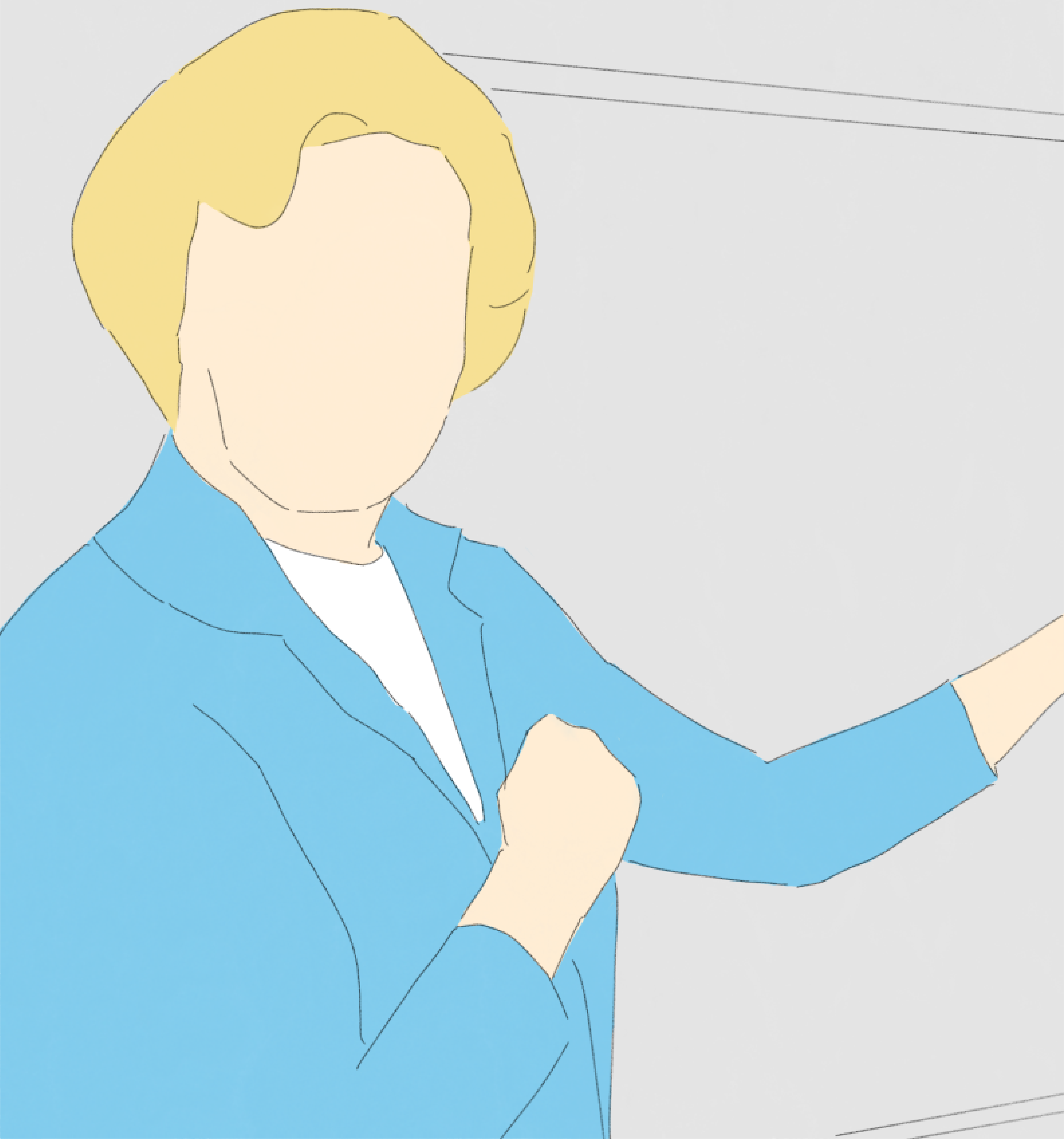
Derive the formula

$$E = mc^2.$$

Hint: The assumption $p = mc$ is false, so do not use it.

From the Universe to the Atom

Origin of the Elements



Big Bang Theory & Expansion of the Universe

An information sheet for the Big Bang is given below. It isn't recommended that this information be memorised, but it can provide some background information.

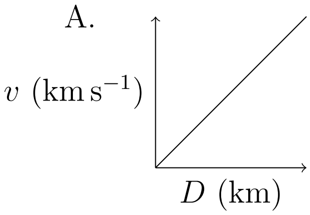
Timeline of the Big Bang

Time (s)	Temperature (K)	Epoch	Mass & Energy	Forces			
0 – 10^{-43}	10^{32}	Planck	Matter and energy constantly interchange in their highly unfamiliar state, however energy dominates.	Single Force			
10^{-43} – 10^{-36}	10^{29}	Grand Unification Theory	Same as Planck epoch.	Grand Unified Force			Gravity
10^{-36} – 10^{-12}	10^{15}	Electro weak	The temperature now allows for matter to exist as a quark-gluon plasma. Leptons constantly produce matter antimatter pairs and annihilate. Energy exists as photons.	Electro weak		Strong	Gravity
10^{-12} – 10^{-6}	10^{12}	Quark	Same as electroweak epoch.	Electro magnetic	Weak	Strong	Gravity
10^{-6} – 10^{0}	10^{11}	Hadron	Quarks now interact to form stable protons and unstable neutrons. Leptons still exist temporarily by matter antimatter pairs.	Electro magnetic	Weak	Strong	Gravity
10^{0} – 10^{1}	10^{10}	Lepton	Leptons now exist in a more stable state with slowed pair production and annihilation.	Electro magnetic	Weak	Strong	Gravity
10^{1} – 10^{13}	10^{9}	Photon	The universe is no longer opaque as the density of free electrons no longer allow for Compton scattering. Photons travel freely, becoming redshifted by the expansion of the universe and being later discovered as CMBR. At the end of the photon epoch, recombination occurs, creating protium, deuterium, helium, and Lithium.	Electro magnetic	Weak	Strong	Gravity
10^{13} – 10^{15}	10^{3}	Dark Ages	After recombination, matter now exists by mass as 75% protium, 24% helium and 1% other elements. The universe is now matter dominated.	Electro magnetic	Weak	Strong	Gravity
10^{15} – Present	10^{0}	Stars & Galaxy	Small areas of concentrated mass gravitationally attract to form galaxies. The universe is still matter dominated (ignoring dark energy).	Electro magnetic	Weak	Strong	Gravity

Question 1

Which graph shows the closest resemblance to Hubble's measurements?

A.



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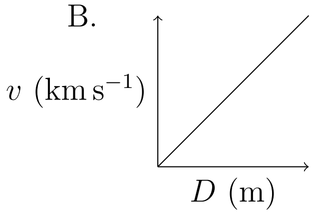

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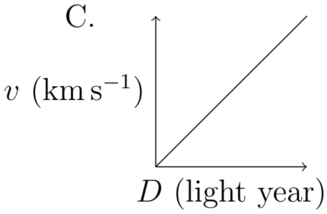
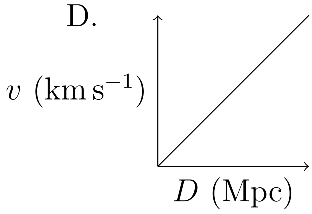
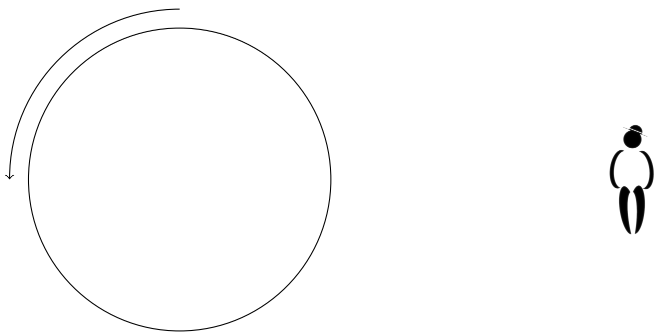


D.



Question 2

Imagine a star and an observer as so.



The star has no velocity other than rotational velocity. Identify the shifting(s) of the wavelength of the light that is emitted from the star to the observer, and why, for 2 marks.

Question 3

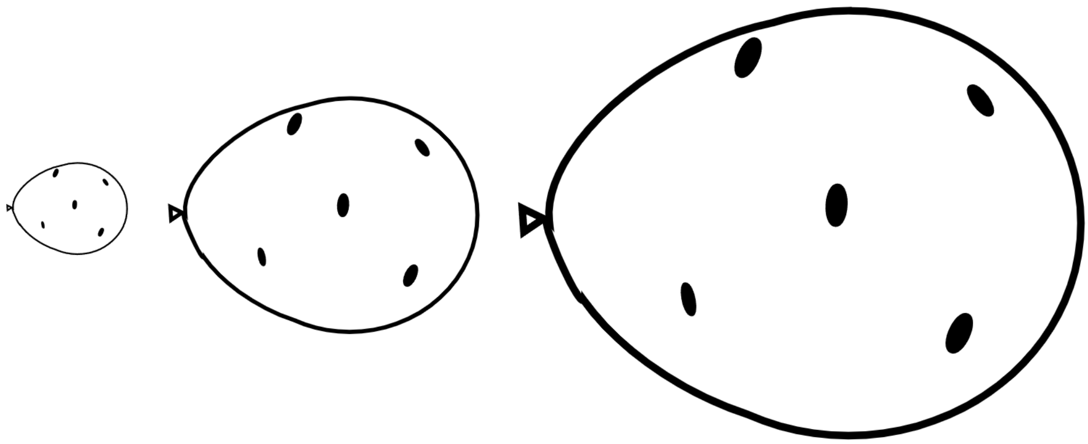
Identify what time period after the Big Bang were nucleons able to form for 1 mark.

Question 4

Describe Leavitt's discovery and assess its importance in the discovery of the expansion of the universe by Hubble for 3 marks.

Question 5

A student performs an investigation with a balloon marked with spots. The balloon is inflated as shown.



The student observes that the spots on the balloon, initially separated at a small distance from each other, are now further away.

Analyse this investigation to assess its effectiveness in modelling the expansion of the universe for 5 marks.

Question 6

Critically evaluate the significance of 3 remnants of the Big Bang that have allowed for its retrospective discovery for 9 marks.

Spectra of Stars

Question 1

Which pair of features will show the largest distortion in a star's emission spectrum?

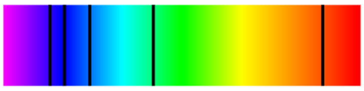
	Density	*Rotational Velocity*
A.	High	High
B.	Low	High
C.	High	Low
D.	Low	Low

Question 2

Hubble used the red shift that Hydrogen absorption spectra lines went through to find the recessional velocity of galaxies. Which simplified spectrum shows the greatest recessional velocity?

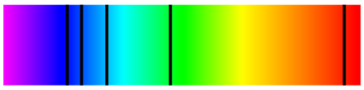
A.

B.



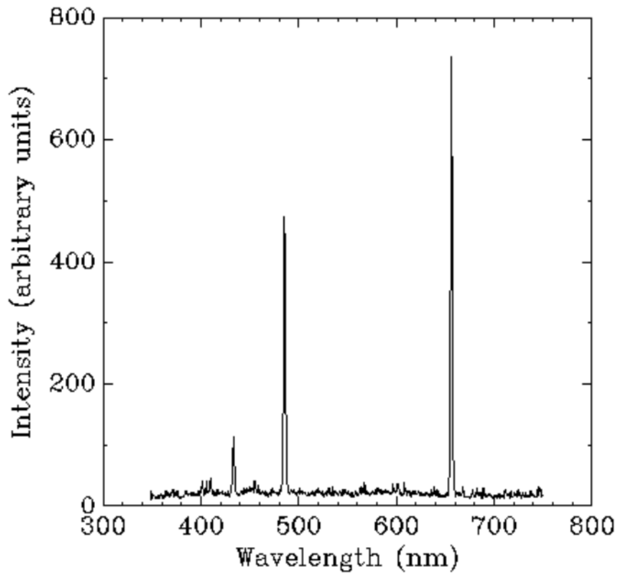
C.

D.



Question 3

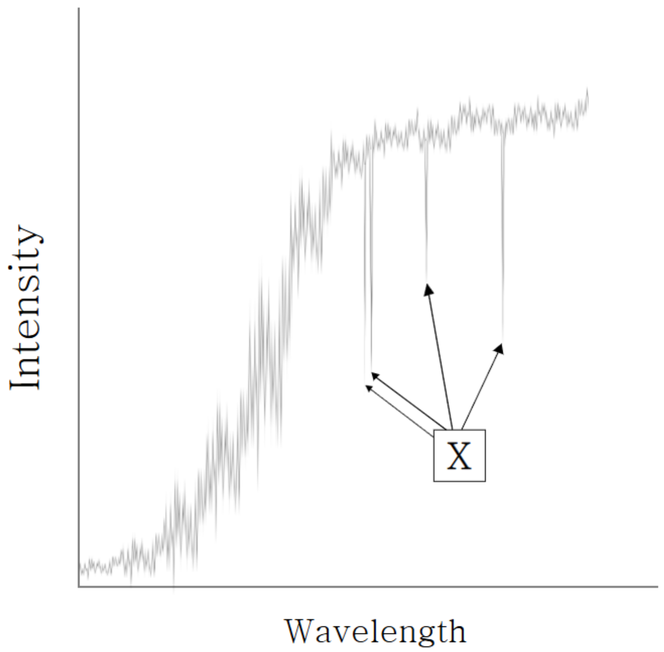
To which energy level in a hydrogen atom will the electrons have transitioned to according to the spectrum shown?



A. n = 1
B. n = 2
C. n = 3
D. n = 4

Question 4

What feature does X represent?



A. Emission Lines
B. Absorption Lines
C. Emission Spectrum
D. Continuous Spectrum

Question 5

Identify the difference between the spectra of light emitted by the hydrogen in the Sun, and the light emitted from the hydrogen in a gas discharge tube and give a reason for the difference for 2 marks.

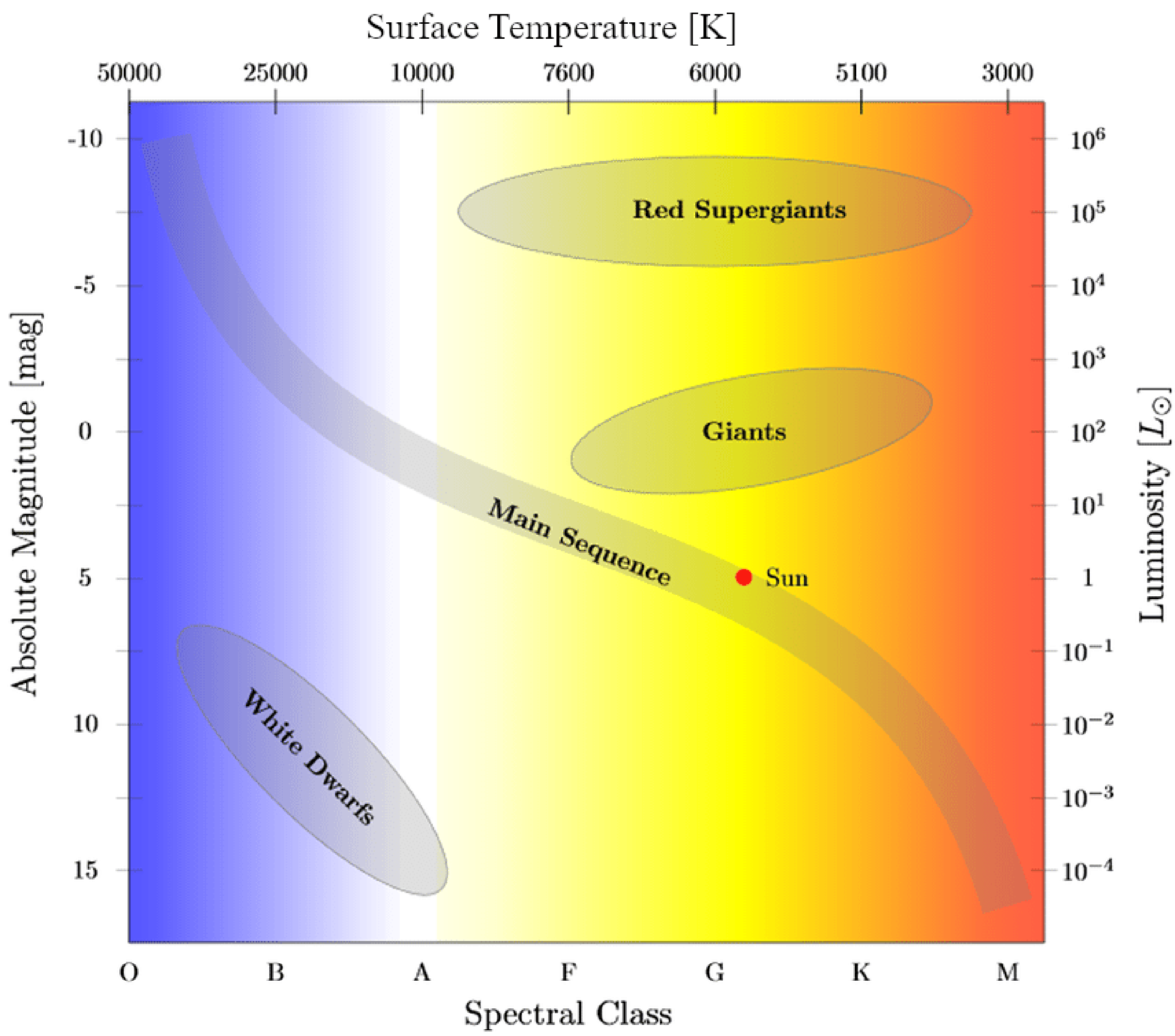
Question 6

Describe continuous, emission and absorption spectra, and evaluate each of their significance to our current scientific understanding, along with examples for 7 marks.

The Hertzsprung-Russell Diagram

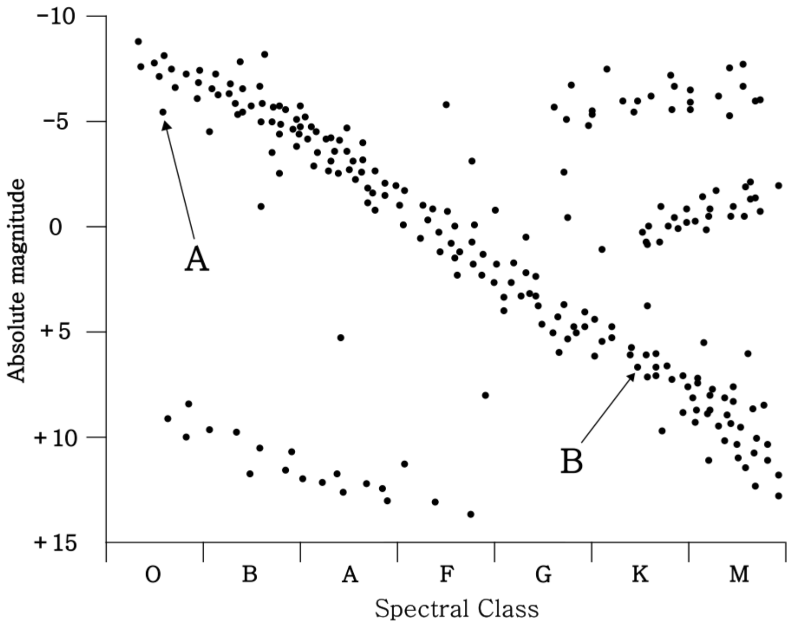
An information sheet for the Hertzsprung-Russell Diagram is given below, it is recommended that this information is memorised to a basic level.

The Hertzsprung-Russell Diagram



Question 1

Compare and contrast stellar bodies A & B for 3 marks.

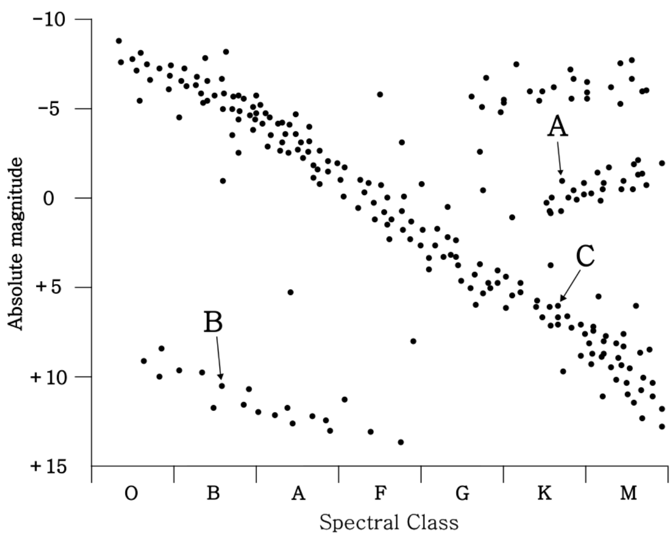


Question 2

Identify and explain how 2 components of a star determine its luminosity for 4 marks.

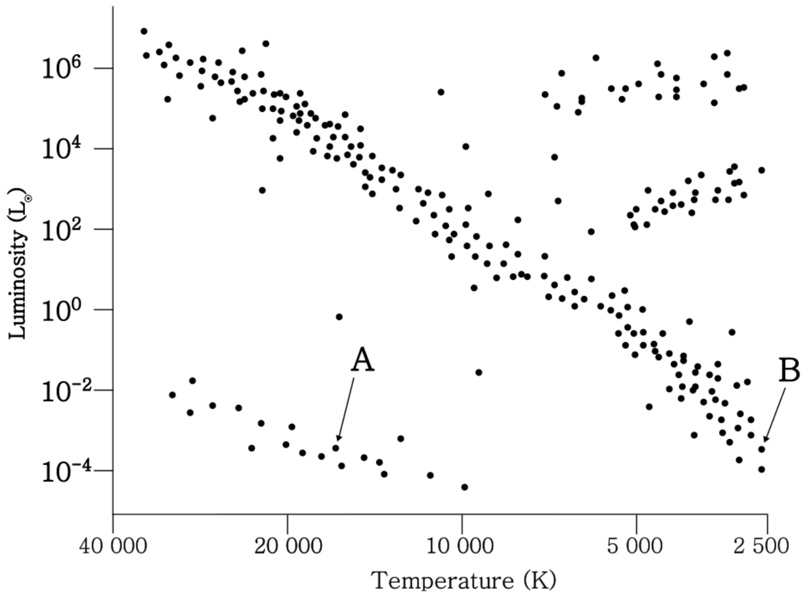
Question 3

Identify which type of stellar bodies are A, B & C, and order them from earliest to latest in the lifespan of a low mass main sequence star for 2 marks.



Question 4

Identify what type of stellar bodies A & B are, and compare their temperature & luminosity, giving reasons for the differences and/or similarities for 4 marks.

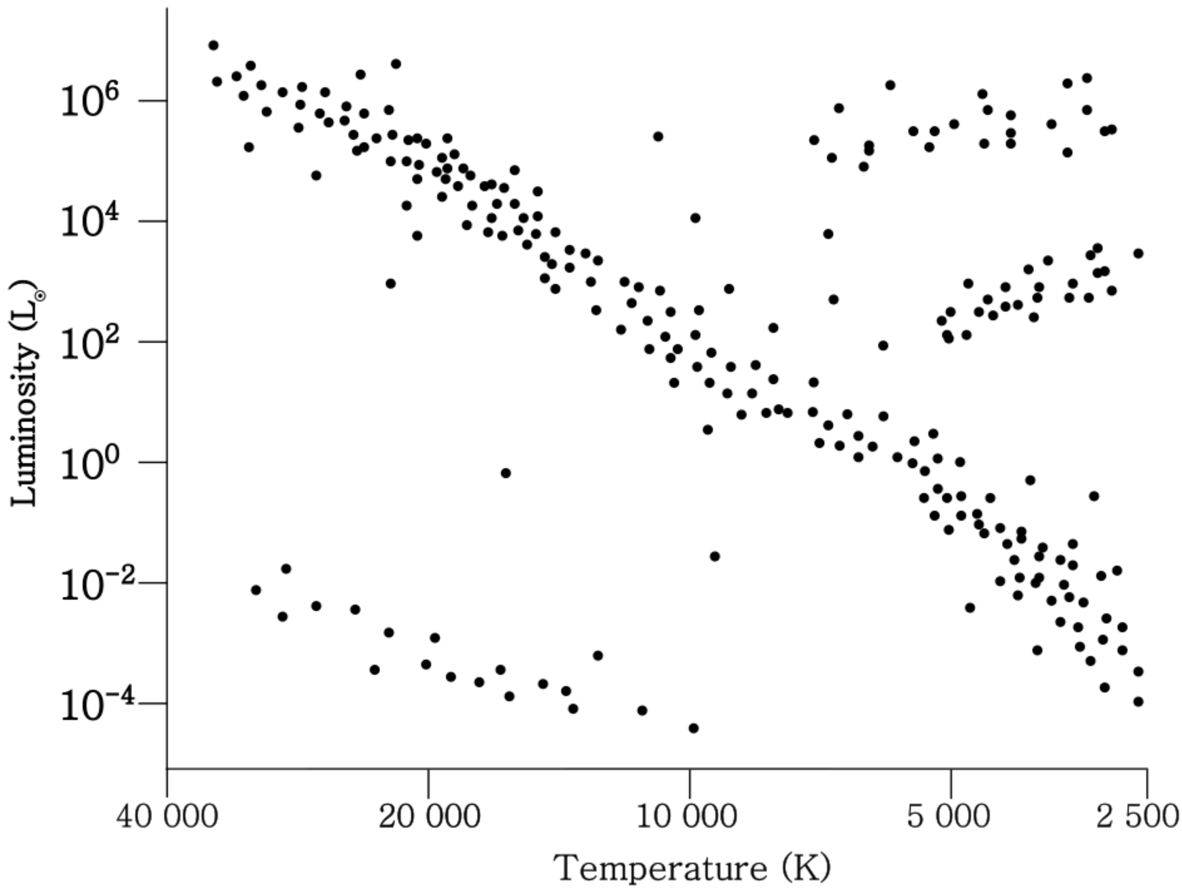


Question 5

Explain how 2 stellar bodies of spectral class M can differ highly in their luminosity for 2 marks.

Question 6

A Hertzsprung-Russell diagram is shown.



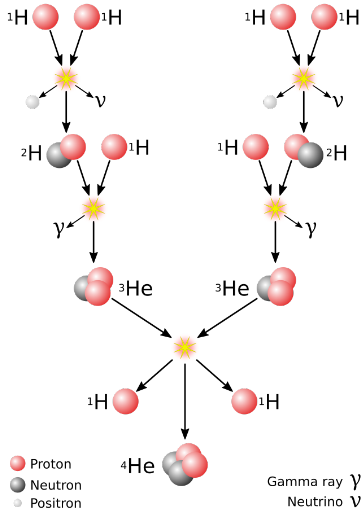
Using the diagram above to estimate the surface temperature of the sun, what will be the estimated luminosity of a main sequence star with half of the Sun's surface temperature in terms of $L_\odot$ to 2 significant figures for 3 marks?

Energy Sources of Stars

Information on the proton-proton chain and the CNO cycle is given below. It is recommended that this information is memorised.

Proton-Proton Chain

The proton-proton chain that is studied in the HSC course is shown on the right. The proton-proton chain is the main process that produces energy in main sequence stars under 1.3 solar masses.



The processes that occur are:

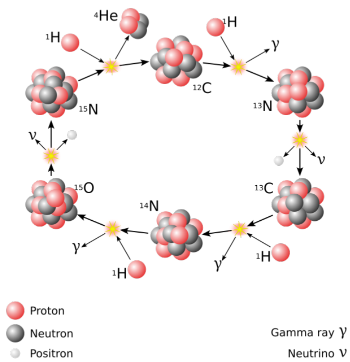
$$^{1}_{1}\text{H} + ^{1}_{1}\text{H} \longrightarrow ^{2}_{1}\text{H} + ^{\;\;0}_{+1}\text{e} + \nu$$
$$^{2}_{1}\text{H} + ^{1}_{1}\text{H} \longrightarrow ^{3}_{2}\text{He} + \gamma$$
$$^{3}_{2}\text{He} + ^{3}_{2}\text{He} \longrightarrow ^{4}_{2}\text{He} + 2\,^{1}_{1}\text{H}$$

This entire process can be simplified to the following reaction.

$$4\,^{1}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + 2\,^{\;\;0}_{+1}\text{e} + 2\nu + 2\gamma + \text{Energy}(26.73\text{ MeV})$$

CNO Cycle

The CNO cycle that is studied in the HSC course is shown on the right. Note that the initial carbon-12 nucleus only acts as a catalyst as it undergoes a sequence of changes, then is reverted to its original form. The CNO cycle is the main process that produces energy in main sequence stars over 1.3 solar masses.



The processes that occur are:

$$^{12}_{\;6}\text{C} + ^{1}_{1}\text{H} \longrightarrow ^{13}_{\;7}\text{N} + \gamma$$
$$^{13}_{\;7}\text{N} \longrightarrow ^{13}_{\;6}\text{C} + ^{\;\;0}_{+1}\text{e} + \nu$$
$$^{13}_{\;6}\text{C} + ^{1}_{1}\text{H} \longrightarrow ^{14}_{\;7}\text{N} + \gamma$$
$$^{14}_{\;7}\text{N} + ^{1}_{1}\text{H} \longrightarrow ^{15}_{\;8}\text{O} + \gamma$$
$$^{15}_{\;8}\text{O} \longrightarrow ^{15}_{\;7}\text{N} + ^{\;\;0}_{+1}\text{e} + \nu$$
$$^{15}_{\;7}\text{N} + ^{1}_{1}\text{H} \longrightarrow ^{12}_{\;6}\text{C} + ^{4}_{2}\text{He}$$

This process can be simplified to the following reaction.

$$4\,^{1}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + 2\,^{\;\;0}_{+1}\text{e} + 2\nu + 3\gamma + \text{Energy}(26.73\text{ MeV})$$

Question 1

Identify the main process that produces energy in our Sun, and account for each step, giving the energy produced for 3 marks.

Question 2

Rigel is a main sequence star with a mass of approximately $21\,M_\odot$. Identify the main process Rigel uses to produce energy, and account for each step, giving the energy produced for 3 marks.

Question 3

Compare and contrast the proton-proton chain and the CNO cycle for 4 marks.

Question 4

Emmy states that because the products of the proton-proton chain and the CNO cycle are very similar, their processes must also be very similar. Assess her statement for 3 marks.

Question 5

Explain why a higher mass main sequence star produces more of its energy via the CNO cycle in comparison to a lower mass sequence star, which produces more of its energy through the proton-proton chain for 4 marks.

Question 6

In a star of mass $2\,M_\odot$, the proton-proton chain and CNO cycle deposit energy into the star. However, the proton-proton chain deposits 26.13 MeV per reaction and the CNO cycle deposits 24.73 MeV due to the neutrino "carrying away" differing energies. If 4.35×10^{27} W is deposited in the star, and 4 fifths of this energy is due to the CNO cycle, what is the mass of hydrogen fused every second? Give your answer in $\mathrm{kg\,s^{-1}}$ to 4 significant figures.

Structure of the Atom

Discovery of the Electron

Question 1

Which of the following correctly explains how the mass of the electron was found?

A. Thomson found the charge of the electron, then Millikan found the charge-to-mass ratio, allowing for the mass to be found.

B. Millikan found the charge of the electron, then Thomson found the charge-to-mass ratio, allowing for the mass to be found.

C. Thomson found the charge-to-mass ratio, then Millikan found the charge of the electron, allowing for the mass to be found.

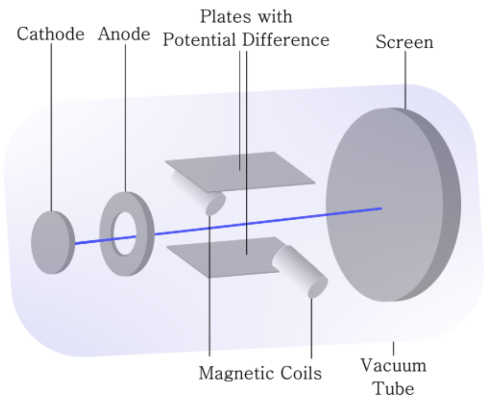
D. Millikan found the charge-to-mass ratio, then Thomson found the charge of the electron, allowing for the mass to be found.

Question 2

Using the Cathode ray experiments, logically infer that negatively charged fundamental particles exist in atoms for 3 marks.

Question 3

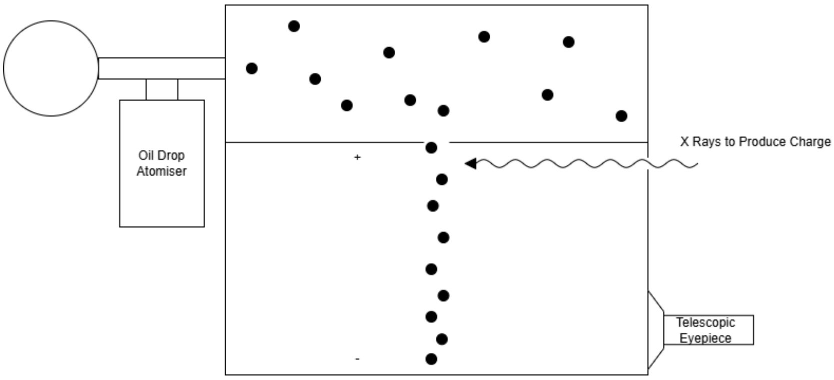
Suppose you are given Thomson's charge-to-mass ratio experiment apparatus as shown.



Explain how you would measure the charge-to-mass ratio of an electron for 4 marks.

Question 4

Suppose you are given Millikan's oil drop experiment apparatus as shown.



Explain how you would find the charge of a single electron for 4 marks.

Question 5

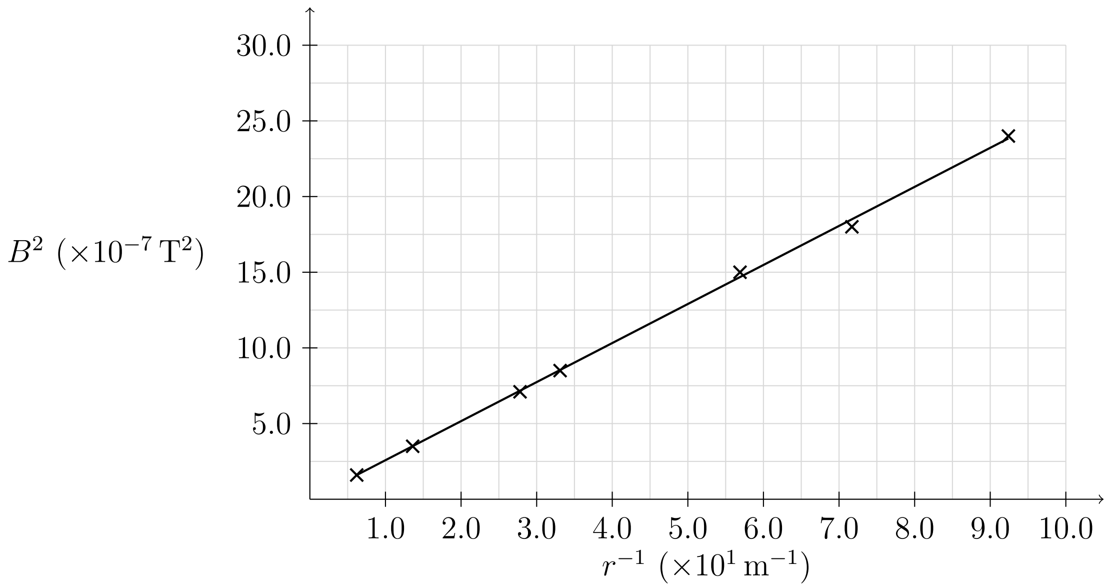
Emmy wants to know the mass of a single die at a store. Fortunately, there is a scale; however, the issue is that the dice come in identical mystery boxes with an unspecified number of specific dice in them. Emmy weighs a few of these mystery boxes and records the data in a table as so.

Mass of Mystery Box (g)
343
389
458
665

What is the likely mass of a single die?

Question 6

Thomson's charge-to-mass ratio experiment is carried out, and after the electric field and the magnetic field are adjusted so that the path of the cathode ray is not deflected, the electric field is deactivated so that the cathode ray travels in a circular path. The magnetic field strength squared is plotted against the reciprocal of the radius of curvature as shown.



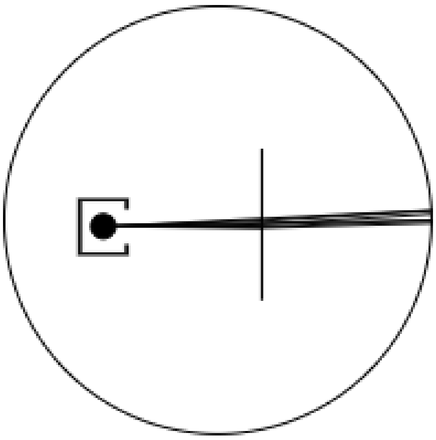
If the velocity of the fired electrons has not changed throughout the experiment, find the strength of the previously used electric field that was adjusted against the magnetic field to leave the cathode ray linear in its path.

Rutherford and Chadwick's Nuclear Discoveries

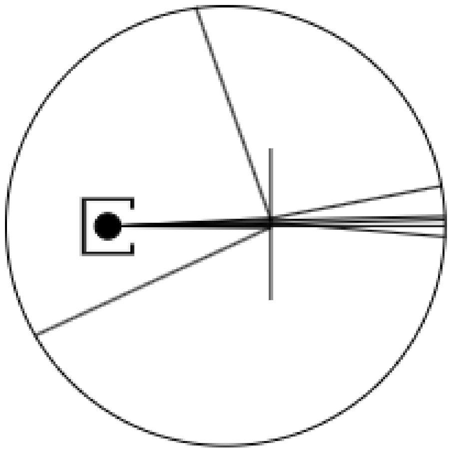
Question 1

Identify which of the given diagrams shows the expected outcome of the Geiger-Marsden experiment previous to Rutherford's model of the atom.

A.



B.



Question 2

In Chadwick's experiment, explain what led physicists to believe the neutral radiation emitted by the Beryllium source was not gamma rays for 3 marks.

Question 3

Describe the components of the Geiger-Marsden experiment, and explain why the observations of the experiment caused a change in the model of the atom for 5 marks.

Question 4

Describe the components of Chadwick's experiment, and explain how the neutron was discovered for 5 marks. Include a nuclear equation in your answer.

Question 5

Emmy creates a model of the Geiger-Marsden experiment. Emmy begins by shooting multiple rounds from her hunting rifle at a target, but with a water balloon directly between her and the bullseye. Some of her shots miss the bullseye and therefore miss the water balloon. However, her more accurate shots do hit the water balloon as they travel to the target, but their trajectories are minimally impacted. Then, Emmy shoots multiple rounds towards the target again, but this time, an extremely durable but thin pole is between her and the bullseye. Most of Emmy's shots miss the narrow pole and hit the target around the bullseye; however, her more accurate shots hit the pole. This causes the bullets to ricochet in unexpected trajectories, with some bullets even almost hitting Emmy. Evaluate the effectiveness of Emmy's model for 5 marks. (The lack of safety is not a valid disadvantage for this response.)

Question 6

> "It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong."
>
> — Richard P. Feynman

With reference to the statement, explain how models and experimental results have interacted with each other in physics for 4 marks. Use the examples of the shift to the Rutherford model of the atom and the shift to the Chadwick model of the atom in your response.

Quantum Mechanical Nature of the Atom

Bohr's Atomic Model and Hydrogen Spectra

EXTENSION DERIVATION: $\frac{1}{\lambda} = R\left(\frac{1}{n_{\mathbf{f}}^2} - \frac{1}{n_{\mathbf{i}}^2}\right)$

To derive the Rydberg formula for Hydrogen, first consider an electron orbiting a proton. This scenario can be approximated to a stationary positive charge and an orbiting negative charge due to the proton being much heavier than an electron.

Now the total energy E is the sum of the electron's kinetic energy K and electric potential energy $V = -\frac{q_e^2}{4\pi\varepsilon_0 r}$.

$$E = K + V$$

For a stable electron orbit, let $F_e = F_c$

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

$$\frac{q_e^2}{8\pi\varepsilon_0 r} = \frac{mv^2}{2}$$

$$\frac{V}{2} = -K \tag{1}$$

$$\text{Now } K = \frac{m_e v^2}{2}$$

$$K = \frac{(m_e v r)^2}{2m_e r^2}$$

$$K = \frac{L^2}{2m_e r^2} \qquad \text{(Where } L \text{ is angular momentum)} \tag{2}$$

In nth electron orbit, $C = n\lambda$

$$2\pi r = n\left(\frac{h}{m_e v}\right)$$

$$m_e v r = \frac{nh}{2\pi}$$

$$L = \frac{nh}{2\pi} \qquad \text{(Bohr's 2nd postulate)}$$

$$\text{Using (2), } K = \frac{n^2 h^2}{8\pi^2 m_e r^2}$$

$$\text{Using (1), } -\frac{q_e^2}{8\pi\varepsilon_0 r} = -\frac{n^2 h^2}{8\pi^2 m_e r^2}$$

$$\frac{q_e^2}{\varepsilon_0} = \frac{n^2 h^2}{\pi m_e r}$$

$$r = \frac{n^2 h^2 \varepsilon_0}{\pi m_e q_e^2} \tag{3}$$

$$
\begin{aligned}
E &= \frac{V}{2} \\
E &= -\frac{q_\mathrm{e}^2}{8\pi\varepsilon_0 r} \\
\text{Using (3), } E &= -\frac{q_\mathrm{e}^2}{8\pi\varepsilon_0} \times \frac{\pi m_\mathrm{e} q_\mathrm{e}^2}{n^2 h^2 \varepsilon_0} \\
\therefore E_\mathrm{n} &= -\frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 n^2 h^2}
\end{aligned}
$$

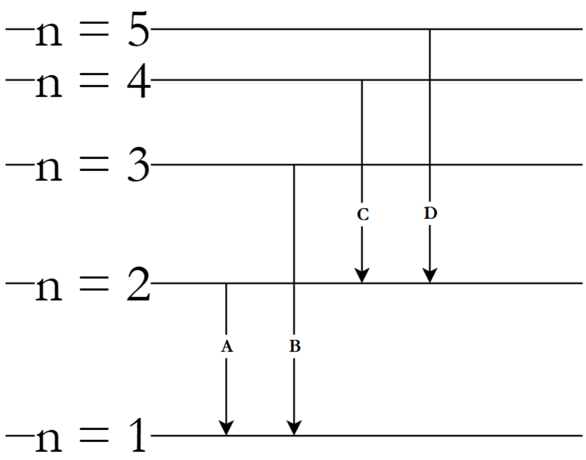
Now let E be the energy of a photon emitted by an electron transitioning from energy level n_i to energy level n_f ($n_\mathrm{i} > n_\mathrm{f}$).

$$
\begin{aligned}
E &= E_\mathrm{i} - E_\mathrm{f} \\
E &= -\frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 n_\mathrm{i}^2 h^2} + \frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 n_\mathrm{f}^2 h^2} \\
E &= \frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right) \\
\frac{hc}{\lambda} &= \frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right) \\
\frac{1}{\lambda} &= \frac{m_\mathrm{e} q_\mathrm{e}^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right) \\
\frac{1}{\lambda} &= R\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right).
\end{aligned}
$$

Note: For a more accurate calculation of Rydberg's constant, use the reduced mass of the electron $\mu = \frac{m_\mathrm{e} m_\mathrm{p}}{m_\mathrm{e} + m_\mathrm{p}} \approx 0.9995 m_\mathrm{e}$. This accounts for the approximation that the proton remains stationary while the electron orbits it.

Question 1

The following diagram shows the electron transitions in a Bohr model hydrogen atom.



Which of these transitions is most likely to produce red light?

Question 2

A physics student states, "The Bohr model of the atom accounts for the discrete spectral lines of hydrogen by the inclusion of discrete energy levels of the electrons around the nucleus. This model satisfies conservation of energy, as when the hydrogen atom absorbs a photon, an electron transitions to a lower energy level to represent the reduction in energy due to absorption, and vice versa for when the atom emits a photon." Assess the student's statement for 3 marks, and include a mathematical model to support your answer.

Question 3

In a Bohr model hydrogen atom, an electron transitions from the second energy level to the sixth energy level. Calculate the wavelength of the photon and determine if the photon is absorbed or emitted.

Question 4

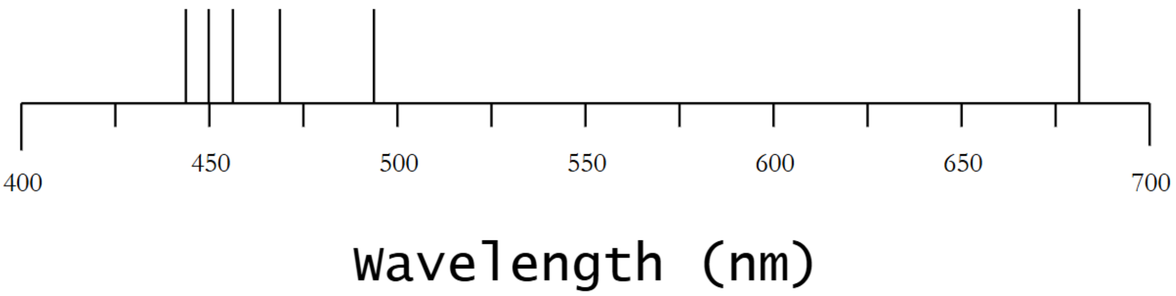
Explain how absorption and emission hydrogen spectra are produced, with a specific explanation for the discrete spectra lines for 6 marks.

Question 5

Mathematically show that in a Bohr model hydrogen atom, the frequency of a photon emitted in the Balmer series is proportional to the negative inverse square of the initial energy level.

Question 6

Previous to Bohr's model of the atom, the discrete emission spectra lines of hydrogen could not be explained.



By describing the Bohr model of the atom, explain how the observations above were addressed. Include a sample calculation and diagram in your response for 6 marks.

Louis de Broglie's Matter Waves

EXTENSION DERIVATION: $\lambda = \frac{h}{mv}$

In 1924, Louis de Broglie hypothesised that the wave–particle duality of light might be a universal property of matter. The derivation shown starts from relations with the photon, then applies it to general masses.

$$
\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
p &= \gamma m v \\
E &= \gamma m c^2 \\
\therefore E^2 - p^2c^2 &= (\gamma m c^2)^2 - (\gamma m v)^2 c^2 \\
E^2 - p^2c^2 &= \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 \\
E^2 - p^2c^2 &= \gamma^2 m^2 c^4 (1 - \frac{v^2}{c^2}) \\
E^2 - p^2c^2 &= \gamma^2 m^2 c^4 (\frac{1}{\gamma^2}) \\
E^2 - p^2c^2 &= m^2c^4 \\
E^2 &= p^2c^2 + m^2c^4 \\
\therefore E^2 &= p^2c^2 \text{ for photons as } m = 0 \\
E &= pc
\end{aligned}
$$

$$
\begin{aligned}
\text{And } E &= hf \\
\therefore hf &= pc \\
\frac{hc}{\lambda} &= pc \\
\lambda &= \frac{h}{p} \\
\lambda &= \frac{h}{mv} \text{ for masses where } p \approx mv
\end{aligned}
$$

Question 1

Emmy is walking to the kitchen at a constant pace. Without breaking her pace, she picks up a bag of chips and eats them as she paces around the kitchen. Which statement is correct?

A. Emmy's de Broglie wavelength does not change.

B. Emmy's de Broglie wavelength increases.

C. Emmy's de Broglie wavelength decreases.

D. Not enough information is given to determine if the statements above are true.

Question 2

An electron is accelerated through a potential difference of 25 V. Calculate the de Broglie wavelength of the electron.

Question 3

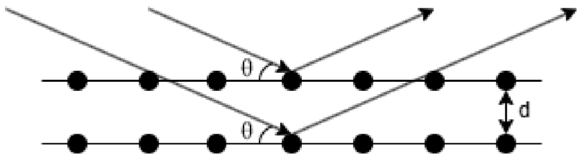
If it exists for all matter in motion, why aren't de Broglie wavelengths apparent in our lives? Mathematically support your answer for 2 marks.

Question 4

Nuclei A and B are in motion. Nucleus B is 3 times heavier than nucleus A, and nucleus A is 4 times faster than nucleus B. Find the de Broglie wavelength of nucleus B in terms of the de Broglie wavelength of nucleus A.

Question 5

Bragg's law is used to describe the relation between the wavelength of a wave, the angle it is reflected to cause constructive interference and the interplanar spacing of the atomic lattice structure of the reflective material. A diagram and a simplified formula are given below.



$\lambda = 2d\sin\theta$
$\lambda =$ Wavelength
$d =$ Interplanar Spacing
$\theta =$ Angle of Reflection

A stream of neutrons at a certain velocity is being shot at a NaCl crystal at an incident angle of 30.0°, resulting in constructive interference. If the interplanar spacing of NaCl is 0.282 nm, find the velocity of the neutrons.

Question 6

Explain how Louis de Broglie addressed fundamental limitations of the Bohr model of the atom for 4 marks.

Schrödinger's Contributions & Quantum Principles

Question 1

Which statement is most correct?

A. Schrödinger was the pioneer of quantum physics.

B. Schrödinger's work influenced Bohr's scientific discoveries.

C. Schrödinger's wave equation has deeply influenced quantum physics.

D. Schrödinger's experiment supported Louis de Broglie's matter wave proposal.

Question 2

> "The Schrödinger wave equation revolutionised quantum mechanics as it was the first mathematical model to be able to calculate definite paths of quantum objects."
>
> — Anonymous Student

Discuss the student's statement's accuracy for 2 marks.

Question 3

Which one of these statements most accurately describes Schrödinger's contribution?

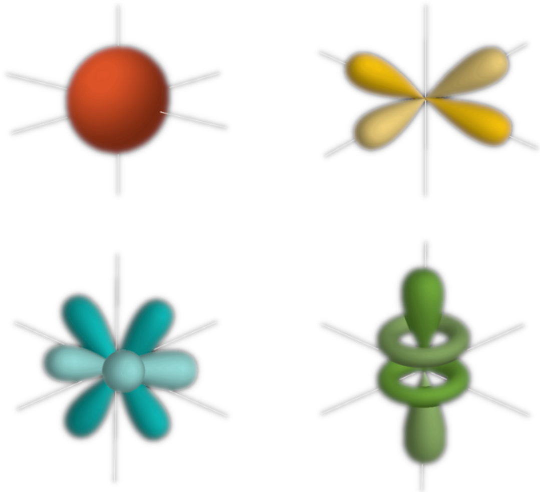
A. Schrödinger's wave equation describes the electrons of the atom as existing in "orbital clouds", which represent the probabilities of the electrons' positions.

B. Schrödinger's wave equation models how electrons exist in quantised energy levels of a nucleus.

C. Schrödinger's wave equation proposes that all matter in motion has a corresponding wavelength.

D. Schrödinger's wave equation shows how objects experience time more slowly the faster they travel.

Question 4



Link how the diagram above relates to electrons, and identify who proposed the model above for 2 marks.

Question 5

Why are the electron orbitals of the Schrödinger model of the atom sometimes called "electron clouds" rather than electron paths? Explain for 2 marks.

Question 6

Bohr, de Broglie and Schrödinger each proposed a model that influenced the scientific understanding of electrons. Explain how the nature of the electron in these three models differs from each other for 4 marks.

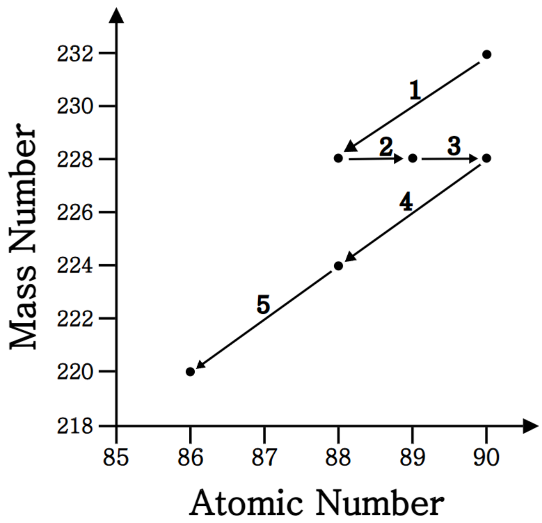
Properties of the Nucleus

Nuclear Decay

Question 1

A laboratory has three radioactive samples. Americium-241, primarily undergoing alpha radiation, Strontium-90, primarily undergoing beta radiation, and Cobalt-60, primarily undergoing gamma radiation. Propose a way to safely shield the radioactivity of each sample and briefly give supporting reasons for 3 marks.

Question 2



From the decay series shown above, write a nuclear equation for each transmutation.

Question 3

Identify and describe 3 types of nuclear decay, and explain what would occur if each of the types' emitted particles entered a cloud chamber for 4 marks.

Question 4

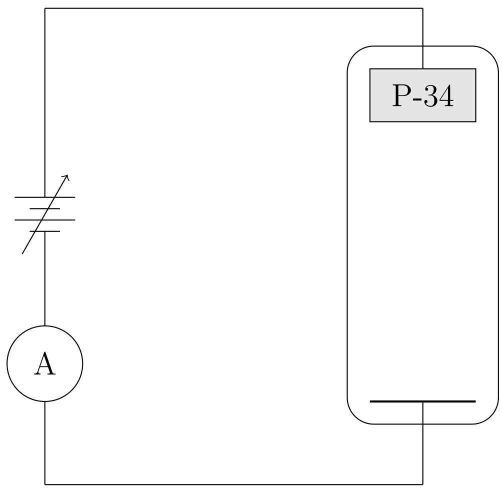
A certain mode of cancer therapy requires a highly precise mode of radiation that can cause severe damage to localised cancer cells with minimal collateral damage to surrounding cells. Propose a type of radiation that can be used for this purpose, and support your proposal with 2 reasons for 3 marks.

Question 5

Suppose an alpha particle emitted from a sample of Radium-226 has an energy of 4.78 MeV. If Emmy is trying to stop the alpha particles emitted with a positively charged electrode, what voltage does he need to set the electrode to?

Question 6

Phosphorus-34 is a radioisotope that emits beta particles, and it has a half-life of 12.4 s. The following apparatus is set, where the P-34 is kept in an evacuated chamber.



Emmy adjusts the voltage so that the beta negative particles emitted by the Phosphorus-34 do not reach the electrode. Then, Emmy makes the statement "Due to Phosphorus-34's relatively short half life, after a few minutes, fewer beta negative particles will be emitted from the Phosphorus-34, and therefore I can reduce the voltage and keep the beta negative particles from reaching the electrode". Assess Emmy's statement for 4 marks.

Modelling Radioactive Decay

Question 1

A sample of Einsteinium-255 has $2.36e21$ atoms. If the half life of this radioisotope is $1.415e5$ s, how many atoms of the radioisotope will remain after a fortnight?

Question 2

15 millicuries of Technetium-99m is administered to Emmy for imaging. After 1 hour, approximately 13.36 millicuries of Technetium-99m is detected in Emmy's body. Determine the half life of Technetium-99m to 2 significant figures.

Question 3

A radioactive sample was created in a laboratory. After a second has passed since its creation, its radiation is measured as 400 counts per second. After one further second, its radiation was measured as 250 counts per second. What will be the radiation count of the sample 3 seconds after its creation? Give your answer to 3 significant figures.

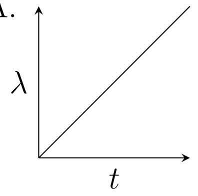
Question 4

A radioactive sample has decayed so its radioactivity is 2025^{-2025} of its original activity. If the sample's half life is 1.349×10^{-4} s, what is the current age of the sample in seconds to 2 significant figures?

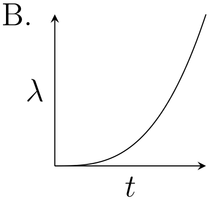
Question 5

Emmy plots the time value, t, on the horizontal axis and the decay constant, λ, on the vertical axis according to the radioactive decay model $N_{\mathrm{t}} = N_0 e^{-\lambda t}$. Which graph most closely resembles the relationship Emmy plots?

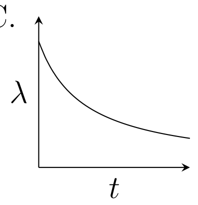
A.



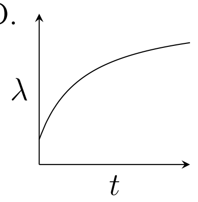
B.



C.



D.



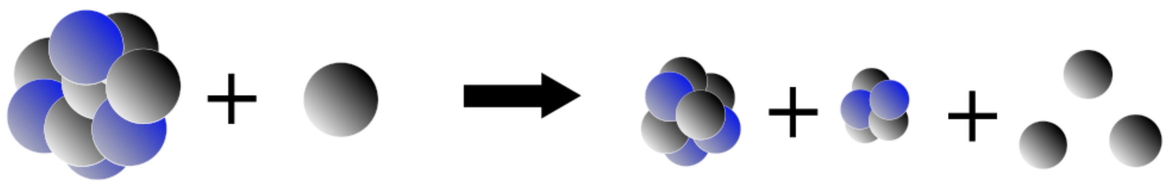
Question 6

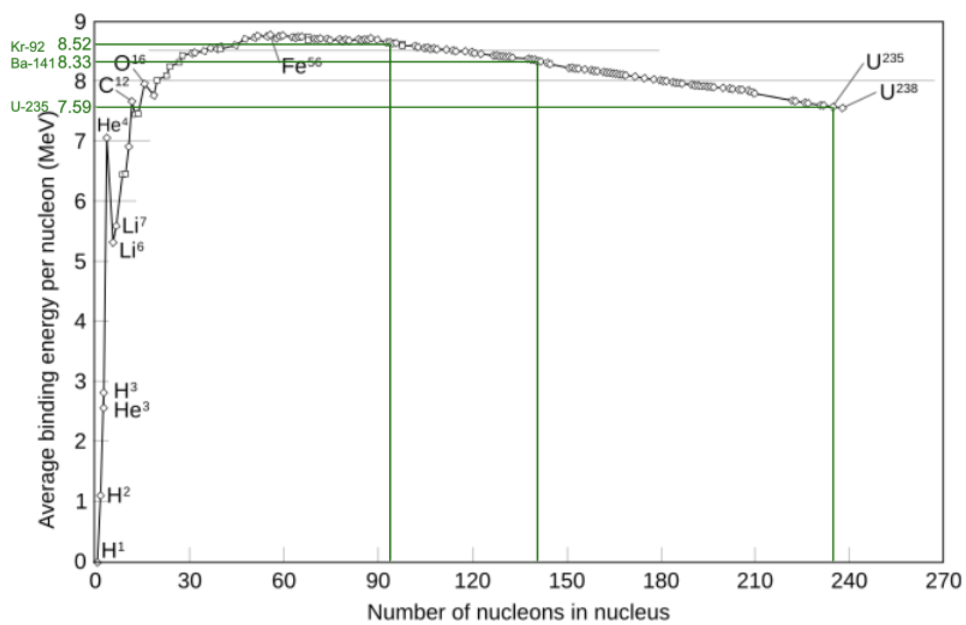
Emmy has found an easy way to determine the fraction of the mass of a remaining radioisotope of its original mass; if she divides a radioisotope's age by its half life, and she raises 2 to the power of the negative of that number, she can determine the fraction of the mass of a remaining radioisotope of its original mass. Mathematically justify her method with the model of decay $N_{\mathrm{t}} = N_0 e^{-\lambda t}$.

Nuclear Energy

An explanation of how the change in binding energy in fission and fusion reactions results in a release of energy is given below.

Suppose a Uranium-235 nucleus. When it absorbs a neutron, suppose it undergoes fission to produce a Barium-141 nucleus, a Krypton-92 nucleus and 3 neutrons as shown.





It can be seen by the binding energy curve given above that both the Barium-141 and Krypton-92 nuclei have a higher binding energy per nucleon than Uranium-235. This leads to the total binding energy of both Barium-141 and Krypton-92 being higher than the total binding energy of the Uranium-235 nucleus. This change in binding energy is the result of the conversion of the nucleons' masses to energy by Einstein's equation $E = mc^2$.

This change in total binding energy is also exactly the amount of energy released in the reaction. How this energy is produced is arguably identical in fission, as the output nuclei have a higher total binding energy, and mass must have been converted to energy, hence the release of energy. In summary, the output nuclei have both higher total binding energy and a higher average binding energy per nucleon. This implies a release of energy and an increase in stability, respectively.

Question 1

In a Uranium-235 nuclear reactor, an error occurs, causing the control rods to keep elevating without stopping, which will soon lead to supercriticality in the system. What action must be taken to prevent a catastrophic meltdown?

A. Remove the coolant

B. Remove the moderator

C. Add coolant

D. Add fuel elements

Question 2

Neptunium-237 absorbs a neutron and undergoes fission, releasing three neutrons in the process. If one of the daughter nuclides is Barium-141, what is the other nuclide?

Question 3

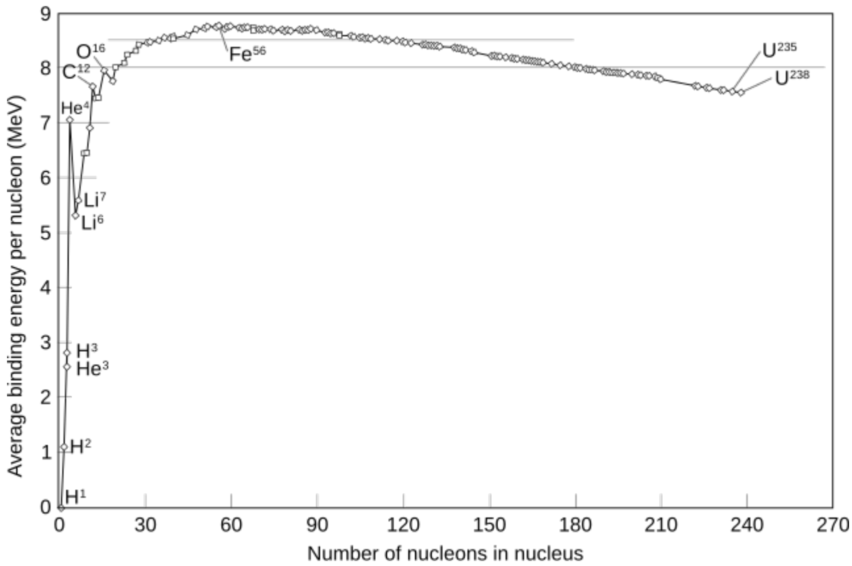
Since the 1940s, research into fusion and fission energy began. By harnessing the strong force, humankind was able to harness energy that was almost 100 000 times more efficient than fossil fuel, and weapons that were millions of times more destructive than any other explosive at the time. In modern times, nuclear fission powers around 10% of the world's electricity, yet fusion has still not achieved net power due to its extreme requirements to sustain a reaction. Still, fusion energy is being researched and developed in hopes of a new form of energy production. The equations for an example fission reaction and fusion reaction are given below.

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \longrightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n} + \text{Energy}(173\,\text{MeV})$$

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n} + \text{Energy}(17.6\,\text{MeV})$$

If a fusion reaction releases less energy than a fission reaction, explain how fusion power still gives an advantage against fission power for 2 marks. Mathematically show your reasoning.

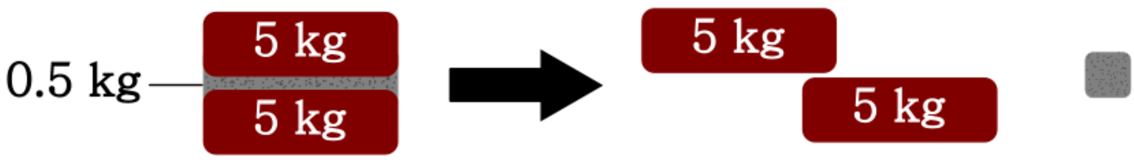
Question 4



Using the diagram above, explain why fission and fusion reactions release energy for 2 marks.

Question 5

Emmy created an analogy that represents how energy is produced in fission reactions using the idea of having two bricks held together by mortar, then separating the bricks apart and having mortar left over as shown.



Explain how Emmy's analogy represents fission reactions, and give a disadvantage of the model for 4 marks.

Question 6

Explain how a nuclear fission reactor produces energy by describing each component's structure and function for 9 marks. Include a nuclear equation and its energy production in your answer.

Mass-Energy Equivalence

Question 1

Nucleus A has a lower total binding energy than nucleus B. Which of the following statements is correct?

A. Nucleus A is less stable than nucleus B

B. It takes less energy to separate nucleus A into its nucleons than nucleus B, and nucleus A has a higher mass defect than nucleus B

C. It takes less energy to separate nucleus A into its nucleons than nucleus B, and nucleus A has a lower mass defect than nucleus B

D. Not enough information is given to deduce the statements above

Question 2

Consider the following nuclear reaction.

$$^{12}_{6}\text{C} + ^{12}_{6}\text{C} \longrightarrow ^{20}_{10}\text{Ne} + ^{4}_{2}\text{He}$$

The masses of the nuclides are given below.

Nuclide	Mass (u)
$^{12}_{6}\text{C}$	12.000
$^{20}_{10}\text{Ne}$	19.992
$^{4}_{2}\text{He}$	4.003

Calculate the energy released in this reaction.

Question 3

If the total binding energy of a $^{2}_{1}\text{H}$ nucleus is 1.714 MeV, determine if energy is absorbed or released in the given reaction, and the amount of energy the gamma particle had.

$$^{2}_{1}\text{H} + \gamma \longrightarrow ^{1}_{1}\text{H} + ^{1}_{0}\text{n}$$

Question 4

A radioisotope thermoelectric generator is powered by the alpha decay of Polonium-210. A section of an info sheet of nuclides and their masses is shown.

Nuclide	Mass (u)
$^{210}_{84}Po$	209.983
$^{208}_{83}Bi$	207.980
$^{206}_{82}Pb$	205.974
$^{204}_{81}Tl$	203.974
$^{202}_{80}Hg$	201.971
$^{4}_{2}He$	4.003
$^{1}_{1}H$	1.008

If the half life of Polonium-210 is 138.376 days, how much energy will the radioisotope thermoelectric generator produce in the first week of it initially has 2.868×10^{23} atoms of Polonium-210 in joules?

Question 5

A nucleus at rest undergoes fission, resulting in two kinetic daughter nuclei. Daughter nucleus A is n times as massive as daughter nucleus B. Mathematically prove that daughter nucleus B has n times the kinetic energy of daughter nucleus A.

Question 6

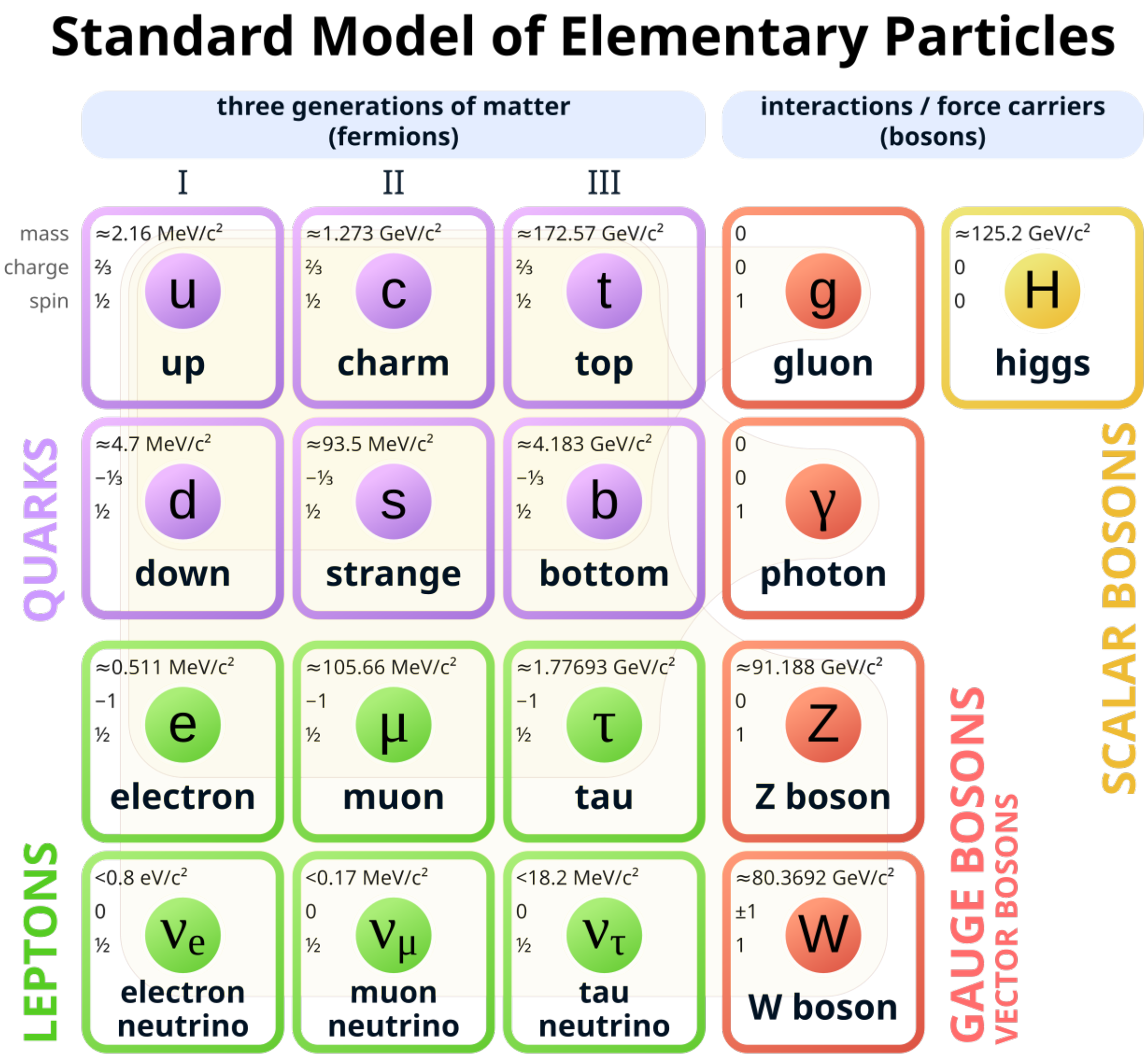
A cargo ship can use up to 200 tonnes of fuel per day. Suppose in the future, a cargo ship that utilises fusion fuel is created. This cargo ship requires 9 terajoules (10^{12} J) a day. If the ship uses fusion fuel that is composed of a 1:1 ratio of $^{2}_{1}H$ and $^{3}_{1}H$, and the ship produces energy by the equation $^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$, with the given information, find how much mass of fuel the cargo ship will fuse by the end of a 50 day voyage to 3 significant figures.

Nuclide	Mass (u)
$^{4}_{2}He$	4.0026032
$^{3}_{1}H$	3.0160493
$^{2}_{1}H$	2.0141018

Deep Inside the Atom

Standard Model of Matter

The standard model is given below as a reference.



Question 1

Identify the quark composition of protons and neutrons.

Question 2

Which of the following are elementary particles in the Standard Model of matter?

A. Hadrons

B. Baryons

C. Leptons

D. Mesons

Question 3

A particle with the quark composition of *uud* undergoes a certain process to become a particle with the quark composition of *udd*. It is known that this process releases two other particles. Determine the two other particles.

Question 4

Describe a neutral helium atom in terms of the Standard Model of matter. Mention all elementary particles for 4 marks.

Question 5

Determine the charge of the following pentaquark: $uudc\bar{c}$

Question 6

The simplified model below shows the balanced reactants and products of a neutron and antineutron annihilation reaction.

$$n + \bar{n} \rightarrow \pi^+ + \pi^0 + \pi^-$$

Pions are mesons, meaning they are composed of a quark and an anti-quark. π^+ are positive pions with a charge of +1, π^- are negative pions with a charge of -1 and π^0 are neutral pions with a charge of 0. Determine the quark compositions of each pion.

Particle Accelerators

It is often asked to describe how particle accelerators provided evidence for the Standard Model of matter. The following 2 examples may be useful in answering future questions.

ELECTRON-PROTON SCATTERING EXPERIMENTS

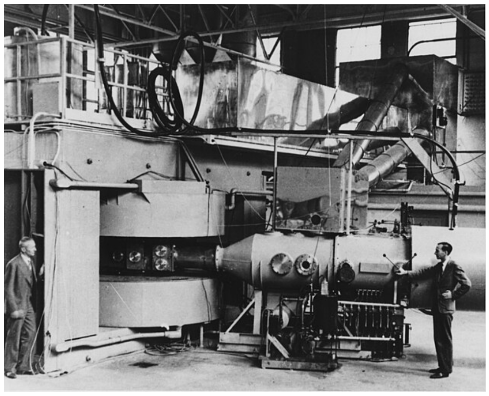
Date	1968
Location	SLAC (Stanford Linear Accelerator Centre)
Description	An accelerated beam of electrons was directed at protons. If the proton were an elementary particle, meaning it would not have an internal structure, the scattering of the electrons upon colliding with the proton would result in predictable scattering patterns. However, the results showed that some electrons were deflecting at significant angles, and some electrons unpredictably experienced minimal deflection. This can be compared with the Geiger-Marsden gold foil experiment; some alpha particles experienced minimal deflection whilst other alpha particles experienced deflection at significant angles, implying that the atom had an internal structure of empty space and dense particles (the nucleus). Similarly, as some electrons were minimally deflected and some electrons were deflected at significant angles, this implied that protons had an internal structure as well, with further analysis showing protons consisted of three smaller particles now called quarks. However, it should be noted that it should not be thought that protons are three quarks in an empty region of space, but more as three quarks that exist in a much more complex space.

DETECTION OF THE HIGGS BOSON

Date	2012
Location	LHC (Large Hadron Collider)
Description	The Higgs particle has been a proposed boson since 1964 by Peter Higgs. However, due to the Higgs particle being 125 GeV (the second heaviest particle of the Standard Model), the particle can only be created in extreme environments such as the collisions in the LHC. Even then, the particle has a half-life of approximately 10^{-22} seconds, meaning the particle can only be observed by the particles it decays into - say, a pair of photons. However, pairs of photons are produced extremely commonly in LHC collisions, but all pairs of photons that have been produced have a quantity called invariant mass. This invariant mass CAN equal the mass of the Higgs boson, or may equal an essentially random number. However, after an astronomical amount of experimental data was gathered, the slight surplus of the measurement of the Higgs boson's mass confirmed its existence with a confidence of 5 sigma. For more information about the statistical confidence of the discovery, search up "empirical rule".

Question 1

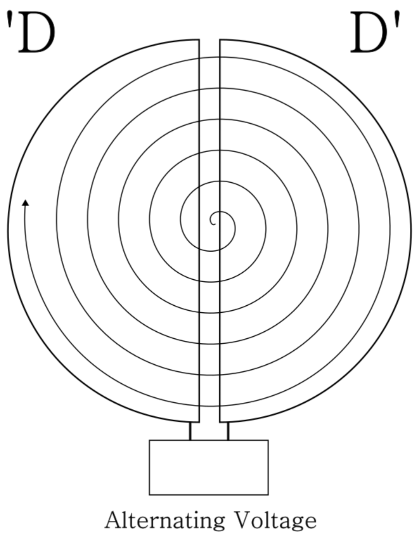
NSLS, Department of Energy



Cyclotron, University of California

The image on the left shows the NSLS, a synchrotron. The image on the right shows the 60-inch Cyclotron from the University of California. Provide a similarity and difference in their operations from a generalised perspective of synchrotrons and cyclotrons for 2 marks.

Question 2

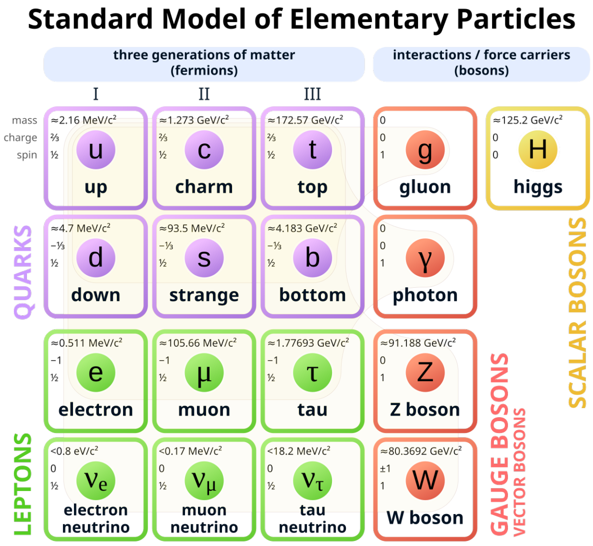


A cyclotron is shown above. Find an expression for the frequency of the AC voltage in terms of the charge of the particle, mass of the particle and the strength of the magnetic field.

Question 3

Describe how a specific experiment with particle accelerators has contributed to our understanding of ONE fundamental particle for 3 marks.

Question 4



Why are some scientific discoveries, such as the Higgs boson, essentially impossible without particle accelerators? Using another scientific discovery that would be impossible without particle accelerators, explain for 3 marks.

Question 5

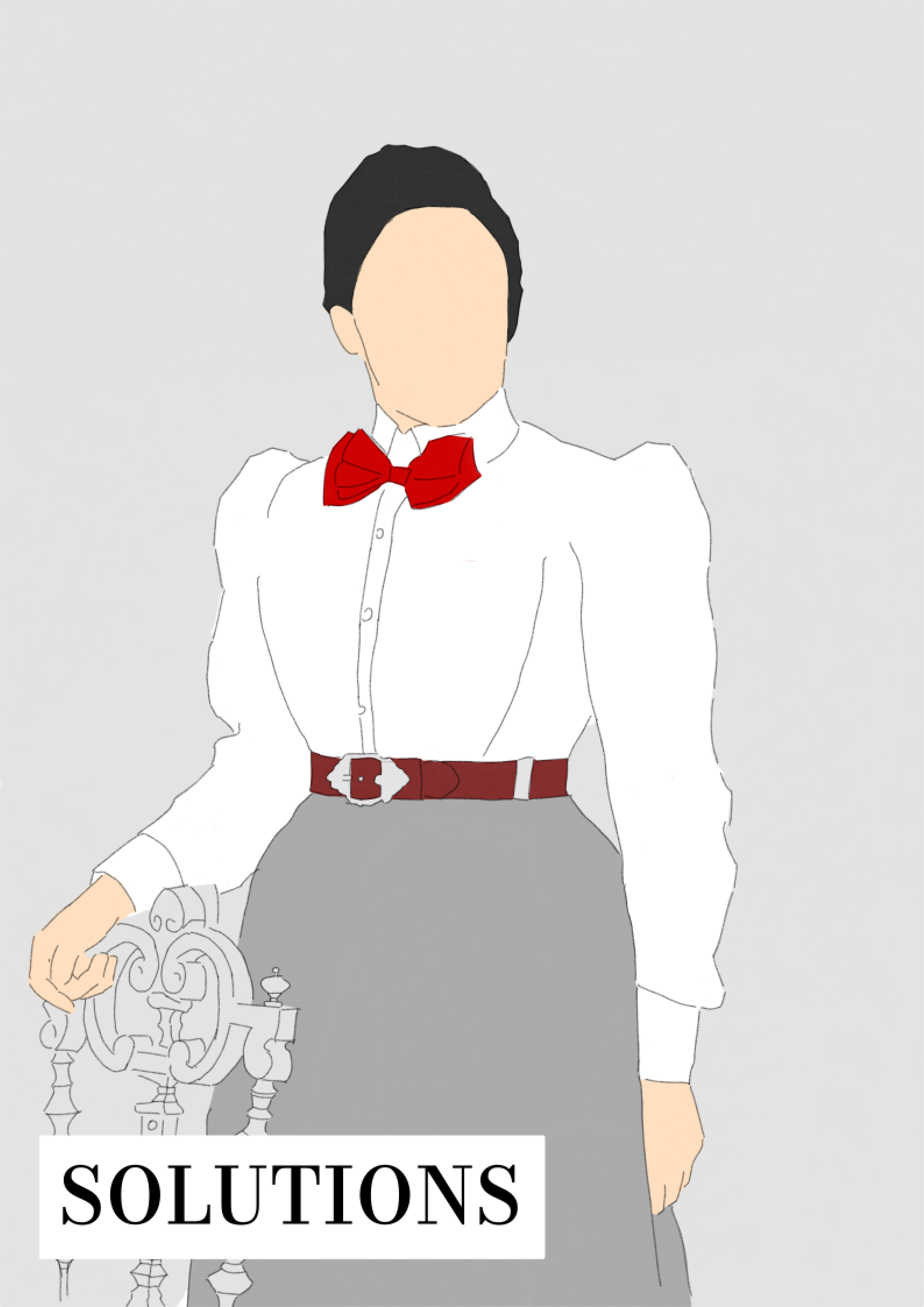
> Particle accelerators have been essential for our advancement in physics, and there are absolutely no disadvantages in their usage.
>
> — Physics Student

Assess the student's statement for 4 marks.

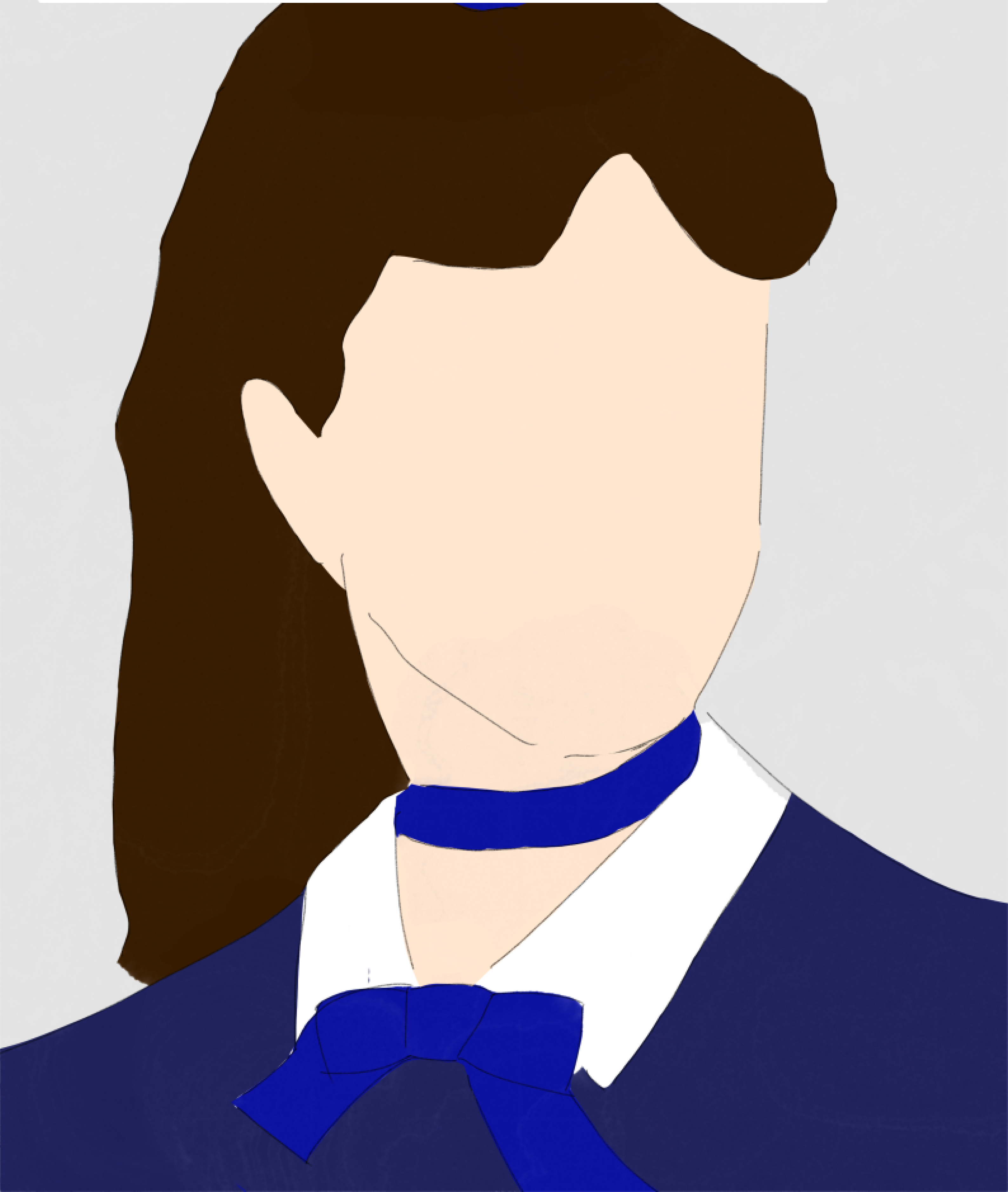
Question 6

With 2 specific examples, demonstrate how particle accelerators have validated scientific theories to develop the Standard Model of matter for 7 marks.

SOLUTIONS



Advanced Mechanics SOLUTIONS



Projectile Motion SOLUTIONS

Individual Components of Projectile Motion SOLUTIONS

Question 1 Solution

The only force acting on the arrow once it is shot will be gravity, which will accelerate it downwards. Because the horizontal component of the arrow's velocity will be unaffected, its horizontal velocity will not change, meaning the answer cannot be A or B, limiting the potential answers to C and D. However, as the gravity will accelerate the arrow downwards, the arrow's vertical velocity will increase downwards as it travels, meaning the answer must be D.

Question 2 Solution

Solution to (a)
Use the vertical component of the basketball's motion to find the solution.

Data List:

- $u = 0\,\text{m}\,\text{s}^{-1}$
- $a = 9.8\,\text{m}\,\text{s}^{-2}$
- $s = 63.0\,\text{m}$
- $t = ?\,\text{s}$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$s = \frac{1}{2}at^2 \qquad (u = 0)$$
$$t^2 = \frac{2s}{a}$$
$$t = \sqrt{\frac{2s}{a}} \qquad (t \geq 0)$$
$$t = \sqrt{\frac{2(63.0)}{(9.8)}}$$
$$t = 3.585\ldots$$
$$t = 3.59\,\text{s}$$

Solution to (b)
Use the time found in part (a) to find the range.

Data List:

- $t = 3.585\ldots$
- $v = 25.0\,\text{m}\,\text{s}^{-1}$
- $s = ?\,\text{m}$

Solution:

$$s = vt$$
$$s = (25.0)(3.585\ldots)$$
$$s = 89.642\ldots$$
$$s = 89.6\,\text{m}$$

Solution to (c)
The same as part (a), however, the time when it is 20.0 m above is being found, meaning it would have a vertical displacement downwards 43.0 m.

Data List:

- $u = 0\,\text{m}\,\text{s}^{-1}$
- $a = 9.8\,\text{m}\,\text{s}^{-2}$
- $s = 43.0\,\text{m}$
- $t = ?\,\text{s}$

Solution:

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
s &= \frac{1}{2}at^2 && (u = 0) \\
t^2 &= \frac{2s}{a} \\
t &= \sqrt{\frac{2s}{a}} && (t \geq 0) \\
t &= \sqrt{\frac{2(43.0)}{(9.8)}} \\
t &= 2.962\ldots \\
t &= 2.96\,\text{s}
\end{aligned}
$$

Question 3 Solution

To find the acute angle that the javelin makes with the ground, the direction of the final velocity of the javelin must be found. The direction of this final velocity can be calculated by the horizontal and vertical components of the javelin's final velocity. Firstly, since the javelin is first thrown with a horizontal velocity of $32.0\,\text{m}\,\text{s}^{-1}$ to the right, the final velocity will also have a horizontal velocity of $32.0\,\text{m}\,\text{s}^{-1}$ as there is no force acting in the horizontal direction. However, there is a force acting in the vertical direction; the gravitational force. Therefore, the final vertical velocity must be calculated as so:

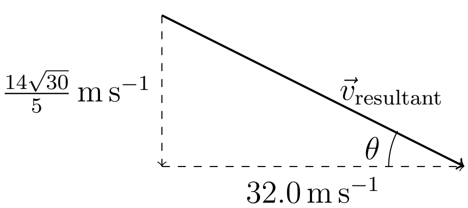
Data List:

- $u = 0\,\text{m}\,\text{s}^{-1}$
- $a = 9.8\,\text{m}\,\text{s}^{-2}$
- $s = 12.0\,\text{m}$
- $v = ?\,\text{m}\,\text{s}^{-1}$

Solution:

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
v^2 &= 2as && (u = 0) \\
v &= \sqrt{2as} && (v \geq 0) \\
v &= \sqrt{2(9.8)(12.0)} \\
v &= \frac{14\sqrt{30}}{5}\,\text{m}\,\text{s}^{-1}
\end{aligned}
$$

Therefore, the final velocity will have a horizontal component to the right of $32.0\,\text{m}\,\text{s}^{-1}$, and a vertical component downwards of $\frac{14\sqrt{30}}{5}\,\text{m}\,\text{s}^{-1}$ as shown:



Now, the acute angle the javelin makes with the ground can be found:

$$\begin{aligned}
\tan\theta &= \frac{v_y}{v_x} \\
\theta &= \tan^{-1}(\frac{v_y}{v_x}) \\
\theta &= \tan^{-1}(\frac{\frac{14\sqrt{30}}{5}}{32.0}) \\
\theta &= \tan^{-1}(\frac{7\sqrt{30}}{80}) \\
\theta &= 25.606\ldots \\
\theta &= 25.6^\circ
\end{aligned}$$

Question 4 Solution

Firstly, we must find the time it takes for the projectile to hit the wall of the target using the vertical displacement:

Data List:

- $u = 0\,\text{m}\,\text{s}^{-1}$
- $a = 9.8\,\text{m}\,\text{s}^{-2}$
- $s = 176.4\,\text{m}$
- $t = ?\,\text{s}$

Solution:

$$\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
s &= \frac{1}{2}at^2 && (u = 0) \\
t^2 &= \frac{2s}{a} \\
t &= \sqrt{\frac{2s}{a}} && (t \geq 0) \\
t &= \sqrt{\frac{2(176.4)}{(9.8)}} \\
t &= 6\,\text{s}
\end{aligned}$$

Now we use the time along with the horizontal component of the projectile to find horizontally how far the projectile travels before colliding with the wall.

Data List:

- $t = 6\,\text{s}$
- $v = 16000\,\text{m}\,\text{s}^{-1}$
- $s = ?\,\text{m}$

Solution:

$$\begin{aligned}
s &= vt \\
s &= (16000)(6) \\
s &= 96000 \\
s &= 9.6e4\,\text{m}
\end{aligned}$$

Question 5 Solution

First, the time the ball takes to reach the ground must be found via using the vertical displacement, keeping in mind that the ball has an initial downwards velocity due to the elevator descending at $4.2\,\mathrm{m\,s^{-1}}$.

Data List:

- $u = 4.2\,\mathrm{m\,s^{-1}}$
- $a = 9.8\,\mathrm{m\,s^{-2}}$
- $s = 56\,\mathrm{m}$
- $t = ?\,\mathrm{m}$

Solution:

$$s = ut + \frac{1}{2}at^2$$

$$\frac{1}{2}at^2 + ut - s = 0$$

$$\therefore t = \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)}$$

$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$

$$t = \frac{-(4.2) \pm \sqrt{(4.2)^2 + 2(9.8)(56)}}{(9.8)}$$

$$t = \frac{-3 + \sqrt{569}}{7}\,\mathrm{s} \qquad (t \geq 0)$$

Now use the time found along with the horizontal component of its initial velocity to find the range of the ball, along with its direction as horizontal displacement is a vector quantity.

Data List:

- $t = \frac{-3+\sqrt{569}}{7}\,\mathrm{s}$
- $v = 2.0\,\mathrm{m\,s^{-1}}$
- $s = ?\,\mathrm{m}$

Solution:

$$s = vt$$

$$s = (2.0)(\frac{-3 + \sqrt{569}}{7})$$

$$s = \frac{-6 + 2\sqrt{569}}{7}$$

$$s = 5.958\ldots$$

$$s = 6.0\,\mathrm{m} \text{ to the right}$$

Question 6 Solution

First off, find the time it takes for the lead ball to reach the ground. Also, it should be known that the fact that the ball is made of lead and is 10 times heavier than the rubber ball is irrelevant and can be ignored in reaching the solution.

Data List:

- $u = -12\,\mathrm{m\,s^{-1}}$
- $a = 9.8\,\mathrm{m\,s^{-2}}$
- $s = 56\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 + ut - s = 0$$
$$\therefore t = \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)}$$
$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$
$$t = \frac{-(-12) \pm \sqrt{(-12)^2 + 2(9.8)(56)}}{(9.8)}$$
$$t = \frac{60 + 8\sqrt{485}}{49}\,\mathrm{s} \qquad (t \geq 0)$$

And since the range is known, the horizontal component of the velocity can be found.

Data List:

- $t = \frac{60+8\sqrt{485}}{49}\,\mathrm{s}$
- $s = (\frac{-6+2\sqrt{569}}{7})^2\,\mathrm{m}$
- $v = ?\,\mathrm{m\,s^{-1}}$

Solution:

$$s = vt$$
$$v = \frac{s}{t}$$
$$v = \frac{(\frac{-6+2\sqrt{569}}{7})^2}{(\frac{60+8\sqrt{485}}{49})}$$
$$v = 7.365\ldots\,\mathrm{m\,s^{-1}}$$

Now find the time(s) it takes for the ball to be 1 m above its starting position.

Data List:

- $u = 12\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = 1\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

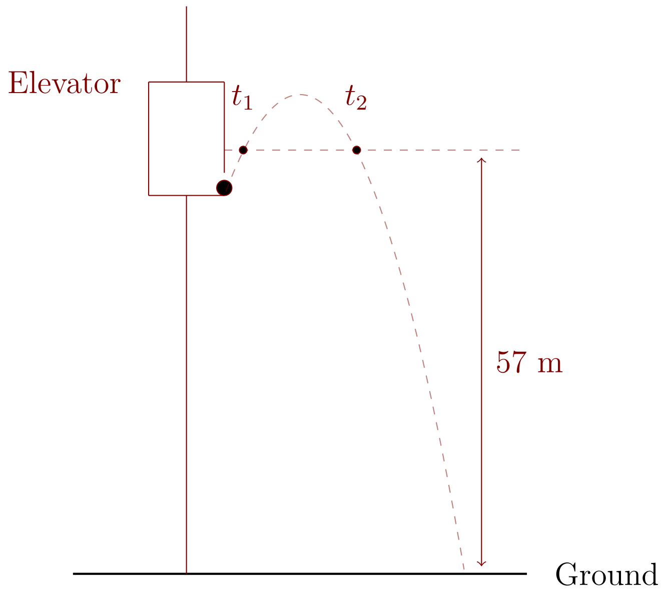
Solution:

$$s = ut + \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 + ut - s = 0$$
$$\therefore t = \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)}$$
$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$
$$t = \frac{-(12) \pm \sqrt{(12)^2 + 2(-9.8)(1)}}{(-9.8)}$$
$$t = 0.086\ldots \text{ OR } t = 2.362\ldots$$
$$t = 2.362\ldots\,\mathrm{s}$$

!?

Multiple Valid Answers

Although both solutions for t are possible, as they both satisfy the constraint that $t \geq 0$, we are only interested in one of the two scenarios. Since the question specifically states the furthest horizontal displacement, only the scenario where the ball has been travelling horizontally for longer (i.e. when t is greatest) is important. This is further clarified visually below.



Now find the range using the found t value and the found v value, also giving the direction as horizontal displacement is a vector quantity.

Data List:

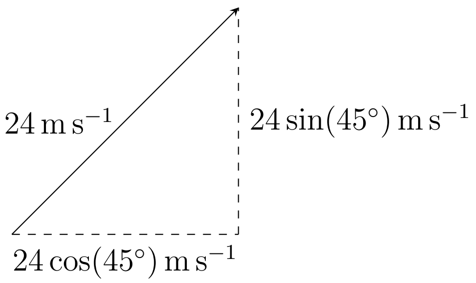
- $t = 2.362\ldots\,\text{s}$
- $v = 7.365\ldots\,\text{m}\,\text{s}^{-1}$
- $s = ?\,\text{m}$

Solution:

$$s = vt$$
$$s = (7.365\ldots)(2.362\ldots)$$
$$s = 17.400\ldots$$
$$s = 17.4\,\text{m to the right}$$

Question 1 Solution

Solution to (a)
The velocity that Emmy kicks the ball at is shown below:



Therefore, the ball's velocity has a vertical component of $24\sin(45°)\,\mathrm{m\,s^{-1}}$ and a horizontal velocity of $24\sin(45°)\,\mathrm{m\,s^{-1}}$.

Vector Resolution

It is very common to break down a vector to its vertical and horizontal components, called vector resolution. If you struggle to resolve vectors, a way to start is to know the rule that is a projectile launched at velocity $x\,\mathrm{m\,s^{-1}}$ at θ to the horizontal, the vertical component will be $x\sin\theta\,\mathrm{m\,s^{-1}}$ and the horizontal velocity will be $x\cos\theta\,\mathrm{m\,s^{-1}}$. Keep this in mind, as this will be assumed knowledge from now on.

At the ball's maximum height, the ball will have no vertical velocity. We can use this to find the time it takes for the soccer ball to reach its peak.

Data List:

- $u = 24\sin(45°)\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $v = 0\,\mathrm{m\,s^{-1}}$
- $t = ?\,\mathrm{s}$

Solution:

$$\begin{aligned}
v &= u + at \\
u + at &= 0 \qquad (v = 0) \\
at &= -u \\
t &= \frac{-u}{a} \\
t &= \frac{-(24\sin(45°))}{(-9.8)} \\
t &= 1.731\ldots \\
t &= 1.7\,\mathrm{s}
\end{aligned}$$

Solution to (b)
Remember to give the direction as vertical displacement is a vector quantity.

Data List:

- $u = 24\sin(45°)\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $v = 0\,\mathrm{m\,s^{-1}}$
- $s = ?\,\mathrm{m}$

Solution:

$$\begin{aligned} v^2 &= u^2 + 2as \\ u^2 + 2as &= 0 && (v = 0) \\ 2as &= -u^2 \\ s &= \frac{-u^2}{2a} \\ s &= \frac{-(24\sin(45°))^2}{2(-9.8)} \\ s &= 14.693\ldots \\ s &= 15\,\mathrm{m}\text{ upwards} \end{aligned}$$

Question 2 Solution

Let this initial velocity Amanda kicked the ball at be $x\,\mathrm{m\,s^{-1}}$. As it is kicked at the same angle of 45° to the horizontal, its vertical component will be $x\sin(45°)\,\mathrm{m\,s^{-1}}$ and its horizontal component will be $x\cos(45°)\,\mathrm{m\,s^{-1}}$. Now, using the vertical component of the velocity, a function of t can be found in terms of v, keeping in mind that the displacement s will equal to 0 as the soccer ball will start and end on the same elevation. !?

!? **Finding Expressions**

Finding functions, or expressions of a certain value in terms of another, is a fundamental skill. For example, in a square where the area is A and the side length is l, we know that $A = l^2$. This can be said that we have a **function for area A in terms of side length l**, or as an **expression for area A in terms of side length l**.

The solution continues over the page.

Data List:

- $u = x\sin(45°)\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = 0\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

Solution:

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
\frac{1}{2}at^2 + ut &= 0 && (s = 0) \\
t(\frac{1}{2}at + u) &= 0 \\
\therefore t = 0 \text{ OR } t &= -\frac{u}{\frac{1}{2}a} \\
t = 0 \text{ OR } t &= -\frac{2u}{a} \\
t &= -\frac{2u}{a} && (t = 0 \text{ is at origin}) \\
t &= \frac{2(x\sin(45°))}{(-9.8)} \\
t &= \frac{5x\sqrt{2}}{49}\,\mathrm{s}
\end{aligned}
$$

Alone, this expression of t in terms of x is trivial. However, another function for t found by the horizontal component will allow for simultaneous equations.

Data List:

- $v = x\cos(45°)\,\mathrm{m\,s^{-1}}$
- $s = 280\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

Solution:

$$
\begin{aligned}
s &= vt \\
t &= \frac{s}{v} \\
t &= \frac{280}{x\cos(45°)} \\
t &= \frac{280\sqrt{2}}{x}\,\mathrm{s}
\end{aligned}
$$

Now the functions of t can be equated as they both express the same t.

$$
\begin{aligned}
t = \frac{5x\sqrt{2}}{49} \text{ AND } t &= \frac{280\sqrt{2}}{x} \\
\therefore \frac{5x\sqrt{2}}{49} &= \frac{280\sqrt{2}}{x} \\
x^2 &= 2744 \\
x &= \sqrt{2744} && (x \geq 0) \\
x &= 52.383\ldots \\
x &= 52\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Question 3 Solution

First, find the time the projectile is in motion by the vertical component.

Data List:

- $u = 65.0\sin(30.0°)\,\text{m}\,\text{s}^{-1}$
- $a = -9.8\,\text{m}\,\text{s}^{-2}$
- $s = -10.0\,\text{m}$
- $t\ ?\,\text{s}$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 + ut - s = 0$$
$$\therefore t = \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)}$$
$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$
$$t = \frac{-(65.0\sin(30.0°)) \pm \sqrt{(65.0\sin(30.0°))^2 + 2(-9.8)(-10.0)}}{(-9.8)}$$
$$t = 6.927\ldots \qquad (t \geq 0)$$

And now use the found t value along with the horizontal component to find the range.

Data List:

- $t = 6.927\ldots\,\text{s}$
- $v = 65.0\cos(30.0°)\,\text{m}\,\text{s}^{-1}$
- $s = ?\,\text{m}$

Solution:

$$s = vt$$
$$s = (65.0\cos(30.0°))(6.927\ldots)$$
$$s = 389.946\ldots$$
$$s = 3.90 \times 10^2\,\text{m}$$

Question 4 Solution

First, find Emmy's vertical velocity when she is 6 m above the ground, or in other words, when her vertical displacement is 4 m.

Data List:

- $u = 0\,\text{m}\,\text{s}^{-1}$
- $a = 9.8\,\text{m}\,\text{s}^{-2}$
- $s = 4\,\text{m}$
- $v = ?\,\text{m}\,\text{s}^{-1}$

Solution:

$$v^2 = u^2 + 2as$$
$$v^2 = 2as \qquad (u = 0)$$
$$v = \sqrt{2as} \qquad (v \geq 0)$$
$$v = \sqrt{2(9.8)(4)}$$
$$v = \frac{14\sqrt{10}}{5}\,\text{m}\,\text{s}^{-1}$$

Because Emmy shoots the arrow as he is falling, the arrow will also have an initial downwards velocity of $\frac{14\sqrt{10}}{5}\,\text{m}\,\text{s}^{-1}$, and also its $65.0\,\text{m}\,\text{s}^{-1}$ velocity that is at 30.0° to the horizontal. Therefore, the true initial velocity's components can be found using a vector addition of $\frac{14\sqrt{10}}{5}\,\text{m}\,\text{s}^{-1}$ and $65.0\,\text{m}\,\text{s}^{-1}$ 30.0° to the horizontal.

$$\Sigma_{v_x} = 65.0\cos(30.0°) = \frac{65\sqrt{3}}{2}\,\text{m}\,\text{s}^{-1}$$

$$\Sigma_{v_y} = 65.0\sin(30.0°) - \frac{14\sqrt{10}}{5} = \frac{325 - 28\sqrt{10}}{10}\,\text{m}\,\text{s}^{-1}$$

With the true vertical component, find the time that the projectile takes to stop, keeping in mind that the vertical displacement s is now 6 m.

Data List:

- $u = \frac{325-28\sqrt{10}}{10}\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = -6\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$\frac{1}{2}at^2 + ut - s = 0$$
$$\therefore t = \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)}$$
$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$
$$t = \frac{-(\frac{325-28\sqrt{10}}{10}) \pm \sqrt{(\frac{325-28\sqrt{10}}{10})^2 + 2(-9.8)(-6)}}{(-9.8)}$$
$$t = 5.067\ldots\,\mathrm{s} \qquad (t \geq 0)$$

Now use the time and the horizontal component to find the range.

Data List:

- $t = 5.067\ldots\,\mathrm{s}$
- $v = \frac{65\sqrt{3}}{2}\,\mathrm{m\,s^{-1}}$
- $s = ?\,\mathrm{m}$

Solution:

$$s = vt$$
$$s = (\frac{65\sqrt{3}}{2})(5.067\ldots)$$
$$s = 285.245\ldots$$
$$s = 2.85 \times 10^2\,\mathrm{m}$$

Question 5 Solution

Let the original velocity of the shot javelin be x. Now use the horizontal component to find a function for t in terms of x.

Data List:

- $v = x\cos(70.0^\circ)\,\mathrm{m\,s^{-1}}$
- $s = 124\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

Solution:

$$s = vt$$
$$t = \frac{s}{v}$$
$$t = \frac{(124)}{(x\cos(70.0^\circ))}$$
$$t = \frac{124}{x\cos(70^\circ)}\,\mathrm{s}$$

Now use the found function for t in terms of x to find the time taken for the projectile to finish its motion.

Data List:

- $u = x\sin(70.0°)\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = 56.0\,\mathrm{m}$
- $t = \frac{124}{x\cos(70°)}\,\mathrm{s}$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$(56.0) = (x\sin(70.0°))(\frac{124}{x\cos(70°)}) + \frac{1}{2}(-9.8)(\frac{124}{x\cos(70°)})^2$$
$$56 = 124\tan(70°) - \frac{376712}{5x^2\cos^2(70°)}$$
$$\frac{376712}{5x^2\cos^2(70°)} = 124\tan(70°) - 56$$
$$5x^2\cos^2(70°) = \frac{376712}{124\tan(70°) - 56}$$
$$x^2 = \frac{376712}{(5\cos^2(70°))(124\tan(70°) - 56)}$$
$$x = \sqrt{\frac{376712}{(5\cos^2(70°))(124\tan(70°) - 56)}} \qquad (x \geq 0)$$
$$x = 47.564\ldots$$
$$x = 47.6\,\mathrm{m\,s^{-1}}$$

Question 6 Solution

Use the information about the peak of the motion to find the initial velocity. Let the initial velocity be x.

Data List:

- $u = x\sin(65.0°)\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = 5.00\,\mathrm{m}$
- $v = 0\,\mathrm{m\,s^{-1}}$
- $x = ?\,\mathrm{m\,s^{-1}}$

Solution:

$$v^2 = u^2 + 2as$$
$$u^2 + 2as = 0 \qquad (v = 0)$$
$$u^2 = -2as$$
$$u = \sqrt{-2as} \qquad (u \geq 0)$$
$$(x\sin(65.0°)) = \sqrt{-2(-9.8)(5.00)}$$
$$x = \frac{\sqrt{-2(-9.8)(5.00)}}{\sin(65°)}$$
$$x = \frac{7\sqrt{2}}{\sin(65°)}$$

Therefore, the vertical velocity of x is equal to $(\frac{7\sqrt{2}}{\sin(65°)})\sin(65.0°) = 7\sqrt{2}\,\mathrm{m\,s^{-1}}$. Now find the time when it reaches the peak/wall.

Data List:

- $u = 7\sqrt{2}\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $v = 0\,\mathrm{m\,s^{-1}}$
- $t = ?\,\mathrm{s}$

Solution:

$$\begin{aligned} v &= u + at \\ u + at &= 0 \\ at &= -u \\ t &= \frac{-u}{a} \\ t &= \frac{-(7\sqrt{2})}{(-9.8)} \\ t_{\text{peak}} &= \frac{5\sqrt{2}}{7}\,\mathrm{s} \end{aligned}$$

And now find the time when Amanda catches the ball.

Data List:

- $u = 7\sqrt{2}\,\mathrm{m\,s^{-1}}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = 3.00\,\mathrm{m}$
- $t = ?\,\mathrm{s}$

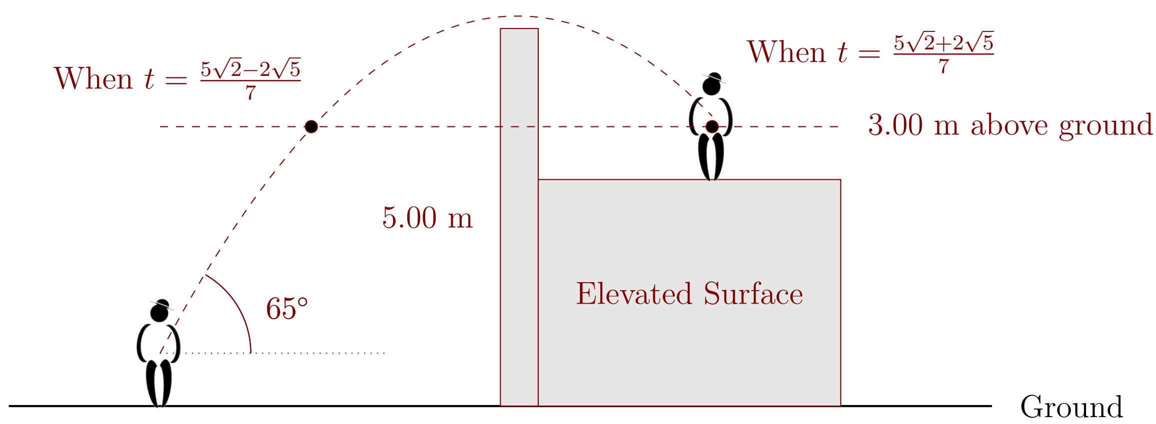
Solution:

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ \frac{1}{2}at^2 + ut - s &= 0 \\ \therefore t &= \frac{-u \pm \sqrt{u^2 - 4(\frac{1}{2}a)(-s)}}{2(\frac{1}{2}a)} \\ t &= \frac{-u \pm \sqrt{u^2 + 2as}}{a} \\ t &= \frac{-(7\sqrt{2}) \pm \sqrt{(7\sqrt{2})^2 + 2(-9.8)(3.00)}}{(-9.8)} \\ t = \frac{5\sqrt{2} - 2\sqrt{5}}{7} &\text{ OR } t = \frac{5\sqrt{2} + 2\sqrt{5}}{7} \\ t_{\text{Amanda}} &= \frac{5\sqrt{2} + 2\sqrt{5}}{7} \end{aligned}$$

⁉

Multiple Valid Answers Again

Again, both solutions of t are possible, similar to Question 6 of Individual Components of Projectile Motion. As the diagram below shows, only the greater t value is important.



Now find the time difference between the ball at the peak/wall, and when Amanda catches the ball. With this time difference and the horizontal component of the ball, find the ball's horizontal displacement. Because this will show how far the ball travels horizontally between the wall and being caught by Amanda, this will consequently show the horizontal distance between the wall and Amanda.

Data List:

- $\Delta t = |t_{\text{peak}} - t_{\text{Amanda}}|\,\text{s}$
- $t_{\text{peak}} = \frac{5\sqrt{2}}{7}\,\text{s}$
- $t_{\text{Amanda}} = \frac{5\sqrt{2}-2\sqrt{5}}{7}\,\text{s}$
- $v = x\cos(65.0°)\,\text{m}\,\text{s}^{-1}$
- $x = \frac{7\sqrt{2}}{\sin(65.0°)}\,\text{m}\,\text{s}^{-1}$
- $s = ?\,\text{m}$

Solution:

$$s = vt$$
$$s = (x\cos(65.0°))(|t_{\text{peak}} - t_{\text{Amanda}}|)$$
$$s = (\left(\frac{7\sqrt{2}}{\sin(65.0°)}\right)\cos(65.0°))(|\left(\frac{5\sqrt{2}}{7}\right) - \left(\frac{5\sqrt{2}-2\sqrt{5}}{7}\right)|)$$
$$s = \left(\frac{7\sqrt{2}}{\tan(65°)}\right)\left(\frac{2\sqrt{5}}{7}\right)$$
$$s = \frac{2\sqrt{10}}{\tan(65°)}$$
$$s = 2.949\ldots$$
$$s = 2.95\,\text{m}$$

Circular Motion SOLUTIONS

Centripetal Motion on a Flat Plane SOLUTIONS

Question 1 Solution

Solution to (a)
The frictional force between the truck and the road is what keeps the truck in circular motion. In other words, the frictional force, F_f is equivalent to the centripetal force F_c. Also, keep in mind that centripetal force is a vector quantity, and although stating its direction doesn't provide much information, it should still be said the centripetal force is acting towards the centre of circular motion.

Data List:

- $m = 150 \times 10^3\,\text{kg}$
- $v = 11\,\text{m}\,\text{s}^{-1}$
- $r = 40\,\text{m}$
- $F_f = ?\,\text{N}$

Solution:

$$F_f = F_c$$
$$F_f = \frac{mv^2}{r}$$
$$F_f = \frac{(150 \times 10^3)(11)^2}{(40)}$$
$$F_f = 453750$$
$$F_f = 4.5 \times 10^5\,\text{N towards centre of circular motion}$$

Solution to (b)
For centripetal acceleration, although stating its direction doesn't provide much information, it should still be said the centripetal acceleration is acting towards the centre of circular motion.

Data List:

- $v = 11\,\text{m}\,\text{s}^{-1}$
- $r = 40\,\text{m}$
- $a_c = ?\,\text{m}\,\text{s}^{-2}$

Solution:

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{(11)^2}{(40)}$$
$$a_c = 3.025$$
$$a_c = 3.0\,\text{m}\,\text{s}^{-2}\text{ towards centre of circular motion}$$

Solution to (c)
A more useful expression of angular velocity can be found as so.

$$\omega = \frac{2\pi}{T}$$

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$

$$\therefore \omega = \frac{2\pi}{\left(\frac{2\pi r}{v}\right)}$$
$$\omega = \frac{v}{r}$$

Now use this expression to find the angular velocity, however, keep in mind that the answer will be found in radians, and must be converted to degrees when answering by multiplying $\frac{180}{\pi}$. Also, the direction must be given by the right hand grip rule.

Right Hand Grip Rule

The right hand grip rule is an easy intuitive method to determine the direction of certain quantities that run in a circular direction. For example, the direction of an angular velocity is determined by the spinning direction of the object being the way the fingers rotate, and the thumb being the direction of the angular velocity. This can also be used for other quantities, such as magnetic fields which will be explored later in **Electromagnetism**.

Data List:

- $v = 11\,\mathrm{m\,s^{-1}}$
- $r = 40\,\mathrm{m}$
- $\omega = ?\,^\circ\,\mathrm{s^{-1}}$

Solution:

$$\omega = \frac{v}{r}$$
$$\omega = \frac{11}{40} \qquad \text{(radians)}$$
$$\omega = 15.756\ldots \qquad \text{(degrees)}$$
$$\omega = 16\,^\circ\,\mathrm{s^{-1}} \text{ out of the page}$$

Question 2 Solution

Solution to (a)
Firstly, derive a more useful expression for linear velocity as so.

$$\omega = \frac{2\pi}{T}$$

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$

$$\therefore \omega = \frac{2\pi}{\left(\frac{2\pi r}{v}\right)}$$
$$\omega = \frac{v}{r}$$
$$\omega r = v$$

Now use this expression to find a linear velocity.

Data List:

- $\omega = 36\,\mathrm{rad\,s^{-1}}$
- $r = 1.4\,\mathrm{m}$
- $v = ?\,\mathrm{m\,s^{-1}}$

Solution:

$$v = \omega r$$
$$v = (36)(1.4)$$
$$v = 50.4$$
$$v = 5.0 \times 10^{1}\,\mathrm{m\,s^{-1}}$$

Solution to (b)
The tension of the string is what keeps the rock in circular motion. In other words, the tension of the string, T, is equal to the centripetal force F_c. Use the previously determined value for velocity.

Data List:

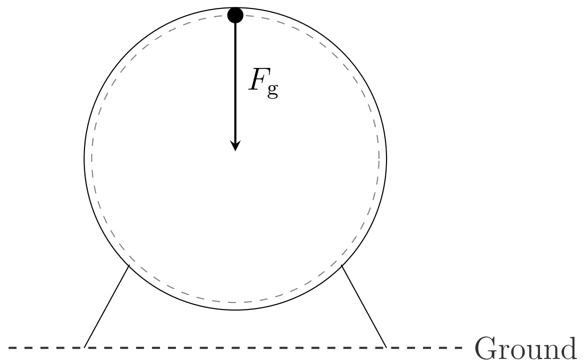
- $m = 0.80\,\mathrm{kg}$
- $v = 50.4\,\mathrm{m\,s^{-1}}$
- $r = 1.4\,\mathrm{m}$
- $T = ?\,\mathrm{s}$

Solution:

$$T = F_c$$
$$T = \frac{mv^2}{r}$$
$$T = \frac{(0.80)(50.4)^2}{(1.4)}$$
$$T = 1451.52$$
$$T = 1.5 \times 10^{3}\,\mathrm{N}$$

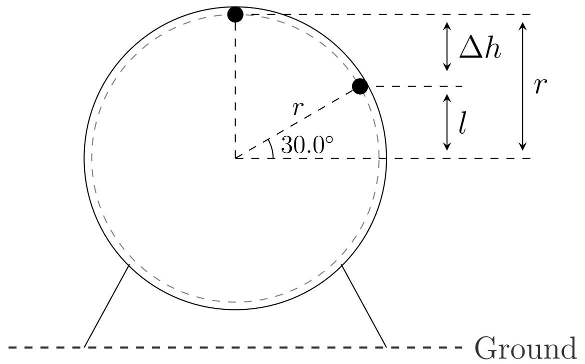
Question 3 Solution

If the ball is travelling at the minimum possible speed, the centripetal force will be supplied by the gravitational force at the top of the path as shown.



$$
\begin{aligned}
F_c &= F_g \\
\frac{mv^2}{r} &= mg \\
v_{\text{Top}}^2 &= gr
\end{aligned}
$$

To find the speed at the position shown, calculate the change in height from the top to that position, since total mechanical energy is conserved.



$$
\begin{aligned}
\Delta h + l &= r \\
\Delta h &= r - l \\
\Delta h &= r - r\sin(30^\circ) \\
\Delta h &= \frac{r}{2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Now } E_{\text{Position}} &= E_{\text{Top}} \\
K_{\text{Position}} &= K_{\text{Top}} + \Delta U \\
\frac{1}{2}mv_{\text{Position}}^2 &= \frac{1}{2}mv_{\text{Top}}^2 + mg\Delta h \\
v_{\text{Position}}^2 &= v_{\text{Top}}^2 + 2g\Delta h \\
v_{\text{Position}} &= \sqrt{gr + gr} \\
v_{\text{Position}} &= \sqrt{2gr} \\
v_{\text{Position}} &= \sqrt{2(9.8)(2)} \\
v_{\text{Position}} &= 6.260\ldots \\
v_{\text{Position}} &= 6.26\,\text{m}\,\text{s}^{-1}
\end{aligned}
$$

Question 4 Solution

Firstly, Emmy's speed must be found when he enters circular motion. This can be done through the conservation of mechanical energy. If you do not know how, visit the Conservation of Mechanical Energy chapter of the textbook. Also keep in mind Δh, the change in height before Emmy enters circular motion is $381 - 66 = 315\,\text{m}$.

Data List:

- $\Delta h = 315\,\text{m}$
- $v = ?\,\text{m}\,\text{s}^{-1}$

Solution:

$$\begin{aligned}
E_{\text{kinetic}} &= E_{\text{potential}} \\
\frac{1}{2}mv^2 &= mg\Delta h \\
v^2 &= 2g\Delta h \\
v &= \sqrt{2g\Delta h} && (v \geq 0) \\
v &= \sqrt{2(9.8)(315)} \\
v &= 21\sqrt{14}\,\text{m}\,\text{s}^{-1}
\end{aligned}$$

Now find the centripetal acceleration with the found speed, while keeping in mind the radius of circular motion is $66 - 1 = 65\,\text{m}$.

Data List:

- $v = 21\sqrt{14}\,\text{m}\,\text{s}^{-1}$
- $r = 65\,\text{m}$
- $a_c = ?\,\text{m}\,\text{s}^{-2}$

Solution:

$$\begin{aligned}
a_c &= \frac{v^2}{r} \\
a_c &= \frac{(21\sqrt{14})^2}{(65)} \\
a_c &= \frac{6174}{65}\,\text{m}\,\text{s}^{-2}
\end{aligned}$$

Now divide the centripetal acceleration by 9.8 to find the amount of g forces Emmy experiences.

$$\begin{aligned}
\text{g forces} &= \frac{6174}{65} \div 9.8 \\
&= \frac{126}{13} \\
&= 9.692\ldots
\end{aligned}$$

As the g forces are above 9, Emmy will not survive, and Tarzan will fail in saving Emmy.

Question 5 Solution

For object A, the only force that will be acting is the gravitational force. The magnitude of the gravitational force, or the net force, can be found as so.

Data List:

- $m = 2.50\,\text{kg}$
- $F_{\text{net}} = ?\,\text{N}$

Solution:

$$F_{\text{net}} = F_{\text{g}}$$
$$F_{\text{net}} = mg$$
$$F_{\text{net}} = (2.50)(9.8)$$
$$F_{\text{net}} = 24.5\,\text{N towards the Earth's centre}$$

Therefore, the magnitude of the net force acting on object A is 24.5 N
For object B, the net force is represented by the centripetal force as it is the resultant of all forces acting on object B, such as the gravitational force or the tension of the robotic arm. ⁉

⁉ Net Forces

The term "Net Force" may start to seem a little confusing now. The question states to compare the difference of the net forces that are experienced by both forces, however it may be confusing to determine the net force of each object. It is easier to tell that the only force acting on object A is the gravitational force; thus, it is the net force itself. However, it is difficult to determine what the net force is on object B, as there can be multiple forces that can be identified to be acting on object B, such as gravitational force or the tension of the robotic arm. However, it should be known that the net force always manifests itself in the motion of the system. This means that no matter the multiple forces acting on, say a 1 kg mass in space, if it is at a constant velocity, the net force must be equal to 0. Furthermore, even if potentially a 1000 different forces were acting on the same mass, but it was accelerating at $1\,\text{m}\,\text{s}^{-2}$, the net force's magnitude must be equal to 1 N. This can be slightly counter intuitive, as the 1 kg mass itself may be experiencing a tremendous amount of different forces, attempting to rip it apart, the net force will only show itself in the motion of the mass. Same to how after you stand for 24 hours, you may feel tired due to having to resist the gravitational force with the tension of your muscles in your legs to keep you up, but because you are not accelerating, the net force acting on you must be equal to 0, although it may not feel like it.

Data List:

- $m = 2.50\,\text{kg}$
- $v = 20.0\,\text{m}\,\text{s}^{-1}$
- $r = 3.00\,\text{m}$
- $F_{\text{c}} = ?\,\text{N}$

Solution:

$$F_{\text{c}} = \frac{mv^2}{r}$$
$$F_{\text{c}} = \frac{(2.50)(20.0)^2}{(3.00)}$$
$$F_{\text{c}} = \frac{1000}{3}\,\text{N towards centre of circular motion}$$

As the centripetal force is the net force acting on object B, the magnitude of the net force is $\frac{1000}{3}$ N. Now the difference in magnitude can be found.

$$\begin{aligned}\text{Difference} &= |F_A - F_B| \\ &= |24.5 - \frac{1000}{3}| \\ &= 308.833\ldots \\ &= 3.09 \times 10^2\,\text{N}\end{aligned}$$

Question 6 Solution

If the wheels rotate without slipping, the tangential velocity of two adjacent wheels must be equal. Because all wheels are next to one another, all wheels must have the same tangential velocity. Furthermore, the leftmost wheel has a radius of 9 m, and since the rightmost wheel is 4 places next to the leftmost wheel, the radius will be $9 - 2(4) = 1$ m. Now, we will derive a most useful definition for angular velocity.

$$\omega = \frac{2\pi}{T}$$

$$\begin{aligned}v &= \frac{2\pi r}{T} \\ T &= \frac{2\pi r}{v}\end{aligned}$$

$$\begin{aligned}\therefore \omega &= \frac{2\pi}{\left(\frac{2\pi r}{v}\right)} \\ \omega &= \frac{v}{r} \\ \omega r &= v\end{aligned}$$

Now let the leftmost wheel be wheel A, and the rightmost wheel be wheel B. Because their tangential velocity will be equal;

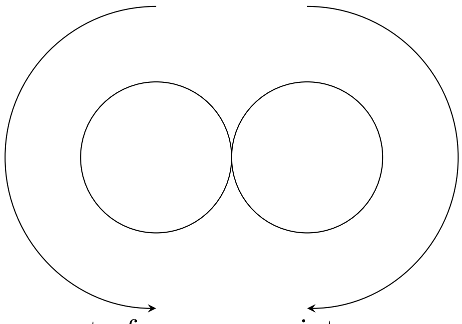
Data List:

- $r_A = 9\,\text{m}$
- $\omega_A = 1\,\text{rad}\,\text{s}^{-1}$
- $r_B = 1\,\text{m}$
- $\omega_B = ?\,\text{rad}\,\text{s}^{-1}$

Solution:

$$\begin{aligned}v_A &= v_B \\ \omega_A r_A &= \omega_B r_B \\ (1)(9) &= \omega_B(1) \\ 9 &= \omega_B \\ \omega_B &= 9\,\text{rad}\,\text{s}^{-1}\end{aligned}$$

However, we cannot finish here as we do not have the direction. This can be found quite easily by the fact that the direction wheels spin invert when adjacent to each other.



Because wheel B is 4 positions to the right, the spinning direction will invert 4 times, or an even number of times. Much like how multiplying a number by -1, an even number of times, inverting the spinning direction an even number of times will result in the direction of the angular velocity not changing. Therefore, $\omega_{\mathrm{B}} = 9\,\mathrm{rad\,s^{-1}}$ out of the page.

Question 1 Solution

Solution to (a)

Data List:

- $m = 2.50 \times 10^3\,\mathrm{kg}$
- $g = 1.67\,\mathrm{m\,s^{-2}}$
- $\theta = 60.0°$
- $F_c = ?\,\mathrm{N}$

Solution:

$$F_c = mg\tan\theta$$
$$F_c = (2.50 \times 10^3)(1.67)\tan(60.0)$$
$$F_c = 7231.312\ldots$$
$$F_c = 7.23 \times 10^3\,\mathrm{N}\text{ towards centre of circular motion}$$

Solution to (b)

Data List:

- $r = 1260\,\mathrm{m}$
- $g = 1.67\,\mathrm{m\,s^{-2}}$
- $\theta = 60.0°$
- $v = ?\,\mathrm{m\,s^{-1}}$

Solution:

$$F_c = \frac{mv^2}{r}$$
$$mg\tan\theta = \frac{mv^2}{r}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta} \qquad (v \geq 0)$$
$$v = \sqrt{(1260)(1.67)\tan(60.0)}$$
$$v = 60.370\ldots \qquad (\mathrm{m\,s^{-1}})$$
$$v = 217.333\ldots \qquad (\mathrm{km\,h^{-1}})$$
$$v = 217\,\mathrm{km\,h^{-1}}$$

Solution to (c)

First find the expression for maximum safe speed.

$$F_c = mg\tan\theta$$
$$\frac{mv^2}{r} = mg\tan\theta$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$

Therefore, the only factors affecting v is the radius of the track, the angle of inclination of the track, and the gravitational field strength. None of these factors is influenced by the vehicle used, and therefore, the maximum safe speed for a track does not change for different vehicles.

Question 2 Solution

Data List:

- $v = 8.00\,\text{m}\,\text{s}^{-1}$
- $r = 40.0\,\text{m}$
- $\theta = ?^\circ$

Solution:

$$\begin{aligned}
F_c &= mg\tan\theta \\
\frac{mv^2}{r} &= mg\tan\theta \\
\tan\theta &= \frac{v^2}{rg} \\
\theta &= \tan^{-1}(\frac{v^2}{rg}) \\
\theta &= \tan^{-1}(\frac{(8.00)^2}{(40.0)(9.8)}) \\
\theta &= 9.272\ldots \\
\theta &= 9.27^\circ
\end{aligned}$$

Question 3 Solution

Find an expression for the maximum safe velocity for the Earth & Moon.

Earth

$$\begin{aligned}
F_c &= mg\tan\theta \\
\frac{mv^2}{r} &= mg\tan\theta \\
v^2 &= rg\tan\theta \\
v &= \sqrt{rg\tan\theta} \\
v &= \sqrt{(45)(9.8)\tan\theta} \\
v_E &= 21\sqrt{\tan\theta}\,\text{m}\,\text{s}^{-1}
\end{aligned}$$

Moon

$$\begin{aligned}
F_c &= mg\tan\theta \\
\frac{mv^2}{r} &= mg\tan\theta \\
v^2 &= rg\tan\theta \\
v &= \sqrt{rg\tan\theta} \\
v &= \sqrt{(117)(1.67)\tan\theta} \\
v_M &= \sqrt{\frac{19539}{100}}\sqrt{\tan\theta}\,\text{m}\,\text{s}^{-1}
\end{aligned}$$

Now find the ratio between these two velocities.

$$\begin{aligned}
\text{Ratio} &= \frac{v_E}{v_M} \\
&= \frac{21\sqrt{\tan\theta}}{\sqrt{\frac{19539}{100}}\sqrt{\tan\theta}} \\
&= \frac{21\sqrt{100}}{\sqrt{19539}} \\
&= 1.502\ldots \\
&\approx 1.5, \text{ OR } \frac{3}{2}
\end{aligned}$$

Therefore, the answer is 3:2, or B.

Question 4 Solution

Find an expression for F_c in terms of momentum, and an expression for angular velocity that will be useful to the solution.

$$F_c = \frac{mv^2}{r}$$
$$F_c = \frac{pv}{r} \qquad (p = mv)$$
$$F_c = p \times \frac{v}{r}$$

$$\omega = \frac{2\pi}{T}$$
$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$
$$\therefore \omega = \frac{2\pi}{(\frac{2\pi r}{v})}$$
$$\omega = \frac{v}{r}$$

Now substitute $\omega = \frac{v}{r}$ into $F_c = p \times \frac{v}{r}$ to find the solution.

Data List:

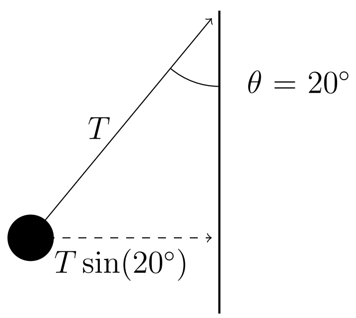
- $F_c = 3150\,\text{N}$
- $p = 1050\,\text{N}$
- $\omega = ?\,\text{rad s}^{-1}$

Solution:

$$F_c = p \times \frac{v}{r}$$
$$F_c = p\omega$$
$$\omega = \frac{F_c}{p}$$
$$\omega = \frac{3150}{1050}$$
$$\omega = 3$$
$$\omega = 3.00\,\text{rad s}^{-1}$$

Question 5 Solution

From the front view, the tension of the string acting on the mass will look like so.



The solution continues over the page.

Since the horizontal component of the tension force, $T\sin(20°)$ is acting towards the centre of circular motion, and is the force that is keeping the mass in circular motion, $T\sin(20°)$ is the centripetal force F_c. Using the fact that $T\sin(20°)$ and F_c are equivalent, the linear velocity can be found.

Data List:

- $T = 800\,\text{N}$
- $r = 14\,\text{m}$
- $m = 15\,\text{kg}$
- $v = ?\,\text{m}\,\text{s}^{-1}$

Solution:

$$T\sin(20°) = F_c$$
$$T\sin(20°) = \frac{mv^2}{r}$$
$$v^2 = \frac{Tr\sin(20°)}{m}$$
$$v = \sqrt{\frac{Tr\sin(20°)}{m}} \qquad (v \geq 0)$$
$$v = \sqrt{\frac{(800)(14)\sin(20°)}{(15)}}$$
$$v = 15.980\ldots$$
$$v = 16\,\text{m}\,\text{s}^{-1}$$

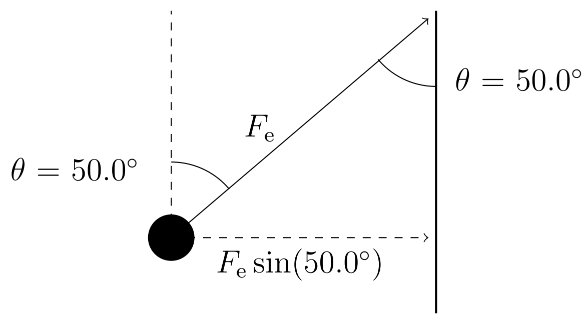
Question 6 Solution

Prior to the solution, take note of these expressions.

$$F_c = \frac{mv^2}{r} \qquad\qquad v = \frac{2\pi r}{T}$$
$$F_c = \frac{pv}{r} \quad (p = mv) \qquad\qquad v = 2\pi r f \quad (f = \frac{1}{T})$$
$$F_c = p \times \frac{v}{r} \qquad\qquad \frac{v}{r} = 2\pi f$$

$$\therefore F_c = p \times 2\pi f$$
$$F_c = 2\pi p f$$

Now, similar to Question 5, from the front view, the electrostatic attractive force (F_e) will look like so.



Therefore, $F_e \sin(50.0°)$ is equivalent to the centripetal force as it is the force keeping the mass in circular motion.
Using our previous expression of $F_c = 2\pi pf$, the magnitude of the electrostatic attractive force can be found.

Data List:

- $p = 0.800\,\text{N s}$
- $f = 2.50\,\text{Hz}$
- $F_e = ?\,\text{N}$

Solution:

$$F_e \sin(50.0°) = F_c$$
$$F_e \sin(50.0°) = 2\pi pf$$
$$F_e = \frac{2\pi pf}{\sin(50.0°)}$$
$$F_e = \frac{2\pi(0.800)(2.50)}{\sin(50.0°)}$$
$$F_e = 16.404\ldots$$
$$F_e = 16.4\,\text{N}$$

Torque SOLUTIONS

Question 1 Solution

Data List:

- $F = 72\,\mathrm{N}$
- $r = 0.36\,\mathrm{m}$
- $\tau = ?\,\mathrm{N\,m}$

Solution:

$$\tau = Fr$$
$$\tau = (72)(0.36)$$
$$\tau = 25.92$$
$$\tau = 26\,\mathrm{N\,m}$$

Question 2 Solution

First, find the torque the 500.0 kg mass applies, keeping in mind the force due to gravity, $F_g = mg$.

Data List:

- $m = 500.0\,\mathrm{kg}$
- $r = 1.00\,\mathrm{m}$
- $\tau = ?\,\mathrm{N\,m}$

Solution:

$$\tau = Fr$$
$$\tau = mgr$$
$$\tau = (500.0)(9.8)(1.00)$$
$$\tau_{\text{mass}} = 4900\,\mathrm{N\,m}\ \text{anticlockwise}$$

Now, if the system is at rest, the net torque must be equal to 0.

$$\tau_{\text{net}} = 0$$
$$\therefore \Sigma\tau = 0$$
$$\tau_{\text{mass}} + \tau_{\text{force}} = 0$$
$$\tau_{\text{force}} = -\tau_{\text{mass}} \qquad \text{(Negative due to opposite directions)}$$

Now find an expression for τ_{force}, letting the distance of the force be x.

Data List:

- $F = 25.0\,\mathrm{N}$
- $r = x\,\mathrm{m}$
- $\tau = ?\,\mathrm{N\,m}$

Solution:

$$\tau = Fr$$
$$\tau = (25.0)(x)$$
$$\tau = 25x$$

Now, equate the torque of the mass and the torque of the force.

$$|\tau_{\text{force}}| = |\tau_{\text{mass}}|$$
$$25x = 4900$$
$$x = 196$$
$$x = 196\,\mathrm{m}$$

Question 3 Solution

From left to right, let the forces be referred to as force A, force B and force C for convenience. Find an expression for all 3 torques that are applied by each force, remembering to find the perpendicular component (in this case, the vertical component) of each force. Let the unknown force be x.

Force A

$$\tau = F_{\perp}r$$
$$\tau_{\text{A}} = (35\sin(45°))(2.6)$$
$$\tau_{\text{A}} = \frac{91\sqrt{2}}{2}\,\text{N m anticlockwise}$$

Force B

$$\tau = F_{\perp}r$$
$$\tau_{\text{B}} = (25)(1.0)$$
$$\tau_{\text{B}} = 25\,\text{N m anticlockwise}$$

Force C

$$\tau = F_{\perp}r$$
$$\tau_{\text{C}} = (x\sin(60.0°))(2.3)$$
$$\tau_{\text{C}} = \frac{23x\sqrt{3}}{20}\,\text{N m clockwise}$$

If the system is in equilibrium, the net torque, or the sum of the torques, must equal zero.

$$\Sigma\tau = 0 \qquad \text{(Take clockwise as positive)}$$
$$\tau_{\text{A}} + \tau_{\text{B}} + \tau_{\text{C}} = 0$$
$$\tau_{\text{c}} = -\tau_{\text{A}} - \tau_{\text{B}}$$
$$\frac{23x\sqrt{3}}{20} = \frac{91\sqrt{2}}{2} + 25$$
$$\frac{23x\sqrt{3}}{20} = \frac{50 + 91\sqrt{2}}{2}$$
$$x = 44.855\ldots$$
$$x = 45\,\text{N}$$

Question 4 Solution

First, find the torque the barbell is applying, keeping in mind that $F_{\text{g}} = mg$.

Data List:

- $m = 145\,\text{kg}$
- $r = 0.300\,\text{m}$
- $\tau = ?\,\text{N m}$

Solution:

$$\tau = Fr$$
$$\tau = mgr$$
$$\tau = (145)(9.8)(0.300)$$
$$\tau = \frac{4263}{10}\,\text{N m anticlockwise}$$

Therefore, the biceps must be applying a torque of $\frac{4263}{10}$ N m clockwise to statically hold the barbell. However since there are two arms, this torque is the resultant of two biceps, meaning a single bicep's torque is only half of this value: $\frac{4263}{20}$ N m. Using this, the force of the biceps can be found.

The solution continues over the page.

Data List:

- $\tau = \frac{4263}{20}\,\text{N m}$
- $r = 20.0e - 3\,\text{m}$
- $F = ?\,\text{N}$

Solution:

$$\tau = Fr$$
$$F = \frac{\tau}{r}$$
$$F = \frac{\left(\frac{4263}{20}\right)}{(20.0e-3)}$$
$$F = 10657.5$$
$$F = 1.07 \times 10^4\,\text{N}$$

Therefore, the tension in the left bicep is 1.07×10^4 N.

Question 5 Solution

First, find the torque required to turn the fan in the first place. Make sure to find the perpendicular component of the force Llewellyn applies.

Data List:

- $F = 150\,\text{N}$
- $\theta = 10.0°$
- $r = 0.10\,\text{m}$
- $\tau = ?\,\text{N m}$

Solution:

$$\tau = F_{\perp} r$$
$$\tau = F\cos\theta r$$
$$\tau = (150)\cos(10.0°)(0.10)$$
$$\tau_{\text{required}} = 15\cos(10°)\,\text{N m}$$

Now use the found torque value to find the force required to turn the fan.

Data List:

- $\tau = 15\cos(10°)\,\text{N m}$
- $r = 0.20\,\text{m}$
- $F = ?\,\text{N}$

Solution:

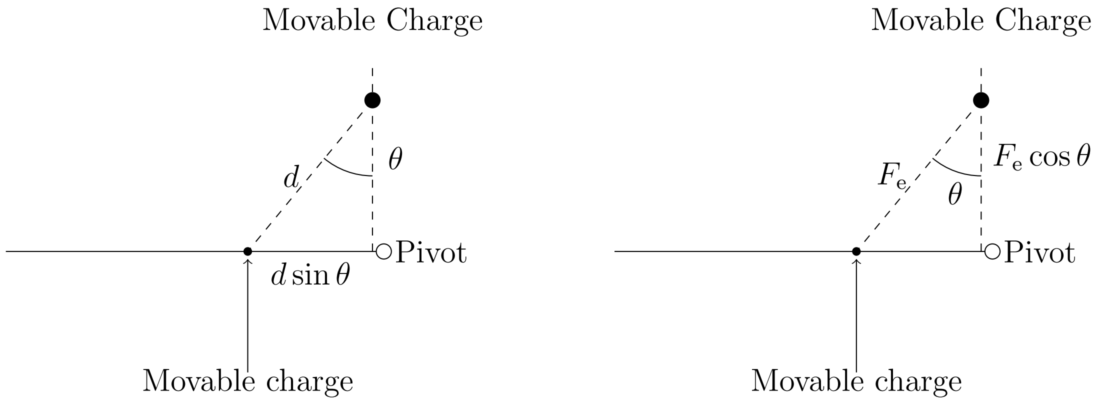
$$\tau = Fr$$
$$F = \frac{\tau}{r}$$
$$F = \frac{(15\cos(10°))}{(0.20)}$$
$$F = 73.860\ldots$$
$$F = 7.4 \times 10^1\,\text{N}$$

Question 6 Solution

Firstly, let the electrostatic force's magnitude be F_e. Since the charges of the charge carriers do not change, and the distance they are separated by does not change, and the magnitude of F_e does not change. However, as the charges are moved, two important variables of torque are changed: the distance between the fulcrum and the application of the force d, and the perpendicular component of the force. Even though the force's magnitude does not change, the force's direction changes, meaning that the perpendicular component (in this case, the vertical component) changes as θ changes.

Solution continues over the page.

From the diagram below, it can be seen that as θ changes, the distance between the fulcrum and the point of application of the force is $d \sin\theta$, and the perpendicular component of the force is $F_e \cos\theta$.



Therefore, the expression for the torque of this system can be written as so.

Data List:

- $r = d \sin\theta \, \mathrm{m}$
- $F = F_e \cos\theta \, \mathrm{N}$
- $\tau = ? \, \mathrm{N\,m}$

Solution:

$$\tau = Fr$$
$$\tau = (F_e \cos\theta)(d \sin\theta)$$
$$\tau = F_e d \sin\theta \cos\theta$$
$$\therefore \tau \propto \sin\theta \cos\theta$$

Proportionality

Proportionality refers to a relationship between two variables in which a change in one results in a proportional change in the other, with a constant ratio (or factor) between the two quantities. The simplest way to identify proportionality is to manipulate equations to the following form: $A = kB$, which shows A is proportional to B or vice versa. In the above case, the proportionality was identified as A was representing τ, k was representing $F_e r$ and B was representing $\sin\theta \cos\theta$.
The symbol $\propto$, often written as α, represents proportionality. For example, $A \propto B$ states A is proportional to B, which is an identical statement to $A = kB$.

Since τ is proportional to $\sin\theta \cos\theta$, and the maximum value of $\sin\theta \cos\theta$ is when $\theta = 45°$, τ will be greatest when $\theta = 45°$, or in other words, the lever will experience the greatest torque when $\theta = 45°$

Motion in Gravitational Fields SOLUTIONS

Conservation of Mechanical Energy SOLUTIONS

Question 1 Solution

Mechanical energy is the sum of kinetic energy and potential energy. To solve this question, let's divide the scenario into three parts:

- Stage 1 - Flight
- Stage 2 - Collision
- Stage 3 - Free Fall

Stage 1 - Flight
Before the birds collide, they are said to be flying horizontally to each other, meaning they are not vertically moving. This means their height is not changing, meaning gravitational potential energy is not lost. Furthermore, since they are both flying, they both have kinetic energy, as a mass with velocity will always have kinetic energy. These deductions imply that the mechanical energy during stage 1 will not be 0, meaning the answer cannot be (B).

Stage 2 - Collision
During the collision, the birds do not change in height. However, they will change in their velocity. By the information that they start to fall directly downwards, it is implied that the collision results in the birds entangling and, for an instantaneous moment, they become motionless. If they do not change in height, they will not change in their gravitational potential energy. However, as they initially have velocity, but then have none during the collision, they lose all of their kinetic energy. As gravitational potential energy is unchanged, and kinetic energy is lost, mechanical energy is lost, meaning the answer cannot be (C). ⁉

Stage 3 - Free Fall
After the collision, the birds are said to start falling directly downwards. That means their height is decreasing, meaning their gravitational potential energy is decreasing. However, their velocity is increasing, meaning their kinetic energy is increasing. If air friction is ignored, lost gravitational potential energy will be completely conserved and changed into kinetic energy, meaning their overall mechanical energy will be conserved and not change, meaning the answer cannot be (A), and therefore must be (D).

Conservation of Energy

The conservation of energy may be confusing, as it is shown that in stage 2, the collision phase, mechanical energy is lost and is not conserved, but in stage 3, the free fall phase, mechanical energy is not lost. How do we tell when energy is conserved and not conserved? In truth, energy is never conserved in the real world, due to the 2nd law of thermodynamics - Entropy; however, entropy is not explicitly studied in the NESA physics course. But what is important is that energy is never conserved, and is only conserved in theoretical ideal environments with a complete absence of heat loss, an impossible scenario. Then how could we tell that in stage 2, mechanical energy was lost, and in stage 3, mechanical energy was not lost? It is only by the step by step ideal reasoning that the solution employed that could clarify the situation.
Escaping the scope of this textbook, if the question pertained to a real situation, the mechanical energy as they are free falling would be decreasing, as their free fall would be impeded by the air molecules. Furthermore, the entropy of this entire system would inevitably increase, as the Kookaburras fly towards each other, their metabolic processes convert ATP into ADP, and expend the concentrated chemical potential energy of the ATP molecule towards the contractions of their muscles, resulting in heat that is dissipated into the atmosphere, which essentially "spreads out" the energy from the Kookaburra to the atmosphere as dictated by the 2nd law of thermodynamics.

Question 2 Solution

First, find the vertical displacement from position A to position B by the conservation of mechanical energy.

Data List:

- $v = 8.00\,\text{m}\,\text{s}^{-1}$
- $\Delta h = ?\,\text{m}$

Solution:

$$\begin{aligned}
\Delta U &= K \\
mg\Delta h &= \frac{1}{2}mv^2 \\
\Delta h &= \frac{v^2}{2g} \\
\Delta h &= \frac{(8.00)^2}{2(-9.8))} \\
\Delta h &= -\frac{160}{49} \\
\therefore s_{\text{A}\rightarrow\text{B}} &= -\frac{160}{49}\,\text{m}
\end{aligned}$$

The solution continues over the page.

Now find the vertical displacement from position B to the ground by projectile motion. Use the vertical component of motion.

Data List:

- $u = 8.00\sin(50.0°)\,\mathrm{m\,s^{-1}}$
- $t = 2.00\,\mathrm{s}$
- $a = -9.8\,\mathrm{m\,s^{-2}}$
- $s = ?\,\mathrm{m}$

Solution:

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
s &= (8.00\sin(50.0°))(2.00) + \frac{1}{2}(-9.8)(2.00)^2 \\
s &= 16\sin(50°) - \frac{98}{5} \\
\therefore s_{\mathrm{B}\to\mathrm{Ground}} &= 16\sin(50°) - \frac{98}{5}\,\mathrm{m}
\end{aligned}
$$

Now to find the vertical displacement from position A to the ground, sum the vertical displacement from position A to position B, and the vertical displacement from position B to the ground. Remember to give direction.

$$
\begin{aligned}
s_{\mathrm{total}} &= s_{\mathrm{A}\to\mathrm{B}} + s_{\mathrm{B}\to\mathrm{Ground}} \\
s_{\mathrm{total}} &= (-\frac{160}{49}) + (16\sin(50°) - \frac{98}{5}) \\
s_{\mathrm{total}} &= -10.608\ldots \\
\therefore s_{\mathrm{total}} &= 10.6\,\mathrm{m}\text{ downwards}
\end{aligned}
$$

Question 3 Solution

Because the drone begins on the ground with no motion, the theoretical energy required for this manoeuvre is simply the final mechanical energy.

Data List:

- $m = 0.150\,\mathrm{kg}$
- $\Delta h = 3.00\,\mathrm{m}$
- $v = 20.0\,\mathrm{m\,s^{-1}}$
- $E_{\mathrm{theoretical}} = ?\,\mathrm{J}$

Solution:

$$
\begin{aligned}
E_{\mathrm{theoretical}} &= U + K \\
E_{\mathrm{theoretical}} &= mg\Delta h + \frac{1}{2}mv^2 \\
E_{\mathrm{theoretical}} &= m(g\Delta h + \frac{1}{2}v^2) \\
E_{\mathrm{theoretical}} &= (0.150)((-9.8)(3.00) + \frac{1}{2}(20.0)^2) \\
E_{\mathrm{theoretical}} &= \frac{3441}{100}\,\mathrm{J}
\end{aligned}
$$

Now, if the manoeuvre was taken out at 36.5% efficiency, that means only 36.5% of the energy taken was required for the manoeuvre, and the rest was losses.

The solution continues over the page.

$$
\begin{aligned}
E_{\text{used}} \times 36.5\% &= E_{\text{theoretical}} \\
E_{\text{used}} &= \frac{E_{\text{theoretical}}}{36.5\%} \\
E_{\text{used}} &= 94.273\ldots \\
E_{\text{used}} &= 94.3\,\text{J}
\end{aligned}
$$

Question 4 Solution

First, find the total energy the boulder can supply without losses by calculating its gravitational potential energy.

Data List:

- $m = 2000.0\,\text{kg}$
- $\Delta h = 310\,\text{m}$
- $E = ?\,\text{J}$

Solution:

$$
\begin{aligned}
E &= U \\
E &= mg\Delta h \\
E &= (2000.0)(9.8)(310) \\
E &= 6076000\,\text{J}
\end{aligned}
$$

Now apply the efficiency of 99.9%.

$$
\begin{aligned}
E_{\text{retrieved}} &= 99.9\% \times E \\
E_{\text{retrieved}} &= 99.9\% \times 6076000 \\
E_{\text{retrieved}} &= 6069924\,\text{J}
\end{aligned}
$$

Now divide this retrieved energy by the energy requirement of a household to see the duration the boulder will power the house for.

$$
\begin{aligned}
t &= \frac{E_{\text{retrieved}}}{E_{\text{per second}}} \\
t &= \frac{6069924}{1500} \\
t &= 4046.616 && \text{(seconds)} \\
t &= 67.443\ldots && \text{(minutes)} \\
t &= 67.4\,\text{min}
\end{aligned}
$$

⁉ Dimensional Analysis

It may be initially confusing why the energy retrieved was divided by the energy required to power the house for a second to get the time the boulder can power the house for in seconds. Along with many other counterintuitive calculations, the steps we take to the solution may seem unexplained. However, something that does show, more intuitively, what the numbers on the page represent is dimensional analysis. Dimensional analysis involves understanding the interactions between units of measurement during calculations, and may be the separating factor between two physics students. Although outside the syllabus, dimensional analysis can be used to show the logic behind calculations such as the one in **Solution 5**.

$$E_{\text{required}} = 1500\,\text{W}$$
$$E_{\text{required}} = 1500\,\text{J}\,\text{s}^{-1}$$

$$E_{\text{retrieved}} = 6069924\,\text{J}$$

$$t = \frac{E_{\text{retrieved}}}{E_{\text{required}}}$$
$$t = \frac{6069924\,\text{J}}{1500\,\text{J}\,\text{s}^{-1}}$$
$$t = 4046.616\,\text{s} \qquad (J\text{'s cancel})$$
$$t = 4046.616\,\text{s} \times \frac{1\,\text{min}}{60\,\text{s}} \qquad \left(\frac{1\,\text{min}}{60\,\text{s}} = 1\right)$$
$$t = 67.433\ldots\ \text{min}$$

As it can be seen, the units can be utilised flexibly within calculations to arrive at the correct unit of measurements, and to even alter the units of measurements the answer is in.

Question 5 Solution

Firstly, find the volume of fuel used:

$$\text{Fuel Used} = \text{Fuel}_{\text{final}} - \text{Fuel}_{\text{initial}}$$
$$\text{Fuel Used} = \frac{3}{4}(60.6) - \frac{1}{5}(60.6)$$
$$\text{Fuel Used} = 33.33\,\text{L}$$

Now the mass of the fuel can be found:

$$\text{Fuel Mass} = \text{Fuel Used} \times \text{Fuel Density}$$
$$\text{Fuel Mass} = (33.33) \times (0.735)$$
$$\text{Fuel Mass} = 24.497\ldots\ \text{kg}$$

And now the energy output can be calculated from the mass of the fuel, along with the efficiency of the Altima:

$$E_{\text{output}} = \text{Efficiency} \times E_{\text{input}}$$
$$E_{\text{output}} = (30\%) \times (\text{Fuel Mass})(\text{Fuel Energy Density})$$
$$E_{\text{output}} = 30\% \times (24.497\ldots)(42.8 \times 10^6)$$
$$E_{\text{output}} = 314548542\,\text{J}$$

Now, an expression of the initial and final energy must be found:
(Note the usage of m_{i} and m_{f}. This is because there is a discrepancy between the initial and final mass of the Altima due to the mass of the fuel that is used.)

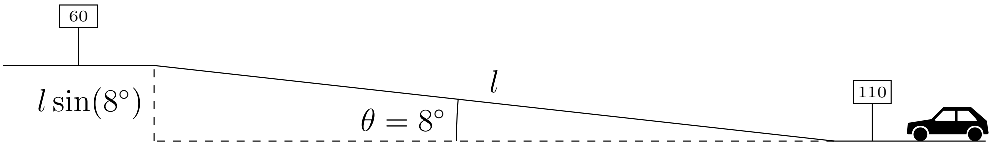
$$E_{\text{i}} = m_{\text{i}} g \Delta h + \frac{m_{\text{i}} v_{\text{i}}^2}{2}$$
$$E_{\text{i}} = \frac{m_{\text{i}} v_{\text{i}}^2}{2} \quad (\Delta h = 0)$$

$$E_{\text{f}} = m_{\text{f}} g \Delta h + \frac{m_{\text{f}} v_{\text{f}}^2}{2}$$
$$E_{\text{f}} = m_{\text{f}}(g \Delta h + \frac{v_{\text{f}}^2}{2})$$
$$E_{\text{f}} = m_{\text{f}}(\frac{2g\Delta h + v_{\text{f}}^2}{2})$$
$$E_{\text{f}} = \frac{m_{\text{f}}(2g\Delta h + v_{\text{f}}^2)}{2}$$

There are also more definitions of variables that must be derived prior to finding the length of the highway:

$$m_{\text{i}} = m_{\text{Altima}} + m_{\text{Initial Fuel}}$$
$$m_{\text{i}} = 1451 + (\text{Fuel Volume})(\text{Fuel Density})$$
$$m_{\text{i}} = 1451 + \left(\frac{3}{4}(60.6)\right)(0.735)$$
$$\therefore m_{\text{i}} = 1484.405\ldots\ \text{kg}$$
$$v_{\text{i}} = 110\,\text{km}\,\text{h}^{-1}$$
$$\therefore v_{\text{i}} = 30.555\ldots\ \text{m}\,\text{s}^{-1}$$
$$m_{\text{f}} = m_{\text{Altima}} + m_{\text{Final Fuel}}$$
$$m_{\text{f}} = 1451 + (\text{Fuel Volume})(\text{Fuel Density})$$
$$m_{\text{f}} = 1451 + (\frac{1}{5}(60.6))(0.735)$$
$$\therefore m_{\text{f}} = 1459.908\ldots\ \text{kg}$$
$$v_{\text{f}} = 60\,\text{km}\,\text{h}^{-1}$$
$$\therefore v_{\text{f}} = 16.666\ldots\ \text{m}\,\text{s}^{-1}$$

And most importantly, an expression for Δh must be found: $\Delta h = l\sin(8°)$ where l is the length of the highway. This may seem clearer in the diagram provided on the following page:



Now that all of the variables are found, the length can be algebraically found.

Data List:

- $E_{\text{output}} = 314548542\,\text{J}$
- $m_\text{i} = 1484.405\ldots\,\text{kg}$
- $m_\text{f} = 1459.908\ldots\,\text{kg}$
- $v_\text{i} = 30.555\ldots\,\text{m}\,\text{s}^{-1}$
- $v_\text{f} = 16.666\ldots\,\text{m}\,\text{s}^{-1}$
- $\Delta h = l\sin(8°)$
- $l = ?\,\text{m}$

Solution:

$$E_{\text{output}} = E_\text{f} - E_\text{i}$$
$$E_{\text{output}} = \frac{m_\text{f}(2g\Delta h + v_\text{f}^2)}{2} - \frac{m_\text{i}v_\text{i}^2}{2}$$
$$E_{\text{output}} = \frac{m_\text{f}(2gl\sin(8°) + v_\text{f}^2) - m_\text{i}v_\text{i}^2}{2}$$
$$2E_{\text{output}} = m_\text{f}(2lg\sin(8°) + v_\text{f}^2) - m_\text{i}v_\text{i}^2$$
$$m_\text{f}(2lg\sin(8°) + v_\text{f}^2) = 2E_{\text{output}} + m_\text{i}v_\text{i}^2$$
$$2lg\sin(8°) + v_\text{f}^2 = \frac{2E_{\text{output}} + m_\text{i}v_\text{i}^2}{m_\text{f}}$$
$$2lg\sin(8°) = \frac{2E_{\text{output}} + m_\text{i}v_\text{i}^2}{m_\text{f}} - v_\text{f}^2$$
$$l = \frac{\frac{2E_{\text{output}} + m_\text{i}v_\text{i}^2}{m_\text{f}} - v_\text{f}^2}{2g\sin(8°)}$$
$$l = \frac{\frac{2(314548542) + (1484.405...)(30.555...)^2}{(1459.908...)} - (16.666\ldots)^2}{2(9.8)\sin(8°)}$$
$$l = 158218.411\ldots \quad (\text{m})$$
$$l = 158.218\ldots \quad (\text{km})$$
$$l = 158\,\text{km}$$

Question 6** Solution

This question is an adaptation of the discovery made by Professor Gregory Galperin in 1995, published in 2003. This startling discovery of colliding blocks computing digits of π was later popularised by Professor Sheldon Glashow, and more recently Grant Sanderson (3Blue1Brown) in his 2019 video "The most unexpected answer to a counting puzzle". For a more elegant and visual explanation of this phenomenon, see 3Blue1Brown's video "Why colliding blocks compute pi" on YouTube.

The two most important equations in examining this scenario are the conservation of (kinetic) energy and the conservation of momentum equations. To allow for data collection (this will become clear later), we should also derive the formula to determine the velocity of two masses after elastic collisions as shown on the following page.

$$
\begin{aligned}
K_\text{i} &= K_\text{f} \\
\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 &= \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \\
m_1u_1^2 - m_1v_1^2 &= m_2v_2^2 - m_2u_2^2 \\
m_1(u_1^2 - v_1^2) &= m_2(v_2^2 - u_2^2) \\
m_1(u_1 - v_1)(u_1 + v_1) &= m_2(v_2 - u_2)(v_2 + u_2) \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
p_\text{i} &= p_\text{f} \\
m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\
m_1u_1 - m_1v_1 &= m_2v_2 - m_2u_2 \\
m_1(u_1 - v_1) &= m_2(v_2 - u_2) \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
&\therefore (5) \text{ into } (4) \\
\Rightarrow \quad m_2(v_2 - u_2)(u_1 + v_1) &= m_2(v_2 - u_2)(v_2 + u_2) \\
u_1 + v_1 &= v_2 + u_2 \\
v_1 &= v_2 + u_2 - u_1 \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\
\text{Using (6), } m_1u_1 + m_2u_2 &= m_1(v_2 + u_2 - u_1) + m_2v_2 \\
m_1u_1 + m_2u_2 &= m_1v_2 + m_1u_2 - m_1u_1 + m_2v_2 \\
(m_1 + m_2)v_2 &= 2m_1u_1 + (m_2 - m_1)u_2 \\
v_2 &= \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \\
\text{Similarly, } v_1 &= \frac{m_1 - m_2}{m_2 + m_1}u_1 + \frac{2m_2}{m_2 + m_1}u_2
\end{aligned}
$$

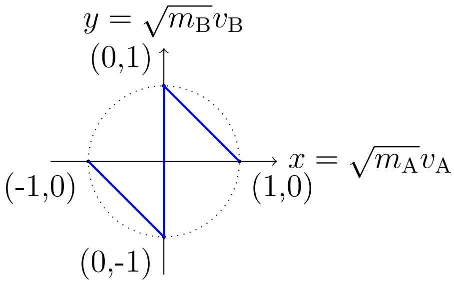
Now let us return to the question. As the system only evolves through elastic collisions, kinetic energy is conserved. Hence, we may state,

$$
\begin{aligned}
\Sigma E_\text{kinetic} &= \text{Constant} \\
\frac{1}{2}m_\text{A}v_\text{A}^2 + \frac{1}{2}m_\text{B}v_\text{B}^2 &= c_1 \\
m_\text{A}v_\text{A}^2 + m_\text{B}v_\text{B}^2 &= c_2 \quad (c_2 = 2c_1)
\end{aligned}
$$

Now note that this equation is close to the equation of a circle; therefore, we may state,

$$
\begin{aligned}
(\sqrt{m_\text{A}}v_\text{A})^2 + (\sqrt{m_\text{B}}v_\text{B})^2 &= c \\
x^2 + y^2 &= c \quad (x = \sqrt{m_\text{A}}v_\text{A}, y = \sqrt{m_\text{B}}v_\text{B})
\end{aligned}
$$

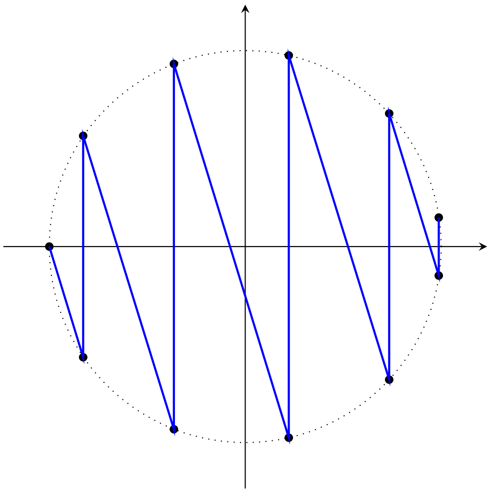
Now let us examine a specific case where $m_\text{A} = m_\text{B} = 1\,\text{kg}$, the initial speed of puck A is unit speed, and the left direction is taken as the negative. Initially, puck A will be travelling to the left at unit speed, then like a Newton's cradle, puck A will transfer all of its kinetic energy to puck B, resulting in puck B travelling left at unit speed, then reflecting off the wall to travel right at unit speed, and finally transferring all of its kinetic energy to puck A for it to travel right at unit speed. If we describe such interactions by points of a 2D space where ($x = \sqrt{m_{\text{A}}}v_{\text{A}}, y = \sqrt{m_{\text{B}}}v_{\text{B}}$), the previously described chain of events can be described by the points $(-1,0),(0,-1),(0,1),(1,0)$. This may be plotted as so,



Although this may not resemble a circle, note that every point lies on a circle of centre $(0,0)$ and radius 1, as clarified by the dotted circle. For a more striking set of results, let us examine the case where $m_{\text{A}} = 10.5\,\text{kg}$. Note that collecting the following data by hand would have been extremely time-consuming if done by hand, so an example Python file to automatically simulate the process has been provided below.

```
import math

# Input Masses
m2 = float(input("Enter␣mass␣"))
m1 = 1.0

# Initial Velocities
v1 = 0.0      # small block (at wall)
v2 = -1.0     # big block (moving left)

data = [(math.sqrt(m2)*v2, math.sqrt(m1)*v1)]
collisions = 0

while True:
  # Block to Block Collision
  u1, u2 = v1, v2
  v1 = ((m1 - m2)/(m1 + m2))*u1 + ((2*m2)/(m1 + m2))*u2
  v2 = ((2*m1)/(m1 + m2))*u1 + ((m2 - m1)/(m1 + m2))*u2
  collisions += 1
  data.append((math.sqrt(m2)*v2, math.sqrt(m1)*v1))

  # Wall Collision
  if v1 < 0:
    v1 = -v1
    collisions += 1
    data.append((math.sqrt(m2)*v2, math.sqrt(m1)*v1))

  # Stopping Condition
  if v2 > 0 and v2 > v1:
    break

print("Collisions:", collisions)
print("Data:", data)
```

For when $m_{\mathrm{A}} = 10.5\,\mathrm{kg}$,

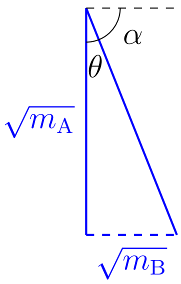


Now, the start of this zig-zag pattern starts at the negative x-axis, as initially only puck A is moving to the left. Then, the pattern moves diagonally to the bottom right, as when both pucks collide, puck A will always accelerate (instantaneously in this model) to the right and puck B will always accelerate to the left, hence the diagonal of the pattern moves to the right, and downwards. The vertical lines represent puck B colliding with the wall, where the sign of its velocity is flipped, causing the plot to reflect about the y-axis. Notice that the final collision appears to occur before the trajectory would "escape" the circle. If another collision were to occur, the diagonal continuation would lie outside the circle. This is not a coincidence: the circle represents the conservation of kinetic energy (with radius proportional to the square root of the system's total energy). Since collisions must obey energy conservation, no point can ever leave the circle. Thus, the final collision happens exactly when the next would otherwise require a violation of energy conservation.

Now let us examine why the gradient of this pattern seems to be constant across all diagonals. Firstly, we will begin with the conservation of momentum.

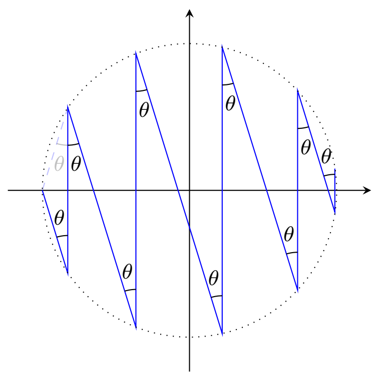
$$\begin{aligned}
\Sigma p &= c_1 \quad \text{arbitrary constant} \\
m_{\mathrm{A}}v_{\mathrm{A}} + m_{\mathrm{B}}v_{\mathrm{B}} &= c_1 \\
\sqrt{m_{\mathrm{A}}}(\sqrt{m_{\mathrm{A}}}v_{\mathrm{A}}) + \sqrt{m_{\mathrm{B}}}(\sqrt{m_{\mathrm{B}}}v_{\mathrm{B}}) &= c_1 \\
\sqrt{m_{\mathrm{A}}}x + \sqrt{m_{\mathrm{B}}}y &= c_1 \\
y &= -\sqrt{\frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}}x + c_2 \quad (c_2 = \frac{c_1}{\sqrt{m_{\mathrm{B}}}}) \\
\therefore \text{slope} &= -\sqrt{\frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}}
\end{aligned}$$

Now that we know the mathematical expression of the slope of the diagonals, we can work with a purely geometric interpretation of this question. In the example previously provided, we can find the angle made by the top of each diagonal as depicted on the following page.

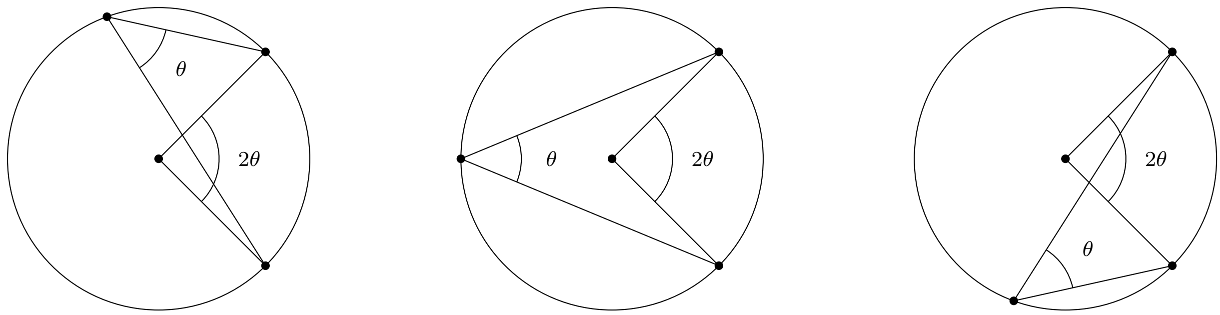


$$\tan\theta = \frac{\sqrt{m_B}}{\sqrt{m_A}}$$
$$\theta = \tan^{-1}\left(\sqrt{\frac{m_B}{m_A}}\right)$$

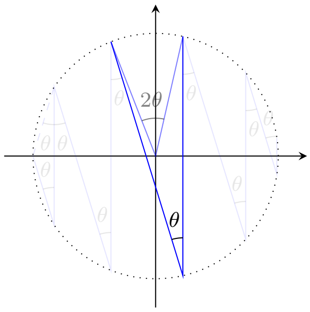
And since the slope is constant for all diagonals, we can deduce that the angle θ subtends all arcs between two subsequent collision points of the circle as shown.



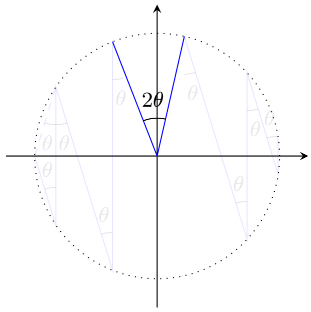
Now we can simplify this diagram using the inscribed angle theorem, which states that for any three points on a circle's circumference A, B and C of centre O, $\angle AOC = 2\angle ABC$. This is shown visually below.

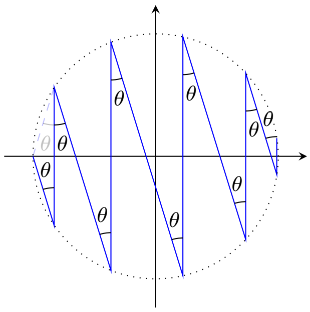


The application of the inscribed angle theorem, isolated on one arc in our example, is shown on the following page.

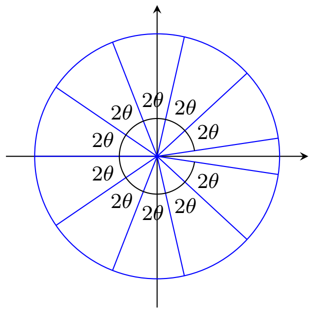


Becomes

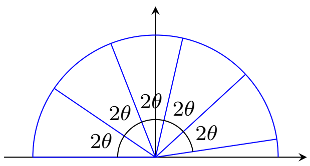




Becomes



To simplify our analysis, let us examine the positive side of the y-axis, as the graph is symmetric about the y-axis.



As can be seen, the number of arcs (or angles) is maximised before the next arc would go below the y-axis. Now, note that for every collision that occurs, an angle is created. Since every collision that is plotted at the top of the circle has a corresponding collision at the bottom half of the circle, the total number of collisions can be mathematically expressed as shown on the following page.

$$\text{Number of collisions} = 2 \times \text{Number of collisions at top half of circle}$$
$$\text{Number of collisions} = 2 \times \text{Number of angles}$$
$$\text{Number of collisions} = 2 \times \frac{\pi}{2\theta} \text{ rounded down}$$
$$\text{Number of collisions} = \frac{\pi}{\theta} \text{ rounded down}$$
$$\text{Number of collisions} = \left\lfloor \frac{\pi}{\theta} \right\rfloor$$
$$\text{Number of collisions} = \left\lfloor \frac{\pi}{\tan^{-1}\left(-\sqrt{\frac{m_\mathrm{B}}{m_\mathrm{A}}}\right)} \right\rfloor$$
$$\text{Now for small } x,\ \tan^{-1} x \approx x \qquad \text{(Small angle approximation)}$$
$$\therefore \text{Number of collisions} = \left\lfloor \frac{\pi}{\left(\sqrt{\frac{m_\mathrm{B}}{m_\mathrm{A}}}\right)} \right\rfloor \qquad (m_\mathrm{A} \text{ is sufficiently large})$$
$$\text{Number of collisions} = \lfloor \pi\sqrt{m_\mathrm{A}} \rfloor \qquad (m_\mathrm{B} = 1) \qquad \text{(First part of question)}$$
$$\text{Now assume that } m_\mathrm{A} = 100^n, n \in \mathbb{Z}^+$$
$$\therefore \text{Number of collisions} = \left\lfloor \pi\sqrt{100^n} \right\rfloor$$
$$\text{Number of collisions} = \left\lfloor \pi(10^2 n)^{\frac{1}{2}} \right\rfloor$$
$$\text{Number of collisions} = \lfloor 10^n \pi \rfloor$$

And thus it naturally follows that when $m_\mathrm{A} = 100^n\,\mathrm{kg}, n \in \mathbb{Z}^+$, the number of collisions follows n digits of the mathematical constant π.

Newton's Law of Universal Gravitation SOLUTIONS

Question 1 Solution

Data List:

- $m_1 = 103\,\text{kg}$
- $m_2 = 82\,\text{kg}$
- $r = 5.00\,\text{m}$
- $F = ?\,\text{N}$

Solution:

$$F = G\frac{m_1 m_2}{r^2}$$
$$F = (6.67 \times 10^{-11})\frac{(103)(82)}{(5.00)}$$
$$F = 2.253\ldots \times 10^{-8}$$
$$F = 2.3 \times 10^{-8}\,\text{N}$$

Question 2 Solution

Find an expression for both pairs' gravitational attraction.

Emmy & Amanda

$$F = G\frac{m_1 m_2}{r^2}$$
$$F = G\frac{(a)(\frac{b}{2})}{(r)^2}$$
$$F = G\frac{ab}{2r^2}$$
$$F_{\text{E\&A}} = \frac{1}{2}(G\frac{ab}{r^2})$$

Enrico & Hans

$$F = G\frac{m_1 m_2}{r^2}$$
$$F = G\frac{(2a)(b)}{(2r)^2}$$
$$F = G\frac{2ab}{4r^2}$$
$$F_{\text{E\&H}} = \frac{1}{2}(G\frac{ab}{r^2})$$

$$\therefore F_{\text{E\&A}} = F_{\text{E\&H}}$$

Therefore, the ratio between their forces is 1:1.

Question 3 Solution

Find an expression for both attractive forces, from planet A and planet B. Let an imaginary mass m be at position C.

Planet A

$$F = G\frac{Mm}{r^2}$$
$$F = G\frac{(a)m}{(\frac{3}{4}r)^2}$$
$$F = G\frac{am}{\frac{9}{16}r^2}$$
$$F = \frac{16}{9}(G\frac{am}{r^2})$$

Planet B

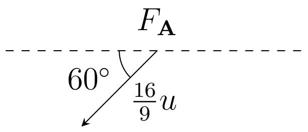
$$F = G\frac{Mm}{r^2}$$
$$F = G\frac{(9a)m}{(r)^2}$$
$$F = G\frac{9am}{r^2}$$
$$F = 9(G\frac{am}{r^2})$$

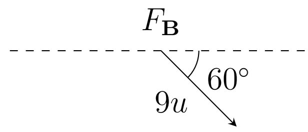
The solution continues over the page.

$$\text{Let } G\frac{am}{r^2} = u$$

$$\therefore F_{\text{A}} = \frac{16}{9}u \qquad\qquad \therefore F_{\text{B}} = 9u$$

Now, we know F_{A} is acting at an angle of $\angle 240°$, and F_{B} at an angle of $\angle 300°$. Since the magnitude and direction of both forces are known, vector addition can be carried out. Firstly, the forces must be resolved to their horizontal and vertical components.





$$F_{\text{A, x}} = \frac{16}{9}\cos(60°) \text{ left}$$
$$F_{\text{A, y}} = \frac{16}{9}\sin(60°) \text{ down}$$

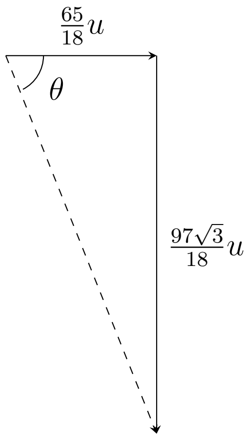
$$F_{\text{B, x}} = 9u\cos(60°) \text{ right}$$
$$F_{\text{B, y}} = 9u\sin(60°) \text{ down}$$

Now, vector addition can be done.

$$\begin{aligned} \Sigma F_{\text{x}} &= F_{\text{A, x}} + F_{\text{B, x}} \\ &= -\frac{16}{9}\cos(60°) + 9u\cos(60°) \\ &= \frac{65}{9}u\cos(60°) \\ &= \frac{65}{18}u \end{aligned} \qquad \text{Let right be positive}$$

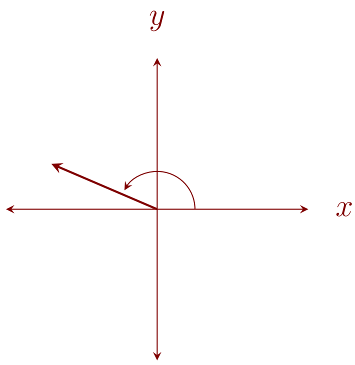
$$\begin{aligned} \Sigma F_{\text{y}} &= F_{\text{A, y}} + F_{\text{B, y}} \\ &= \frac{16}{9}\sin(60°) + 9u\sin(60°) \\ &= \frac{97}{9}u\sin(60°) \\ &= \frac{97\sqrt{3}}{18}u \end{aligned} \qquad \text{Let down be positive}$$

Therefore, the resultant vector can be drawn as shown on the following page, and from the visualisation, the direction of the resultant force can be found.



$$\tan\theta = 97\sqrt{3}18u \div \frac{65}{18}u$$
$$\tan\theta = \frac{97\sqrt{3}}{65}$$
$$\theta = \tan^{-1}(\frac{97\sqrt{3}}{65})$$
$$\therefore \text{Polar Angle} = 360 - \theta$$
$$\text{Polar Angle} = 291.150\ldots$$
$$\text{Polar Angle} = 291^\circ$$

Polar Angles



Polar angles are used to describe the direction of a vector. Polar angles can be thought similar to true bearings, however true bearings are described to be measured from North clockwise. Instead, polar angles are measured in the anticlockwise direction from the positive x axis.

Question 4 Solution

First, find the mass of Jupiter utilising the formula to find the volume of a sphere.

$$m_{\text{Jupiter}} = \text{Density}_{\text{Jupiter}} \times \text{Volume}_{\text{Jupiter}}$$
$$m_{\text{Jupiter}} = 1326 \times (\frac{4}{3}\pi r^3)$$
$$m_{\text{Jupiter}} = 1326 \times \frac{4}{3}\pi(6.991 \times 10^7)^3$$
$$m_{\text{Jupiter}} = 1.897\ldots \times 10^{27} \text{ kg}$$

Now find the gravitational field strength, or gravitational acceleration at the surface of Jupiter ⁉

Data List:

- $M = 1.897\ldots \times 10^{27}$ kg
- $r = 6.991 \times 10^7$ m
- $g = ?\,\text{N kg}^{-1}$

Solution:

$$g = \frac{GM}{r^2}$$
$$g = \frac{(6.67e-11)(1.897\ldots \times 10^{27})}{(6.991 \times 10^7)^2}$$
$$g = 25.899\ldots$$
$$g = 2.590 \times 10^1\,\text{N kg}^{-1}$$

⁉ Gravitational Field Strength

Gravitational field strength is a quantity that is very close to the gravitational force experienced. Let the force experienced due to gravity be F_g. Now, by Newton's second law, $a = \frac{F_g}{m}$, with a representing gravitational acceleration. As $F_g = G\frac{Mm}{r^2}$, a must equal to $\frac{G\frac{Mm}{r^2}}{m}$, or simply $G\frac{M}{r^2}$, measured in m s^{-2}. Finding the gravitational acceleration is important to finding the gravitational field strength as they are equivalent quantities. Much like how the voltage, V is equivalent to the unit J C^{-1}, gravitational acceleration, measured in m s^{-2} is equivalent to gravitational field strength, measured in N kg^{-1}. This is algebraically shown below.

$$\text{N kg}^{-1} = (\text{kg m s}^{-2})(\text{kg}^{-1}) \qquad (\text{N} = \text{kg m s}^{-2})$$
$$\text{N kg}^{-1} = \text{m s}^{-2}$$

Question 5 Solution

From the general shape of the graph, it is clear that the answer is either (C) or (D). However, it cannot be determined if F_g is proportional to $\frac{1}{r}$ or $\frac{1}{r^2}$ graphically. Instead, it can be algebraically determined.

$$F_\mathrm{g} = G\frac{m_1 m_2}{r^2}$$
$$F_\mathrm{g} \propto \frac{1}{r^2}$$
$$\therefore \text{The answer is (D).}$$

Question 6 Solution

This graph has plotted F_g against $\frac{1}{r^2}$. Therefore, the gradient of the graph represents $\frac{F_\mathrm{g}}{\frac{1}{r^2}} = F_\mathrm{g}r^2$. What $F_\mathrm{g}r^2$ represents can be found algebraically.

$$F_\mathrm{g} = G\frac{m_1 m_2}{r^2}$$
$$F_\mathrm{g}r^2 = Gm_1 m_2$$

Therefore, $F_\mathrm{g}r^2$ represents the product of the masses multiplied by the gravitational constant. Now, the gradient of the graph must be found. For this, two easily readable points should be taken from the line, the solution will use the points (3.0, 2.65) and (9.0, 7.95).

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\text{gradient} = \frac{(7.95 \times 10^{21}) - (2.65 \times 10^{21})}{(9.0 \times \frac{1}{9\times 10^{23}}) - (3.0 \times \frac{1}{9\times 10^{23}})}$$
$$\text{gradient} = 7.95 \times 10^{44}\,\mathrm{N\,m^2}$$

Now use this value to find the product of the masses.

Data List:

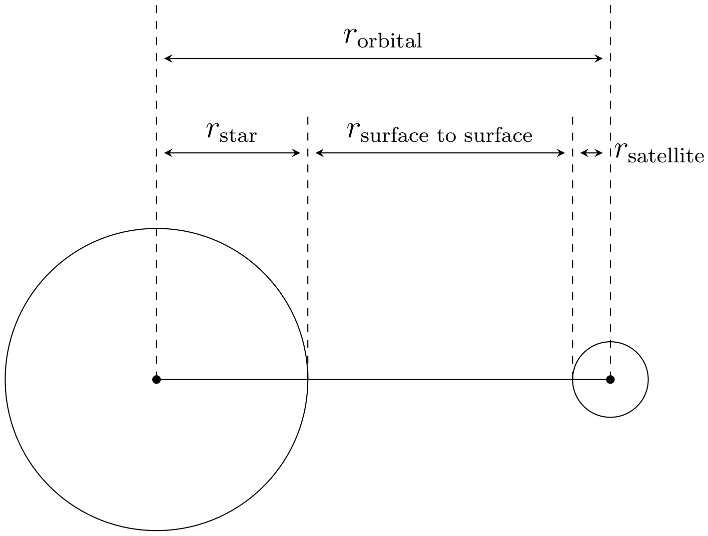
- $F_\mathrm{g}r^2 = 7.95 \times 10^{44}\,\mathrm{kg^2}$
- $m_1 m_2 = ?\,\mathrm{kg^2}$

Solution:

$$F_\mathrm{g}r^2 = Gm_1 m_2$$
$$m_1 m_2 = \frac{F_\mathrm{g}r^2}{G}$$
$$m_1 m_2 = \frac{7.95 \times 10^{44}}{6.67 \times 10^{-11}}$$
$$m_1 m_2 = 1.191\ldots \times 10^{55}$$
$$m_1 m_2 = 1.19 \times 10^{55}\,\mathrm{kg^2}$$

Question 1 Solution

First, find the orbital radius as shown, keeping in mind that the radius is $\frac{1}{2}$ of the diameter.



$$\therefore r_{\text{orbital}} = r_{\text{surface to surface}} + r_{\text{star}} + r_{\text{satellite}}$$
$$r_{\text{orbital}} = 4598390 \times 10^3 + \frac{1894560}{2} \times 10^3 + \frac{9420}{2} \times 10^3$$
$$r_{\text{orbital}} = 5.55038 \times 10^9\,\text{m}$$

Now use the orbital radius to find the orbital period of the satellite.

Data List:

- $r = 5.55038 \times 10^9\,\text{m}$
- $M = 2.83 \times 10^{31}\,\text{kg}$
- $T = ?\,\text{s}$

Solution:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$
$$\frac{1}{T^2} = \frac{GM}{4\pi^2 r^3}$$
$$T^2 = \frac{4\pi^2 r^3}{GM}$$
$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} \qquad (T \geq 0)$$
$$T = \sqrt{\frac{4\pi^2 (5.55038 \times 10^9)^3}{(6.67 \times 10^{-11})(2.83 \times 10^{31})}}$$
$$T = 59800.912\ldots$$
$$T = 5.98 \times 10^4\,\text{s}$$

Question 2 Solution

As Eworb and the satellite of Bvzerk orbit the same large mass, their values for $\frac{GM}{4\pi^2}$ will be equal, as it is only dependent on the large mass.

$$
\begin{aligned}
\frac{r_{\text{Eworb}}^3}{T_{\text{Eworb}}^2} &= \frac{GM}{4\pi^2} = \frac{r_{\text{satellite}}^3}{T_{\text{satellite}}^2} \\
\therefore \frac{r_{\text{Eworb}}^3}{T_{\text{Eworb}}^2} &= \frac{r_{\text{satellite}}^3}{T_{\text{satellite}}^2} \\
T_{\text{satellite}}^2 &= \frac{r_{\text{satellite}}^3 T_{\text{Eworb}}^2}{r_{\text{Eworb}}^3} \\
T_{\text{satellite}}^2 &= \frac{(2.3 \text{ zylecks})^3 (1 \text{ ewa})^2}{(1 \text{ zyleck})^3} \\
T_{\text{satellite}}^2 &= 12.167 \text{ ewas}^2 \\
T_{\text{satellite}} &= \sqrt{12.167} \text{ ewas} \qquad (T \geq 0) \\
T_{\text{satellite}} &= 3.488\ldots \\
T_{\text{satellite}} &= 3.49 \text{ ewas}
\end{aligned}
$$

Question 3 Solution

First, convert the 365.25 days orbital period to seconds.

$$
\begin{aligned}
T_{\text{orbital}} &= 365.25\,\text{d} \\
T_{\text{orbital}} &= 365.25\,\text{d} \times \frac{24\,\text{h}}{1\,\text{d}} \times \frac{60\,\text{min}}{1\,\text{h}} \times \frac{60\,\text{s}}{1\,\text{min}} \\
T_{\text{orbital}} &= 31557600\,\text{s}
\end{aligned}
$$

Now find the mass of the sun, taking the required data values from the reference sheet.

Data List:

- $r = 149.6 \times 10^9\,\text{m}$
- $T = 31557600\,\text{s}$
- $M = ?\,\text{kg}$

Solution:

$$
\begin{aligned}
\frac{r^3}{T^2} &= \frac{GM}{4\pi^2} \\
M &= \frac{4\pi^2 r^3}{GT^2} \\
M &= \frac{4\pi^2 (149.6 \times 10^9)^3}{(6.67 \times 10^{-11})(31557600)^2} \\
M &= 1.989\ldots \times 10^{30} \\
M &= 1.990 \times 10^{30}\,\text{kg}
\end{aligned}
$$

Question 4 Solution

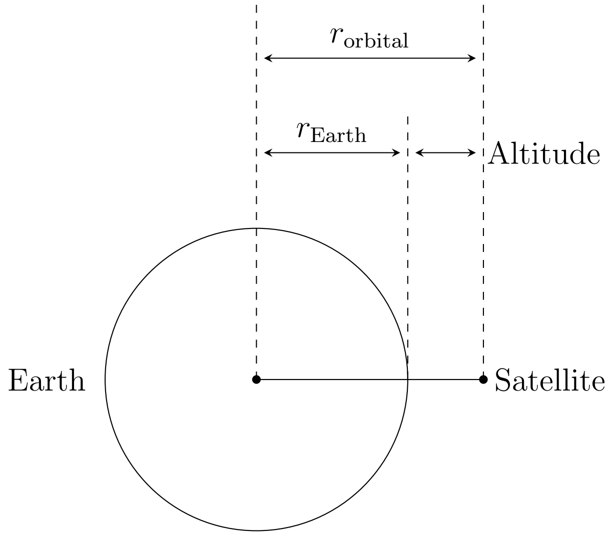
Firstly, find an expression for orbital velocity with Kepler's law.

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$

$$\therefore \frac{r^3}{(\frac{2\pi r}{v})^2} = \frac{GM}{4\pi^2}$$
$$\frac{r^3}{\frac{4\pi^2 r^2}{v^2}} = \frac{GM}{4\pi^2}$$
$$v^2 = \frac{GM}{r}$$
$$v = \sqrt{\frac{GM}{r}} \qquad (v \geq 0)$$

Now find the orbital radius of the ISS.



$$\begin{aligned} r_{\text{orbital}} &= r_{\text{Earth}} + \text{Altitude} \\ &= 6.371 \times 10^6 + 408 \times 10^3 \\ &= 6.779 \times 10^6\,\text{m} \end{aligned}$$

Now use the previous expression for orbital velocity to solve the question.

Data List:

- $M = 6.0 \times 10^{24}\,\text{kg}$
- $r = 6.779 \times 10^6\,\text{m}$
- $v = ?\,\text{m}\,\text{s}^{-1}$

Solution:

$$v = \sqrt{\frac{GM}{r}}$$
$$v = \sqrt{\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.779 \times 10^6)}}$$
$$v = 7683.440\ldots$$
$$v = 7.68 \times 10^3\,\text{m}\,\text{s}^{-1}$$

Question 5 Solution

The ratio of the orbital velocity of the orbital period is 2:1, meaning $\frac{v}{T} = 2$. Even though this is the only given data, by equating Newton's law of universal gravitation and Kepler's third law of planetary motion, the gravitational field strength, $\frac{F_\text{g}}{m}$ or a_g can be found.

$$F_\text{g} = \frac{GMm}{r^2} \qquad\qquad \frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$GM = \frac{F_\text{g} r^2}{m} \qquad\qquad GM = \frac{4\pi^2 r^3}{T^2}$$

$$\begin{aligned}
\therefore \frac{F_\text{g} r^2}{m} &= \frac{4\pi^2 r^3}{T^2} \\
\frac{F_\text{g}}{m} &= \frac{4\pi^2 r}{T^2} \\
\frac{F_\text{g}}{m} &= \frac{2\pi}{T} \times \frac{2\pi r}{T} \\
\frac{F_\text{g}}{m} &= \frac{2\pi v}{T} \\
\frac{F_\text{g}}{m} &= 2\pi(2) \\
a_\text{g} &= 4\pi \,\text{N kg}^{-1}
\end{aligned}$$

Question 6* Solution

Firstly, the orbital period of moon$_{23}$ must be found. Instead of manually multiplying the orbital period up to moon$_{23}$, a pattern can be found.

$$2T_1 = T_2 \implies nT_1 = T_n \text{ if } n = 2$$

$$\begin{aligned}
3T_2 &= T_3 \\
6T_1 &= T_3 \implies n(n-1)T_1 = T_n \text{ if } n = 3
\end{aligned}$$

$$\begin{aligned}
4T_3 &= T_4 \\
24T_1 &= T_4 \implies n(n-1)(n-2)T_1 = T_n \text{ if } n = 4
\end{aligned}$$

$$\therefore n!T_1 = T_n, \quad \forall n \in \mathbb{Z}^+$$

$$\begin{aligned}
\therefore \text{ When } n &= 23: \\
T_{23} &= 23!T_1
\end{aligned}$$

Mathematical Notation

What was used just then is called standard mathematical notation. This level of standard mathematical notation can be seen in the HSC courses of mathematics extension 2, and sometimes in mathematics extension 1. Standard notation at a more advanced level allows for a standard language and set of terms and symbols that allow for the communication of mathematical ideas through a unified and clear method. For example, $\forall$ means "for all", $\in$ means "is an element of", and $\mathbb{Z}^+$ means "set of positive integers". Now, the clause $n!T_1 = T_n, \quad \forall n \in \mathbb{Z}^+$ can be read as "$n!T_1$ equals T_n for all n that is an element of the set of positive integers".

Now that the orbital period of moon$_{23}$ is found in terms of moon$_1$, their orbital radii can be compared. This is because both moon$_1$ and moon$_{23}$ orbit Planet X, meaning they orbit the same large mass. By Kepler's third law of planetary motion, it can be known that the ratio of the cube of the orbital radius to the square of the orbital period ($\frac{r^3}{T^2}$) will be equal for both moons.

$$
\begin{aligned}
\frac{r_1^3}{T_1^2} &= \frac{r_{23}^3}{T_{23}^2} \\
\frac{r_1^3}{T_1^2} &= \frac{r_{23}^3}{(23!T_1)^2} \\
\frac{r_1^3}{T_1^2} &= \frac{r_{23}^3}{(23!)^2 T_1^2} \\
r_{23}^3 &= (23!)^2 r_1^3 \\
r_{23} &= \sqrt[3]{(23!)^2}\ r_1 \\
r_{23} &= 8.743\ldots \times 10^{14} r_1 \\
r_{23} &= 8.74 \times 10^{14} r_1
\end{aligned}
$$

Question 1 Solution

For an object to be able to escape the gravitational field of a mass, the kinetic energy of the escaping mass must be equal to the negative of the gravitational potential energy, meaning $K_{\text{esc}} = -U$. Additionally, the radius used will be the sum of the 100.0 km altitude and the radius of the Earth to find the true distance between the centres of the mass & Earth. Also, the 1.0 kg data value is irrelevant.

$$\begin{aligned} r &= r_{\text{Earth}} + \text{Altitude} \\ &= 6.371 \times 10^6 + 100 \times 10^3 \\ &= 6.471 \times 10^6\,\text{m} \end{aligned}$$

Data List:

- $M = 6.0 \times 10^{24}\,\text{kg}$
- $r = 6.471 \times 10^6\,\text{m}$
- $v_{\text{esc}} = ?\,\text{m s}^{-1}$

Solution:

$$\begin{aligned} K_{\text{esc}} &= -U \\ \frac{1}{2}mv^2 &= \frac{GMm}{r} \\ v^2 &= \frac{2GM}{r} \\ v &= \sqrt{\frac{2GM}{r}} \qquad (v \geq 0) \\ v &= \sqrt{\frac{2(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.471 \times 10^6)}} \\ v &= 11121.614\ldots \\ v &= 1.112 \times 10^4\,\text{m s}^{-1} \end{aligned}$$

Question 2 Solution

As the satellite is in circular orbit:

$$K = \frac{1}{2}mv^2 \qquad\qquad U = -\frac{GMm}{r}$$

However, to compare them, F_c and F_g must be equated. This can be done because the gravitational force is the only force keeping the satellite in orbit.

The solution continues over the page.

$$
\begin{aligned}
F_c &= F_g \\
\frac{mv^2}{r} &= \frac{GMm}{r^2} \\
mv^2 &= \frac{GMm}{r} \\
2(\frac{1}{2}mv^2) &= \frac{GMm}{r} \\
2K &= -U \\
\therefore K &\propto U
\end{aligned}
$$

Because the kinetic energy of the satellite is proportional to the gravitational potential energy, the kinetic energy will also be doubled.

Question 3 Solution

$$
\begin{aligned}
F_g &= F_c \\
\frac{GMm}{r^2} &= \frac{mv^2}{r} \\
\frac{GM}{r} &= v^2 \\
\sqrt{\frac{GM}{r}} &= x \\
\sqrt{\frac{2GM}{r}} &= x\sqrt{2} \\
v_{esc} &= x\sqrt{2} \qquad \text{(See \textbf{Solution 2} for derivation.)}
\end{aligned}
$$

Question 4 Solution

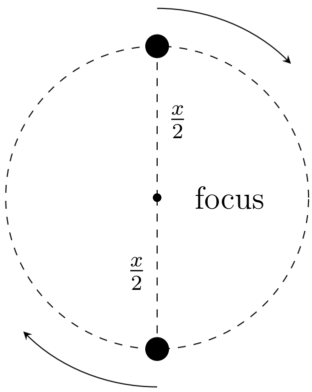
Throughout the entire path, the total mechanical energy $U + K$ will stay constant due to the conservation of mechanical energy. Using this information, the magnitude of the orbital velocity at the perigee can be found. Also note that the terms used, apogee and perigee, mean that the larger mass is the Earth, as the "gee" in both terms refers to "Gaia", meaning Earth. Similarly, the aphelion and the perihelion would mean the larger mass is the Sun, as "helion" in both terms refers to "helios", meaning Sun.

Let A refer to apogee and P refer to perigee.

$$
\begin{aligned}
U_{\mathrm{P}} + K_{\mathrm{P}} &= U_{\mathrm{A}} + K_{\mathrm{A}} \\
-\frac{GMm}{r_{\mathrm{P}}} + \frac{1}{2}mv_{\mathrm{P}}^2 &= -\frac{GMm}{r_{\mathrm{A}}} + \frac{1}{2}mv_{\mathrm{A}}^2 \\
-\frac{2GM}{r_{\mathrm{P}}} + v_{\mathrm{P}}^2 &= -\frac{2GM}{r_{\mathrm{A}}} + v_{\mathrm{A}}^2 && \text{(Multiply through by } \frac{2}{m}\text{)} \\
v_{\mathrm{P}}^2 &= v_{\mathrm{A}}^2 - \frac{2GM}{r_{\mathrm{A}}} + \frac{2GM}{r_{\mathrm{P}}} \\
v_{\mathrm{P}} &= \sqrt{v_{\mathrm{A}}^2 - 2GM\left(\frac{1}{r_{\mathrm{A}}} - \frac{1}{r_{\mathrm{P}}}\right)} && (v \geq 0) \\
v_{\mathrm{P}} &= \sqrt{(3.28 \times 10^3)^2 - 2(6.67 \times 10^{-11})(6.0 \times 10^{24})\left(\frac{1}{(2.23 \times 10^7)} - \frac{1}{(9.56 \times 10^6)}\right)} \\
v_{\mathrm{P}} &= 5.888\ldots \times 10^3\ldots \\
v_{\mathrm{P}} &= 5.89 \times 10^3\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Question 5 Solution

A constant orbital speed implies the two masses are in a circular orbit, as being in an orbit of varying distances to each other would cause them to vary in their velocity. This is because if they have periods where the two masses are closer, their gravitational potential energy will be lowered, and therefore, some of the energy must be converted to kinetic energy, increasing the speed of the mass. Furthermore, as they have equal masses, the focus of their circular path will be exactly in the middle. This then allows for the magnitude of the velocity to be expressed as so.



Gravitational Force

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$F_g = \frac{Ga^2}{x^2}$$

Centripetal Force

$$F_c = \frac{mv^2}{r}$$
$$F_c = \frac{av^2}{(\frac{x}{2})}$$
$$F_c = \frac{2av^2}{x}$$

As the gravitational force is the only force keeping the masses in orbit:

$$F_c = F_g$$
$$\frac{2av^2}{x} = \frac{Ga^2}{x^2}$$
$$v^2 = \frac{Ga}{2x}$$
$$v = \sqrt{\frac{Ga}{2x}} \qquad (v \geq 0)$$

Question 6 Solution

Firstly, find the velocity of the two satellites with the information that they are geosynchronous (orbital period of 24 h).

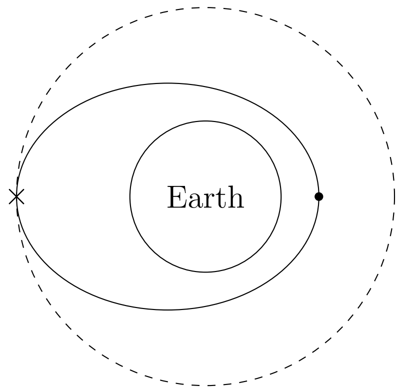
$$\begin{aligned}
\frac{r^3}{T^2} &= \frac{GM}{4\pi^2} \\
r &= \sqrt[3]{\frac{GMT^2}{4\pi^2}} \\
r &= \sqrt[3]{\frac{(6.67\times10^{-11})(6.0\times10^{24})(24\times60\times60)^2}{4\pi^2}} \\
r &= 4.229\times10^{7}\,\mathrm{m}
\end{aligned}$$

$$\begin{aligned}
\text{Now, } v &= \sqrt{\frac{GM}{r}} \qquad \text{(See \textbf{Solution 4} for derivation)} \\
v &= \sqrt{\frac{(6.67\times10^{-11})(6.0\times10^{2}4)}{4.229\ldots\times10^{7}}} \\
\therefore u &= 3.075\ldots\times10^{3}\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Now use the conservation of momentum to find the final velocity of the coalesced mass.

$$\begin{aligned}
p_\mathrm{i} &= p_\mathrm{f} \\
m_{1000}u - m_{500}u &= m_{1500}v \\
1000u - 500u &= 1500v \\
500u &= 1500v \\
v &= \frac{1}{3}u \\
v &= \frac{1}{3}(3.075\ldots\times10^{3}) \\
v &= 1.025\ldots\times10^{3}\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Since we know that at its collision point, the coalesced mass is at its apogee (as it has the smallest speed), we know that the orbital speed at the apogee of radius $4.229\ldots\times10^{7}$ m is $1.025\ldots\times10^{3}\,\mathrm{m\,s^{-1}}$. We also know from the information given that the speed at its perigee is $7.41\times10^{3}\,\mathrm{m\,s^{-1}}$. This is visualised on the following page.



By the conservation of mechanical energy,

$$
\begin{aligned}
E_{\mathrm{A}} &= E_{\mathrm{P}} \\
U_{\mathrm{A}} + K_{\mathrm{A}} &= U_{\mathrm{P}} + K_{\mathrm{P}} \\
-\frac{GMm}{r_{\mathrm{A}}} + \frac{mv_{\mathrm{A}}^2}{2} &= -\frac{GMm}{r_{\mathrm{B}}} + \frac{mv_{\mathrm{B}}^2}{2} \\
-\frac{2GM}{r_{\mathrm{A}}} + v_{\mathrm{A}}^2 &= -\frac{2GM}{r_{\mathrm{P}}} + v_{\mathrm{P}}^2 \\
\frac{2GM}{r_{\mathrm{P}}} &= v_{\mathrm{P}}^2 - v_{\mathrm{A}}^2 + \frac{2GM}{r_{\mathrm{A}}} \\
r_{\mathrm{P}} &= \frac{2GM}{v_{\mathrm{P}}^2 - v_{\mathrm{A}}^2 + \frac{2GM}{r_{\mathrm{A}}}} \\
r_{\mathrm{P}} &= \frac{2(6.67 \times 10^{-11})(6.0 \times 10^24)}{(7.41 \times 10^3)^2 - (1.025\ldots \times 10^3)^2 + \frac{2(6.67\times10^{-11})(6.0\times10^24)}{(4.229\ldots\times10^7)}} \\
r_{\mathrm{P}} &= 1.099\ldots \times 10^7\,\mathrm{m} \\
\therefore a &= \frac{r_{\mathrm{P}} + r_{\mathrm{A}}}{2} \\
a &= \frac{4.229\ldots \times 10^7 + 1.099\ldots \times 10^7}{2} \\
a &= 2.664\ldots \times 10^7\,\mathrm{m} \\
\text{Now, } \frac{r^3}{T^2} &= \frac{GM}{4\pi^2} \\
T &= \sqrt{\frac{4\pi^2 r^3}{GM}} \\
T &= \sqrt{\frac{4\pi^2(2.664\ldots \times 10^7)}{(6.67 \times 10^{-11})(6.0 \times 10^{24})}} \\
T &= 43204.284\ldots\,\mathrm{s} \\
T &= 720.071\ldots\,\mathrm{min} \\
T &= 12.001\ldots\,\mathrm{h} \\
\therefore T &= 12\,\mathrm{h}
\end{aligned}
$$

Electromagnetism SOLUTIONS

Charged Particles in Electric and Magnetic Fields SOLUTIONS

Charged Particles in Electric Fields SOLUTIONS

Question 1 Solution

The direction of the field can be found by finding the direction a positive charge will experience a force towards. Since a negatively charged particle experiences a force East, a positive charge will experience a force West; hence, the direction of the field is West.

Data List:

- $q = -1.6 \times 10^{-6}\,\text{C}$
- $F = 2.72 \times 10^{-2}\,\text{N}$
- $E = ?$

Solution:

$$F = qE$$
$$E = \frac{F}{q}$$
$$E = \frac{2.72 \times 10^{-2}}{1.6 \times 10^{-6}}$$
$$E = 1.7 \times 10^{4}$$
$$E = 1.7 \times 10^{4}\,\text{V m}^{-1}\ \text{West}$$

Question 2 Solution

Data List:

- $q = -1.602 \times 10^{-19}\,\text{C}$
- $m = 9.109 \times 10^{-31}\,\text{kg}$
- $E = 1.76 \times 10^{-6}\,\text{N C}^{-1}$

Solution:

$$F = ma$$
$$a = \frac{F}{m}$$
$$\text{Now } F = qE$$
$$\therefore a = \frac{qE}{m}$$
$$a = \frac{(1.602 \times 10^{-19})(1.76 \times 10^{-6})}{9.109 \times 10^{-31}}$$
$$a = 309\ 531.232\ldots$$
$$a = 3.10 \times 10^{5}\,\text{m s}^{-1}$$

Question 3 Solution

For the alpha particle to be stationary, the net force must equal 0. As the gravitational force will be acting downwards on the alpha particle, the electrostatic force must be acting upwards. As the alpha particle consists of two protons and two neutrons, the alpha particle is positively charged. Thus, the electric field must be acting upwards; hence, the positive plate is plate A.

Question 4 Solution

Data List:

- $q = 1.602 \times 10^{-19}\,\mathrm{C}$
- $E = 2500\,\mathrm{V\,m^{-1}}$
- $d = 4.0 \times 10^{-2}\,\mathrm{m}$
- $m = 1.673 \times 10^{-27}\,\mathrm{kg}$

Solution:

$$\begin{aligned}
W &= qEd \\
\frac{1}{2}mv^2 &= qEd \\
v^2 &= \frac{2qEd}{m} \\
v &= \sqrt{\frac{2qEd}{m}} \\
v &= \sqrt{\frac{2(1.602 \times 10^{-19})(2500)(4.0 \times 10^{-2})}{1.673 \times 10^{-27}}} \\
v &= 1.383\ldots \times 10^5 \\
v &= 1.4 \times 10^5\,\mathrm{m\,s^{-1}}
\end{aligned}$$

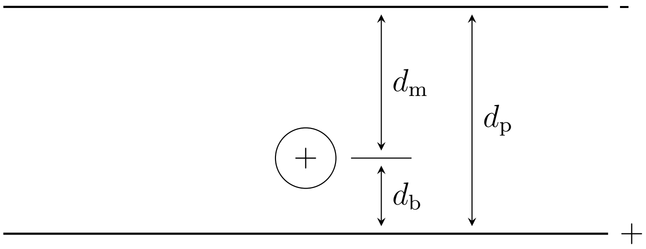
Question 5 Solution

$$\begin{aligned}
W &= qV \\
\frac{1}{2}mv^2 &= qV \\
v^2 &= \frac{2qV}{m} \\
\therefore v^2 &\propto \frac{1}{m}
\end{aligned}$$

$\therefore$ The answer is D.

Question 6 Solution

Let the distance the charge moves through (i.e., the distance between the top plate and the charge) be d_{m}, the distance between the charge and the bottom plate be d_{b} and the distance between the plates be d_{p} as shown below.



Now, the work done on the charge can be expressed as

$$W = qV_{\text{m}}$$

Now, the V_{m} represents the voltage that the charge will move through. This voltage that the charge moves through can be expressed as so

$$V_{\text{m}} = Ed_{\text{m}} \qquad (V = Ed)$$
$$V_{\text{m}} = (\frac{V}{d_{\text{p}}})d_{\text{m}} \qquad (E = \frac{V}{d})$$
$$V_{\text{m}} = \frac{Vd_{\text{m}}}{d_{\text{p}}}$$

Now, d_{m} is $d_{\text{p}} - d_{\text{b}}$ as shown in the diagram. Therefore:

$$V_{\text{m}} = \frac{V(d_{\text{p}} - d_{\text{b}})}{d_{\text{p}}}$$
$$V_{\text{m}} = V(\frac{d_{\text{p}} - d_{\text{b}}}{d_{\text{p}}})$$
$$V_{\text{m}} = V(1 - \frac{d_{\text{b}}}{d_{\text{p}}})$$

Thus,

$$W = qV_{\text{m}}$$
$$W = qV(1 - \frac{d_{\text{b}}}{d_{\text{p}}})$$

Now see $(1 - \frac{d_{\text{b}}}{d_{\text{p}}})$. It is known that d_{b} is a constant from the question's information. However, as Emmy moves the plate upwards, d_{p} will increase, and thus it will make $\frac{d_{\text{b}}}{d_{\text{p}}}$ smaller as d_{b} is a constant. But, $\frac{d_{\text{b}}}{d_{\text{p}}}$ will never equal zero, as there is always some distance between the bottom plate and the charge. Thus, $1 - \frac{d_{\text{b}}}{d_{\text{p}}}$ will approach 1 as Emmy increases the distance between the plates, but $1 - \frac{d_{\text{b}}}{d_{\text{p}}}$ will never equal 1. Thus, as Emmy increases the distance between the plates by elevating the top plate, the work done on the charge will approach qV, but never reach qV.

Trajectories in Electric Fields SOLUTIONS

Question 1 Solution

In scenario A, the baseball will experience a constant downward acceleration due to Earth's gravitational field, thus travelling in a parabolic path. In scenario B, the electron will also experience a constant force and therefore a constant acceleration downwards and will hence also travel in a parabolic path. Therefore, the answer is A.

Question 2 Solution

There are two ways to solve this question, solution 1 being more direct but more cumbersome and solution 2 being more complex but less work overall.

SOLUTION 1
First, find the velocity of the electron.

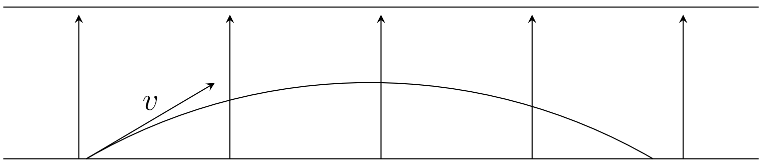
Data List:

- $q = -1.602 \times 10^{-19}\,\mathrm{C}$
- $V = 2000\,\mathrm{V}$
- $m = 9.109 \times 10^{-31}\,\mathrm{kg}$
- $v = ?\,\mathrm{m\,s^{-1}}$

Solution:

$$\begin{aligned}
W &= qV \\
\frac{1}{2}mv^2 &= qV \\
v^2 &= \frac{2qV}{m} \\
v &= \sqrt{\frac{2qV}{m}} \\
v &= \sqrt{\frac{2(-1.602 \times 10^{-19})(2000)}{9.109 \times 10^{-31}}} \\
v &= 2.652\ldots \times 10^{7} \\
v &= 2.652\ldots \times 10^{7}\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Now, we can use this initial velocity to find the electric field as shown.



Notice that as the electron is negatively charged, the force experienced will be opposite to the direction of the electric field, i.e. downwards.

Data List:

- $u = 2.652\ldots \times 10^{7}\,\mathrm{m\,s^{-1}}$
- $s = 0$
- $m = 9.109 \times 10^{-31}\,\mathrm{kg}$
- $q = -1.602 \times 10^{-19}\,\mathrm{C}$
- $t = 5.027 \times 10^{-8}\,\mathrm{s}$
- $E = ?\,\mathrm{N\,C^{-1}}$

Solution:

$$
\begin{aligned}
s &= u_{\mathrm{y}}t + \frac{1}{2}at^2 \\
0 &= u_{\mathrm{y}}t + \frac{1}{2}at^2 \qquad (t \neq 0, \text{ as it is the origin}) \\
0 &= t(u_{\mathrm{y}} + \frac{1}{2}at) \\
u_{\mathrm{y}} + \frac{1}{2}at &= 0 \\
\frac{1}{2}at &= -u_{\mathrm{y}} \\
a &= \frac{-2u_{\mathrm{y}}}{t} \\
\text{Now, } a &= \frac{F_{\mathrm{e}}}{m} \\
a &= \frac{qE}{m} \\
\therefore \frac{qE}{m} &= \frac{-2u_{\mathrm{y}}}{t} \\
E &= \frac{-2mu_{\mathrm{y}}}{qt} \\
E &= \frac{-2mu\sin(30^\circ)}{qt} \\
E &= \frac{-mu}{qt} \\
E &= \frac{-(9.109 \times 10^{-31})(2.652\ldots \times 10^{7})}{(-1.602 \times 10^{-19})(5.027 \times 10^{-8})} \\
E &= 3000.027\ldots \\
E &= 3.0 \times 10^{3}\,\mathrm{N\,kg^{-1}}
\end{aligned}
$$

SOLUTION 2

The velocity of the electron can be found as it was in solution 1. Now, since the path begins and ends at the same vertical height and is a parabolic path, its peak will occur at the middle of the period between the start and the end of the path.

$\therefore t_{\mathrm{peak}} = 2.5135 \times 10^{-8}\,\mathrm{s}$.

The solution continues over the page.

Data List:

- $v_y = 0$
- $u = 2.652\ldots \times 10^7\,\text{m s}^{-1}$
- $m = 9.109 \times 10^{-31}\,\text{kg}$
- $q = -1.602 \times 10^{-19}\,\text{C}$
- $t = 2.5135 \times 10^{-8}\,\text{s}$
- $E = ?\,\text{V m}^{-1}$

Solution:

$$
\begin{aligned}
v &= u_y + at \\
0 &= u_y + at \qquad (v = 0) \\
-u_y &= at \\
a &= -\frac{u_y}{t} \\
\text{Now, } a &= \frac{F_e}{m} \\
a &= \frac{qE}{m} \\
\therefore \frac{qE}{m} &= -\frac{u_y}{t} \\
E &= -\frac{mu_y}{qt} \\
E &= -\frac{mu\sin(30^\circ)}{qt} \\
E &= -\frac{mu}{2qt} \\
E &= -\frac{(9.109 \times 10^{-31})(2.652\ldots \times 10^7)}{2(-1.602 \times 10^{-19})(2.5135 \times 10^{-8})} \\
E &= 3000.027\ldots \\
E &= 3.0 \times 10^3\,\text{V m}^{-1}
\end{aligned}
$$

Question 3 Solution

First, express the velocity of the proton as so.

$$
\begin{aligned}
W &= qV \\
\frac{1}{2}mv^2 &= qV \\
v^2 &= \frac{2qV}{m} \\
v &= \sqrt{\frac{2qV}{m}}
\end{aligned}
$$

Now, find the voltage via the parabolic trajectory as shown on the following page.

Data List:

- $s = 0$
- $m = 1.673 \times 10^{-27}\,\text{kg}$
- $q = 1.602 \times 10^{-19}\,\text{C}$
- $E = 4200\,\text{N}\,\text{C}^{-1}$
- $t = 6.988 \times 10^{-6}\,\text{s}$
- $u = \sqrt{\frac{2qV}{m}}$
- $V = ?\,\text{V}$

Solution:

$$
\begin{aligned}
s &= u_y t + \frac{1}{2}at^2 \\
0 &= u_y t + \frac{1}{2}at^2 \qquad (s = 0) \\
t(\frac{1}{2}at + u_y) &= 0 \\
\frac{1}{2}at + u_y &= 0 \qquad (t \neq 0\text{, as it is origin}) \\
u_y &= -\frac{at}{2} \\
\text{Now, } a &= \frac{F_e}{m} \\
a &= \frac{qE}{m} \\
\therefore u_y &= -\frac{qEt}{2m} \\
u\sin(56^\circ) &= -\frac{qEt}{2m} \\
u &= -\frac{qEt}{2m\sin(56^\circ)} \\
\sqrt{\frac{2qV}{m}} &= -\frac{qEt}{2m\sin(56^\circ)} \\
\frac{2qV}{m} &= \frac{q^2E^2t^2}{4m^2\sin^2(56^\circ)} \\
V &= \frac{qE^2t^2}{8m\sin^2(56^\circ)} \\
V &= \frac{(1.602 \times 10^{-19})(4200)^2(6.988 \times 10^{-6})^2}{8(1.673 \times 10^{-27})\sin^2(56^\circ)} \\
V &= 15\,001.425\ldots \\
V &= 1.5 \times 10^4\,\text{V}
\end{aligned}
$$

Question 4 Solution

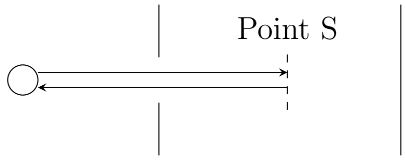
In scenario A, the acceleration supplied by the Earth's gravitational field is given by $a_g = \frac{GM}{r^2}$. Notice how as the object falls, r decreases, and as $a_g \propto \frac{1}{r^2}$, a_g will increase. For scenario B, the acceleration will be supplied by the electric field, and as $a_e = \frac{F_e}{m}$, $a_e = \frac{qE}{m}$. Notice how E, or $\frac{V}{d}$ will not change during the fall, and neither will q or m. Thus, a_e will be constant throughout the fall. As a_g will increase during the fall, while a_e is constant, Emmy in scenario A will fall further.

Question 5 Solution

Firstly, the initial velocity of the charged particle can be expressed as so.

$$W = qV$$
$$\frac{1}{2}mv^2 = qV$$
$$v^2 = \frac{2qV}{m}$$

Now, when the particle enters the electric field, the proton or the electron will experience a constant acceleration to the left. Thus, the particle will travel a path as shown.



Now, at point S, the velocity will be 0 as the velocity changes in direction. Furthermore, it can be visually seen that point S is the point where the particle is furthest from the left plate. Now we can find an expression for this distance as shown.

Data List:

- $u^2 = \frac{2qV}{m}$
- $v = 0$
- $s = ?$

Solution:

$$v^2 = u^2 + 2as$$
$$0 = u^2 + 2as \qquad (v = 0)$$
$$2as = -u^2$$
$$s = -u^2 \div 2a$$
$$\text{Now, } a = \frac{F_\text{e}}{m}$$
$$a = \frac{qE}{m}$$
$$\therefore s = -u^2 \div \frac{2qE}{m}$$
$$s = -\frac{2qV}{m} \times \frac{m}{2qE}$$
$$s = -\frac{V}{E}$$

Now it can be seen that the distance travelled is only affected by the voltage of the gun and the electric field strength. Since such quantities are kept constant in magnitude between the proton and the electron, both particles will travel the same distance.

Question 6 Solution

First, find an expression for the range of trial A as s_{A}.

Data List:

- $s_{\mathrm{y}} = \frac{d}{2}$
- $u_{\mathrm{y}} = 0$
- $s_{\mathrm{A}} = ?$

Solution:

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}at^2 \qquad (u = 0)$$

$$t^2 = \frac{2s}{a}$$

$$t^2 = \frac{d}{a}$$

$$\text{Now, } a = \frac{F_{\mathrm{e}}}{m}$$

$$a = \frac{qE}{m}$$

$$\text{Now, } E = \frac{V}{d}$$

$$\therefore a = \frac{qV}{md}$$

$$\therefore t^2 = \frac{md^2}{qV}$$

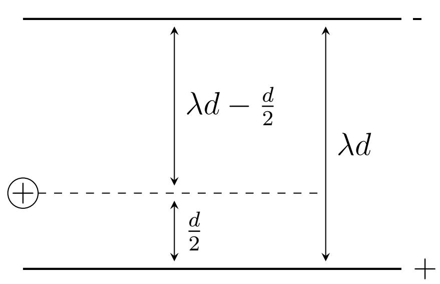
$$t = \sqrt{\frac{md^2}{qV}}$$

$$t = d\sqrt{\frac{m}{qV}}$$

$$\therefore s_{\mathrm{A}} = v_{\mathrm{x}}t$$

$$s_{\mathrm{A}} = v_{\mathrm{x}}d\sqrt{\frac{m}{qV}}$$

Now find an expression for the range of trial B as s_{B}. A diagram is used to help visualise the values.



Trial B

Data List:

- $s_{\text{y}} = \lambda d - \frac{d}{2}$
- $u = 0$
- $s_{\text{B}} = ?$

Solution:

$$s = ut + \frac{1}{2}at^2$$
$$s = \frac{1}{2}at^2 \qquad (u = 0)$$
$$t^2 = \frac{2s}{a}$$
$$t^2 = \frac{2(\lambda d - \frac{d}{2})}{a}$$
$$t^2 = \frac{2d(\lambda - \frac{1}{2})}{a}$$
$$\text{Now, } a = \frac{F_{\text{e}}}{m}$$
$$a = \frac{qE}{m}$$
$$\text{Now, } E = \frac{V}{\lambda d}$$
$$\therefore a = \frac{qv}{\lambda md}$$
$$\therefore t^2 = \frac{2md^2\lambda(\lambda - \frac{1}{2})}{qV}$$
$$t = \sqrt{\frac{2md^2\lambda(\lambda - \frac{1}{2})}{qV}}$$
$$t = d\sqrt{\frac{2m\lambda(\lambda - \frac{1}{2})}{qV}}$$
$$t = d\sqrt{\frac{m}{qV}}\sqrt{2\lambda(\lambda - \frac{1}{2})}$$
$$\therefore s_{\text{B}} = v_{\text{x}}t$$
$$s_{\text{B}} = v_{\text{x}}d\sqrt{\frac{m}{qV}}\sqrt{2\lambda(\lambda - \frac{1}{2})}$$

Now, it is known that s_{B} is 2 times that of s_{A}.

The solution continues over the page.

$$\therefore s_{\mathrm{B}} = 2s_{\mathrm{A}}$$

$$v_{\mathrm{x}}d\sqrt{\frac{m}{qV}}\sqrt{2\lambda(\lambda - \frac{1}{2})} = 2v_{\mathrm{x}}d\sqrt{\frac{m}{qV}}$$

$$\sqrt{2\lambda(\lambda - \frac{1}{2})} = 2$$

$$2\lambda(\lambda - \frac{1}{2}) = 4$$

$$\lambda(\lambda - \frac{1}{2}) = 2$$

$$\lambda^2 - \frac{1}{2}\lambda - 2 = 0$$

$$\therefore \lambda = \frac{\frac{1}{2} \pm \sqrt{(-\frac{1}{2})^2 - 4(-2)}}{2}$$

$$\lambda = \frac{\frac{1}{2} \pm \sqrt{\frac{33}{4}}}{2}$$

$$\lambda = \frac{\frac{1}{2} \pm \frac{\sqrt{33}}{2}}{2}$$

$$\lambda = \frac{1 \pm \sqrt{33}}{4}$$

$$\lambda = \frac{1 + \sqrt{33}}{4} \qquad (\lambda > 0)$$

Thus, if the horizontal range of trial B is twice that of trial A, $\lambda = \frac{1+\sqrt{33}}{4}$. Demonstrated as required.

Question 1 Solution

Remember to account for the direction of the charge by using the direction of the force.

Data List:

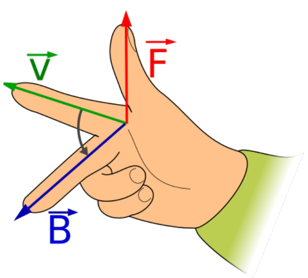
- $F = 2.7 \times 10^{-2}\,\text{N}$
- $v = 6.3 \times 10^{5}\,\text{m}\,\text{s}^{-1}$
- $B = 0.40\,\text{T}$
- $q = ?\,\text{C}$
- $\theta = 40°$

Solution:

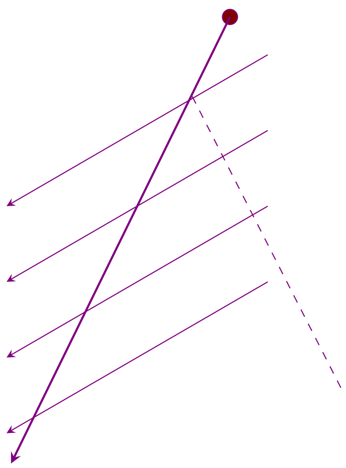
$$F = qvB\cos\theta$$
$$q = \frac{F}{vB\cos\theta}$$
$$q = \frac{(2.7 \times 10^{-2})}{(6.3 \times 10^{5})(0.40)(\cos(40°))}$$
$$q = 1.398\ldots \times 10^{-7}$$
$$q = -1.4 \times 10^{-7}\,\text{C}$$

Moving Charges in Mag. Fields

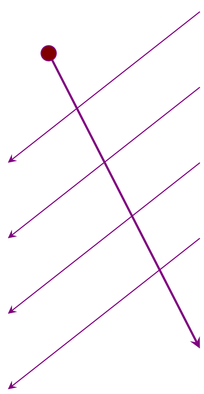
Directions of forces are easier to tell if the charged particle is moving in a perpendicular direction to the magnetic field by the right hand rule as shown below.



However, to determine the force of a moving charge that is not travelling in a perpendicular direction to the field is slightly more difficult. To determine the direction of the force, it is best to temporarily "snap" the direction of the moving charge to the closest perpendicular distance to determine the direction of the force. An example is given below.



"Snaps" to:



Question 2 Solution

Data List:

- $m = 1.673 \times 10^{-27}\,\text{kg}$
- $v = 0.999c\,\text{m}\,\text{s}^{-1}$
- $q = 1.602 \times 10^{-19}\,\text{C}$
- $r = 4 \times 10^{3}\,\text{m}$
- $B = ?\,\text{T}$

Solution:

$$F_\text{B} = F_\text{C}$$
$$qvB = \frac{mv^2}{r}$$
$$qB = \frac{mv}{r}$$
$$B = \frac{mv}{qr}$$
$$B = \frac{(1.673 \times 10^{-27})(0.999c)}{(1.602 \times 10^{-19})(4 \times 10^{3})}$$
$$B = 7.824\ldots \times 10^{-4}$$
$$B = 7.8 \times 10^{-4}\,\text{T}$$

Question 3 Solution

Since the electron will be deflected upwards by the electric field, the magnetic force must act downwards, thus the magnetic field must be directed into the page.

Data List:

- $v = 2 \times 10^{6}\,\text{m}\,\text{s}^{-1}$
- $V = 2 \times 10^{1}\,\text{V}$
- $d = 5 \times 10^{-2}\,\text{m}$
- $B = ?\,\text{T}$

Solution:

$$F_\text{B} = F_\text{e}$$
$$qvB = qE$$
$$vB = E$$
$$B = \frac{E}{v}$$
$$B = \frac{V}{vd} \qquad (E = \frac{V}{d})$$
$$B = \frac{(2 \times 10^{1})}{(2 \times 10^{6})(5 \times 10^{-2})}$$
$$B = 2 \times 10^{-4}$$
$$B = 2.0 \times 10^{-4}\,\text{T into the page}$$

Question 4 Solution

As the particle exits in the opposite direction to the direction it has entered the magnetic field, it can be known that the particle has travelled in a semicircular path. Thus, the 0.3 m distance is the diameter of the semicircular path, meaning the radius is 0.15 m.

Data List:

- $v = 7.2 \times 10^5\,\text{m}\,\text{s}^{-1}$
- $B = 0.1\,\text{T}$
- $r = 0.15\,\text{m}$

Solution:

$$F_\text{B} = F_\text{c}$$
$$qvB = \frac{mv^2}{r}$$
$$qB = \frac{mv}{r}$$
$$\frac{q}{m} = \frac{v}{Br}$$
$$\frac{q}{m} = \frac{7.2 \times 10^5}{(0.1)(0.15)}$$
$$\frac{q}{m} = 4.8 \times 10^7$$
$$\frac{q}{m} = 4.8 \times 10^7\,\text{C}\,\text{kg}^{-1}$$

Question 5 Solution

To find the radius of curvature of the particle in the field:

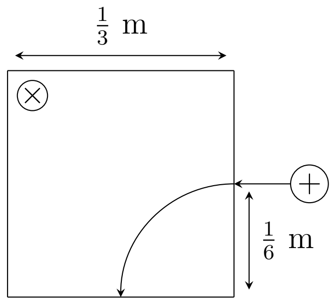
Data List:

- $\frac{q}{m} = 1\,\text{C}\,\text{kg}^{-1}$
- $v = 1\,\text{m}\,\text{s}^{-1}$
- $B = 6\,\text{T}$
- $r = ?\,\text{m}$

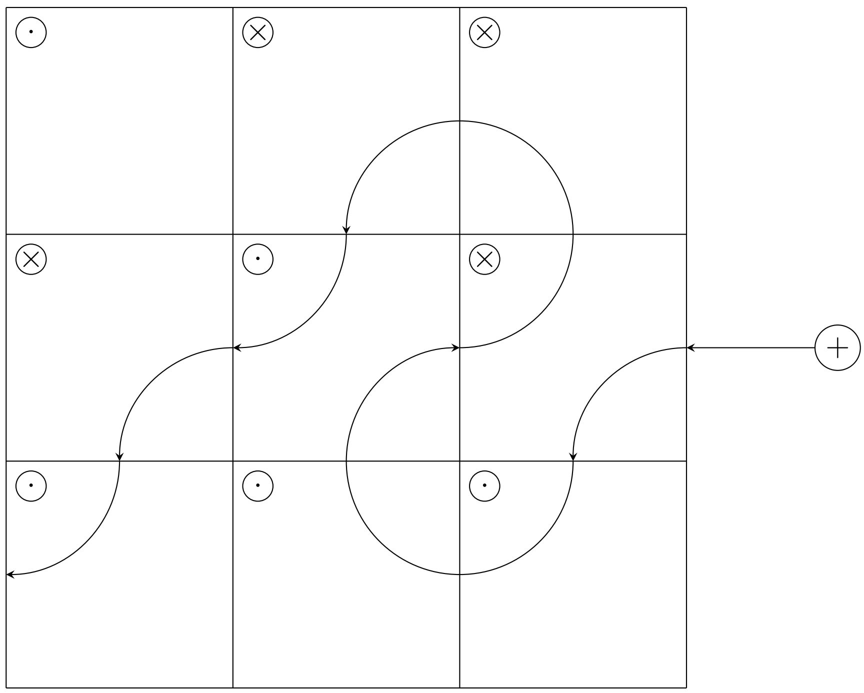
Solution:

$$F_\text{B} = F_\text{c}$$
$$qvB = \frac{mv^2}{r}$$
$$qB = \frac{mv}{r}$$
$$r = \frac{mv}{qB}$$
$$r = \frac{m}{q} \times \frac{v}{B}$$
$$r = (\frac{q}{m})^{-1} \times \frac{v}{B}$$
$$r = (1)^{-1} \times \frac{1}{6}$$
$$r = \frac{1}{6}\,\text{m}$$

Now, notice how each square has a side length of $\frac{1}{3}$ m. Thus, the particle will follow a quadrant path as shown on the following page.



Stringing together these predictable trajectories within each square, we get a path like so.



Now the each time the particle enters and exits a square, the particle travels a quarter of the circumference of its circular path. Thus, the distance the particle travels in the path can be found as so:

Data List:

- $n_{\text{squares}} = 10$
- $r = \frac{1}{6}$ m
- $d = ?$

Solution:

$$d = \text{Number of squares} \times \text{quarter of circumference}$$
$$d = n_{\text{squares}} \times \frac{1}{4}(2\pi r)$$
$$d = \frac{n_{\text{squares}}\pi r}{2}$$
$$d = \frac{(10)\pi(\frac{1}{6})}{2}$$
$$d = \frac{5\pi}{6}\text{ m}$$

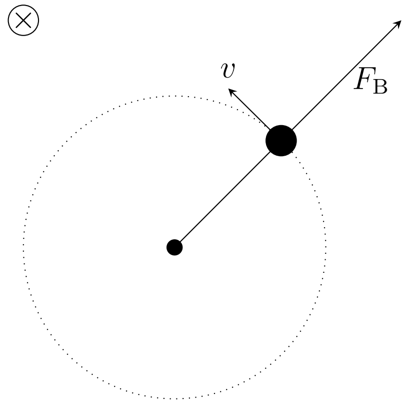
Because the particle travels at $1\,\text{m}\,\text{s}^{-1}$, the particle will travel in the magnetic field for $\frac{5\pi}{6}$ s. Demonstrated as required.

Question 6 Solution

To find an expression for the maximum tension the rope can withstand, let the frequency of 6 Hz be f_n for "no field".

$$
\begin{aligned}
F_\text{c} &= \frac{mv^2}{r} \\
\text{Now, } v &= \frac{2\pi r}{T} \\
v &= 2\pi r f \\
\therefore F_\text{c} &= \frac{m}{r} \times (2\pi r f_\text{n})^2 \\
F_\text{c} &= \frac{m}{r} \times 4\pi^2 r^2 f_\text{n}^2 \\
T_\text{breaking} &= 4\pi^2 m r f_\text{n}^2
\end{aligned}
$$

Now, if the rope snaps in the magnetic field at 5.9 Hz, that must mean the rope increases in tension faster. This is because $F_\text{c} = 4\pi^2 m r f^2 \implies F_\text{c} \propto f^2$. Thus, the magnetic field must be supplying a force to the negatively charged particle outwards of the centre of its circular motion, increasing the tension. This can occur through a magnetic field directed into the page as shown below.



Thus, the tension of the rope is the centripetal force AND the magnetic force combined. With the previously determined expression of the breaking tension of the rope, the magnetic field strength can be found. Let the frequency of 5.9 Hz be f_m for "magnetic field".

The solution continues over the page.

Data List:

- $m = 3\,\text{kg}$
- $f_n = 6\,\text{Hz}$
- $f_m = 5.9\,\text{Hz}$
- $q = -5.0 \times 10^{-1}\,\text{C}$
- $B = ?\,\text{T}$

Solution:

$$\begin{aligned}
T_{\text{breaking}} &= F_{\text{c}} + F_{\text{B}} \\
F_{\text{B}} &= T_{\text{breaking}} - F_{\text{c}} \\
qvB &= 4\pi^2 mrf_{\text{n}}^2 - \frac{mv^2}{r} \\
\text{Now } v &= \frac{2\pi r}{T} \\
v &= 2\pi rf \\
\therefore 2\pi rf_{\text{m}}qB &= 4\pi^2 mrf_{\text{n}}^2 - \frac{m}{r} \times (2\pi rf_{\text{m}})^2 \\
2\pi rf_{\text{m}}qB &= 4\pi^2 mrf_{\text{n}}^2 - \frac{m}{r} \times 4\pi^2 r^2 f_{\text{m}}^2 \\
2\pi rf_{\text{m}}qB &= 4\pi^2 mrf_{\text{n}}^2 - 4\pi^2 mrf_{\text{m}}^2 \\
2\pi r(f_{\text{m}}qB) &= 4\pi^2 mr(f_{\text{n}}^2 - f_{\text{m}}^2) \\
f_{\text{m}}qB &= 2\pi m(f_{\text{n}}^2 - f_{\text{m}}^2) \\
B &= \frac{2\pi m(f_{\text{n}}^2 - f_{\text{m}}^2)}{f_{\text{m}}q} \\
B &= \frac{2\pi(3)((6)^2 - (5.9)^2)}{(5.9)(-5.0 \times 10^{-1})} \\
B &= -7.603\ldots \\
B &= 7.6\,\text{T into page}
\end{aligned}$$

The Motor Effect SOLUTIONS

Currents in Magnetic Fields SOLUTIONS

Question 1 Solution

If the force is going into the page, the current must be running from A to B. This is further explained in the solution for **Question 1** of **Charged Particles and Magnetic Fields**.

Ensure that you are finding the component of the current perpendicular to the field lines.

Data List:

- $\frac{F}{l} = 7.6 \times 10^{-2}\,\mathrm{N\,m^{-1}}$
- $B = 0.45\,\mathrm{T}$
- $\theta = 42°$
- $I = ?\,\mathrm{A}$

Solution:

$$F = lIB\cos\theta$$
$$I = \frac{F}{lB\cos\theta}$$
$$I = \frac{F}{l} \times \frac{1}{B\cos\theta}$$
$$I = 7.6 \times 10^{-2} \times \frac{1}{0.45\cos(42°)}$$
$$I = 2.272\ldots \times 10^{-1}$$
$$I = 2.3 \times 10^{-1}\,\mathrm{A\ from\ A\ to\ B}$$

Question 2 Solution

$$F = qvB$$
$$\text{Now, } I = \frac{q}{t}$$
$$q = It$$
$$\therefore F = ItvB$$
$$\text{Now, } v = \frac{s}{t}$$
$$v = \frac{l}{t} \qquad \text{(Let the distance travelled in the wire be } l\text{)}$$
$$\therefore F = \frac{ItlB}{t}$$
$$F = lIB$$

Question 3 Solution

For the wire to be suspended in mid-air, the magnetic force must be acting upwards, thus the magnetic field must be directed out of the page.

Data List:

- $l = 1\,\text{m}$
- $m = 2.5\,\text{kg}$
- $I = 400\,\text{A}$
- $B = ?\,\text{T}$

Solution:

$$\begin{aligned} F_\text{B} &= F_\text{g} \\ lIB &= mg \\ B &= \frac{mg}{lI} \\ B &= \frac{(2.5)(9.8)}{(1)(400)} \\ B &= 6.125 \times 10^{-2} \\ B &= 6.1 \times 10^{-2}\,\text{T out of the page} \end{aligned}$$

Question 4 Solution

Criteria	Marks
• Identifies coil does not vary in measured weight • Justifies identification	2
• Identifies coil does not vary in measured weight	1

Sample answer:
The coil's measured weight will not vary as the magnetic field alternates in direction. This is because opposite sides of the coil will experience an equal force in opposite directions (as they only differ in direction of current), thus cancelling out and causing no variations in measured weight.

Question 5 Solution

For the lever to be stationary, the torque that Emmy applies and the torque that the wire applies must be equal to a net zero. Another point to note is that the force the wire applies to the lever will be the sum of the gravitational force and the magnetic force. Now, since the gravitational force of the wire is much less than the gravitational force of Emmy, the magnetic force of the wire must act downwards so that the wire supplies an equal torque as Emmy. Hence, the magnetic field must act to the right.

Data List:

- $m_{\text{Emmy}} = 78\,\text{kg}$
- $m_{\text{wire}} = 0.3\,\text{kg}$
- $l = 50 \times 10^{-2}\,\text{m}$
- $I = 200\,\text{A}$
- $B = ?\,\text{T}$

Solution:

$$\begin{aligned}
\tau_{\text{Emmy}} &= \tau_{\text{wire}} \\
F_{\text{Emmy}} r &= F_{\text{wire}} r \\
F_{\text{Emmy}} &= F_{\text{wire}} \\
m_{\text{Emmy}} g &= m_{\text{wire}} g + lIB \\
lIB &= m_{\text{Emmy}} g - m_{\text{wire}} g \\
B &= \frac{g(m_{\text{Emmy}} - m_{\text{wire}})}{lI} \\
B &= \frac{(9.8)(78 - 0.3)}{(50 \times 10^{-2})(200)} \\
B &= 7.614\ldots \\
B &= 7.6\,\text{T to the right}
\end{aligned}$$

Question 6 Solution

When the ropes snap, there will be 2 accelerating forces acting on the bar. The gravitational force and the magnetic force. As the bar falls, due to the maintained current, the bar will continually experience both forces downwards. Thus,

Data List:

- $l = 5.0\,\text{m}$
- $I = 300.0\,\text{A}$
- $B = 4.2\,\text{T}$
- $W = 500.0\,\text{N}$
- $a = ?\,\text{m}\,\text{s}^{-2}$

Solution:

$$
\begin{aligned}
a_{\text{bar}} &= a_{\text{g}} + a_{\text{B}} \\
a_{\text{bar}} &= g + \frac{F_{\text{B}}}{m} \\
a_{\text{bar}} &= g + \frac{lIB}{m} \\
\text{Now, } W &= mg \\
m &= \frac{W}{g} \\
\therefore a_{\text{bar}} &= g + \frac{lIBg}{W} \\
a_{\text{bar}} &= g(1 + \frac{lIB}{W}) \\
a_{\text{bar}} &= (9.8)(1 + \frac{(5)(300)(4.2)}{(500)}) \\
a_{\text{bar}} &= 133.28\,\text{m}\,\text{s}^{-2}
\end{aligned}
$$

Now to find the time taken for the bar to reach the floor.

Data List:

- $u = 0$
- $s = 20\,\text{m}$
- $a = 133.28\,\text{m}\,\text{s}^{-2}$
- $t = ?\,\text{s}$

Solution:

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
s &= \frac{1}{2}at^2 && (u = 0) \\
t^2 &= \frac{2s}{a} \\
t &= \sqrt{\frac{2s}{a}} && (t \geq 0) \\
t &= \sqrt{\frac{2(20)}{(133.28)}} \\
t &= 0.547\ldots \\
t &= 0.55\,\text{s}
\end{aligned}
$$

Parallel Wires SOLUTIONS

Question 1 Solution

Criteria	Marks
• Identifies the force between wires • Identifies the effects of increasing the length of the wire	2
• 1 of the above	1

Sample answer:
The force between the wires will be attractive as the currents of both wires run in the same direction.

Data List:

- $l = 2.5\,\text{m}$
- $I_1 = 4.2\,\text{A}$
- $I_2 = 1.0\,\text{A}$
- $r = 1.25\,\text{m}$

Solution:

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$
$$F = \frac{\mu_0 l I_1 I_2}{2\pi r}$$
$$F = \frac{(4\pi \times 10^{-7})(2.5)(4.2)(1)}{2\pi(1.25)}$$
$$F = 1.68 \times 10^{-6}$$
$$F = 1.7 \times 10^{-6}\,\text{N attractive}$$

Now, if Wire X increased in length, the magnitude of the attractive force would increase in magnitude from 1.68×10^{-6} N. This is because $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \implies F = \frac{\mu_0 l I_1 I_2}{2\pi r} \implies F \propto l$.

Question 2 Solution

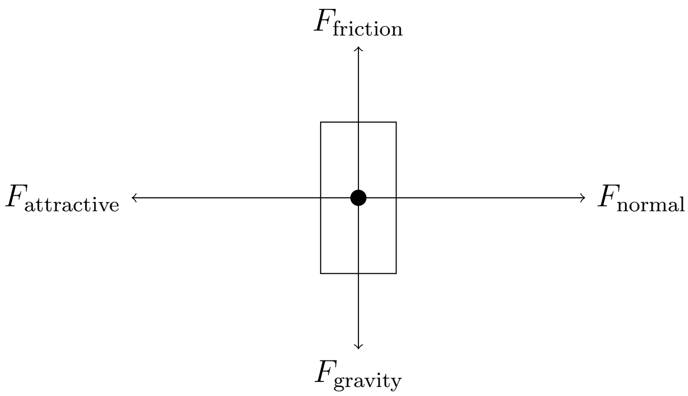
Now $V = IR \implies I + \frac{V}{R} \implies I \propto V$. Notice how the current in sections AB and CD will be equal as the circuit is a series circuit. Thus,

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$
$$F = \frac{\mu_0 l I_1 I_2}{2\pi r}$$
$$F \propto I^2$$
$$\text{Now } I \propto V$$
$$\therefore F \propto V^2$$

Thus, the answer is B.

Question 3 Solution

On each wire, 4 forces are acting, shown by the force body diagram below (the box represents the right wire).



Since the wires are stationary, $F_{\text{net}} = 0$, meaning $F_{\text{normal}} = F_{\text{attractive}}$ and $F_{\text{friction}} = F_{\text{gravity}}$. Let the coefficient of friction be k. Thus,

Data List:

- $r = 3.0 \times 10^{-2}\,\text{m}$
- $m = 7 \times 10^{-3}\,\text{kg}$
- $k = 2.3$
- $l = 2.5 \times 10^{-2}\,\text{m}$

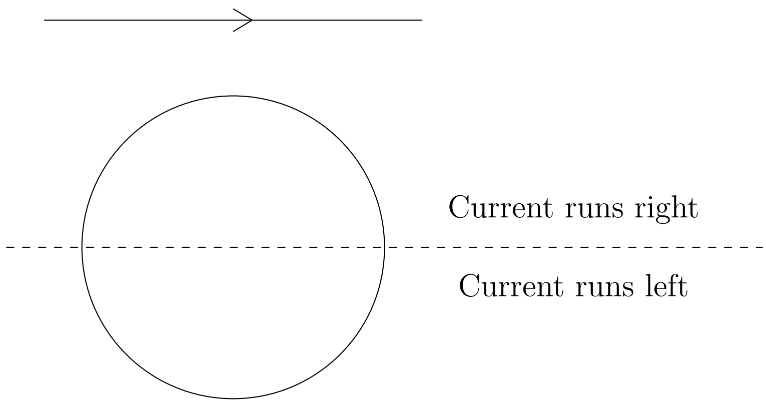
Solution:

$$\begin{aligned}
F_{\text{friction}} &= F_{\text{gravity}} \\
kF_{\text{normal}} &= F_{\text{gravity}} && (F_{\text{friction}} = kF_{\text{normal}}) \\
kF_{\text{attractive}} &= F_{\text{gravity}} && (F_{\text{normal}} = F_{\text{attractive}}) \\
\text{Now } \frac{F_{\text{attractive}}}{l} &= \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \\
F_{\text{attractive}} &= \frac{\mu_0 l I_1 I_2}{2\pi r} \\
\text{And } F_{\text{gravity}} &= mg \\
\therefore \frac{\mu_0 l I_1 I_2}{2\pi r} &= mg \\
\frac{\mu_0 l I^2}{2\pi r} &= mg && (I_1 = I_2) \\
I^2 &= \frac{2\pi r m g}{k\mu_0 l} \\
I &= \sqrt{\frac{2\pi r m g}{k\mu_0 l}} \\
I &= \sqrt{\frac{2\pi(3 \times 10^{-2})(7 \times 10^{-3})(9.8)}{(2.3)(4\pi \times 10^{-7})(2.5 \times 10^{-2})}} \\
I &= 423.032\ldots \\
I &= 4.2 \times 10^{2}\,\text{A}
\end{aligned}$$

Question 4 Solution

Criteria	Marks
• Identifies the attractive and repulsive forces • Identifies that the attractive force will be greater • Identifies circular circuit will experience a force towards straight wire	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Now, the current of the circular circuit with the current running clockwise can be divided as so.

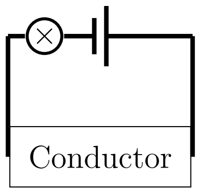


As the current in the straight wire runs to the right, the top half of the circular circuit will experience an attractive force towards the straight wire by the motor effect as the current runs in the same direction. Although the current is not running parallel, a component of the current runs to the right, i.e. the current will still experience an attractive force. The bottom half will similarly experience a repulsive force away from the straight wire. However, the bottom half of the circular circuit is further away from the straight conductor, and as $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \implies F = \frac{\mu_0 l I_1 I_2}{2\pi r} \implies F \propto \frac{1}{r}$, the top half of the circular circuit will experience a greater force than the bottom half, resulting in a net force for the circular circuit towards the straight wire.

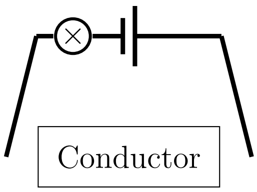
Question 5 Solution

Criteria	Marks
• Identifies the opposite currents of the left and right sections of the circuit • Explains how the globe turns off by the motor effect • Explains how the globe turns on by gravity	4
• All of the 4 mark criteria however, to a lesser degree	3
• 2 of the 4 mark criteria	2
• 1 of the 4 mark criteria	1

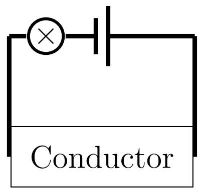
Sample answer:
When the circuit is turned on, the current will run clockwise, thus the globe will be powered and turned on. However, this will result in the current of the right section of the circuit running downwards, and the current of the left section of the circuit running upwards. This will result in a repulsive force by the motor effect, leading to both wires experiencing a force away from the conductor, breaking the circuit and thus resulting in the globe turning off. After the wires have been repelled, there will no longer be a current flowing in the wires, and thus there will be no repulsive force. As the wires on the left and right hang freely, the gravitational force will result in the wires hanging straight down again, thus completing the circuit to turn the globe on again. This cycle is shown in the following diagram.



Complete Circuit
Globe On



Motor Effect Repulsion
Globe Off



Gravity Pulls Wires Down
Globe On

This cycle will continue indefinitely, thus resulting in the globe flickering on and off.

Question 6 Solution

All wires will experience an attractive force as all currents run in the same direction. Now let the wire with the current 5.0 A be w_1, the wire of current 3.0 A (the middle wire) be w_2, the wire of current 2.0 A be w_3 and the wire of current 1.0 A be w_4. Now, see the equation for the force between parallel wires.

$$\begin{aligned}\frac{F}{l} &= \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}\\ F &= \frac{\mu_0 l I_1 I_2}{2\pi r}\\ F &= \frac{\mu_0 l I_1}{2\pi}\times\frac{I_2}{r}\end{aligned}$$

Now notice that l and I_1 are constants, as all wires are 5.0 cm, and the middle wire will constantly have the unknown current. Thus,

$$\begin{aligned}
&\text{Let } k = \frac{\mu_0 l I_1}{2\pi}\\
&\therefore F = k\times\frac{I_2}{r}\\
\text{For } w_1\qquad &F_1 = k\times\frac{5}{6\times 10^{-2}}\\
&F_1 = \frac{250k}{3}\text{ N to the left}\\
\text{For } w_3\qquad &F_3 = k\times\frac{2}{3\times 10^{-2}}\\
&F_3 = \frac{200k}{3}\text{ N to the right}\\
\text{For } w_4\qquad &F_4 = k\times\frac{1}{6\times 10^{-2}}\\
&F_4 = \frac{50k}{3}\text{ N to the right}\\
&\therefore \Sigma F = F_1 + F_3 + F_4 \qquad \text{(Let left be positive)}\\
&\Sigma F = \frac{250k}{3} - \frac{200k}{3} - \frac{50k}{3}\\
&\Sigma F = 0
\end{aligned}$$

Thus, the middle wire experiences a resultant force of 0.

Electromagnetic Induction SOLUTIONS

Magnetic Flux SOLUTIONS

Question 1 Solution

Data List:

- $N = 300$
- $B = 0.15\,\mathrm{T}$
- $\theta = 10°$
- $\phi = 8.7 \times 10^{-2}\,\mathrm{Wb}$
- $r = ?\,\mathrm{m}$

Solution:

$$\phi = NBA\cos\theta$$
$$A = \frac{\phi}{NB\cos\theta}$$
$$\pi r^2 = \frac{\phi}{NB\cos\theta}$$
$$r^2 = \frac{\phi}{\pi NB\cos\theta}$$
$$r = \sqrt{\frac{\phi}{\pi NB\cos\theta}}$$
$$r = \sqrt{\frac{8.7\times10^{-2}}{\pi(300)(0.15)\cos(10°)}}$$
$$r = 2.499\ldots\times10^{-2}$$
$$r = 2.5\times10^{-2}\,\mathrm{m}$$

Question 2 Solution

Data List:

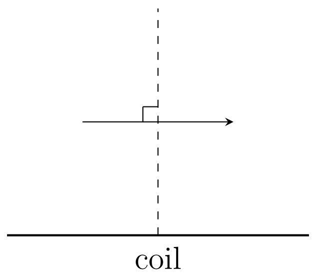
- $\theta = 120°$
- $B = 2\,\mathrm{T}$
- $r = 2\,\mathrm{m}$
- $\phi = ?\,\mathrm{Wb}$

Solution:

$$\phi = BA$$
$$\text{Now, } A = \frac{\theta}{360°}\pi r^2$$
$$\therefore \phi = \frac{\theta}{360°}B\pi r^2$$
$$\phi = \frac{120°}{360°}(2)\pi(2)^2$$
$$\phi = \frac{8\pi}{3}$$
$$\phi = 8.4\,\mathrm{Wb}$$

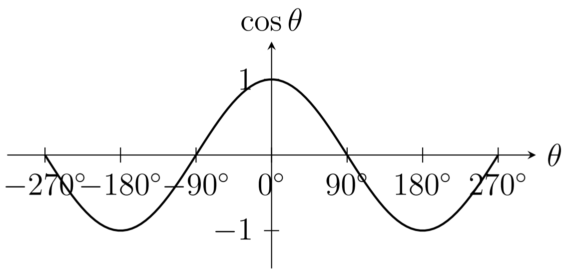
Question 3 Solution

At the beginning, it can be seen that no magnetic flux goes through the coil. In other words, the magnetic field lines run 90° to the normal as shown below.

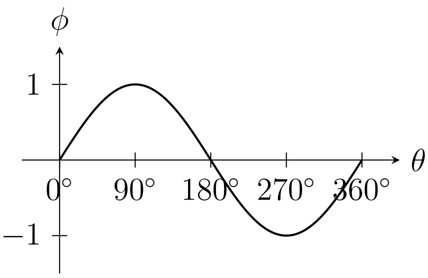


Therefore, let $\theta = -90°$. The reason θ is first declared to be a negative angle is because as the coil rotates clockwise, as depicted in the question, θ will approach 0° where the flux will be at a maximum, and rotate past 0° until the end of the revolution at 270°. The rotation ends at 270° as the rotation begins at $-90°$. Now as $\phi = BA\cos\theta \implies \phi \propto \cos\theta$, the graph will be a cos graph, but starting at $-90°$. This is shown below.

cos graph

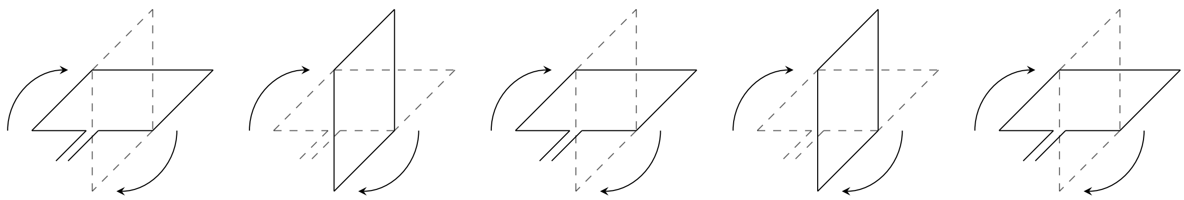


ϕ graph



Question 4 Solution

Note that the magnetic flux through the coil will be at a maximum at 2 states during a revolution. This is shown below.



ϕ minimum	ϕ maximum	ϕ minimum	ϕ maximum	ϕ minimum
0°	90°	180°	270°	360°

Now find the number of rotations the coil does in 3.5 seconds.

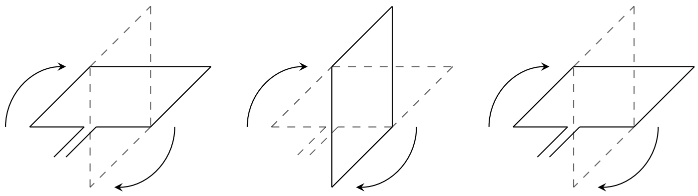
Data List:

- rate = 3400 rpm
- $t = 3.5\,\text{s}$
- $n = ?$

Solution:

$$n = \text{rate} \times t$$
$$n = 3400 \text{ rpm} \times 3.5\,\text{s}$$
$$n = 3400 \text{ rpm} \times \frac{3.5}{60} \text{ min}$$
$$n = 198\frac{1}{3} \text{ rotations}$$

Firstly, the coil does 198 full rotations, meaning 396 maxima of flux will be reached. However the coil will complete a third of a rotation further, which will result in one further maximum of flux as shown below.



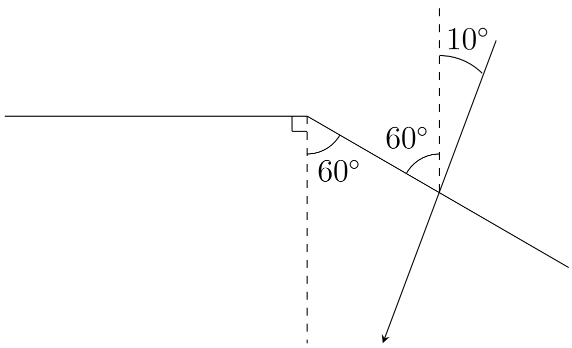
ϕ minimum ϕ maximum ϕ minimum

0° 90° 180°

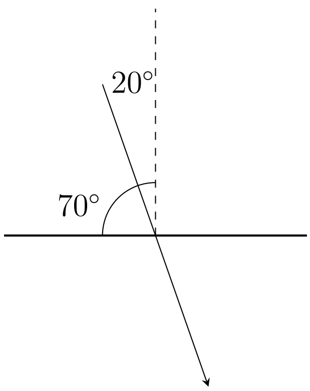
As the rotation will be a further 120°, the coil will reach a maximum of flux and stop before reaching a minimum at 180°. Thus, the coil will reach a maximum of flux $396 + 1 = 397$ times.

Question 5 Solution

Firstly, the flux linkage of the loop can be expressed as so: $\phi = BA\cos\theta_1 + BA\cos\theta_2 \implies \phi = BA(\cos\theta_1 + \cos\theta_2)$, where θ_1 is the angle of the magnetic field lines to the normal in one square section of the coil, and θ_2 is the angle of the magnetic field lines to the normal in the other. Now, θ_1 can be seen to be $10°$, however, θ_2 requires more work to determine as shown:



Thus, the magnetic field lines' angle to the normal in the second section can be found as so:



Meaning that $\theta_2 = 20°$. Now, the flux linkage can be found as shown below.

Data List:

- $B = 0.55\,\text{T}$
- $l = 3 \times 10^{-2}\,\text{m}$
- $\theta_1 = 10°$
- $\theta_2 = 20°$

Solution:

$$\phi = BA(\cos\theta_1 + \cos\theta_2)$$
$$\phi = Bl^2(\cos\theta_1 + \cos\theta_2) \qquad (A = l^2)$$
$$\phi = (0.55)(3 \times 10^{-2})^2(\cos(10°) + \cos(20°))$$
$$\phi = 9.526\ldots \times 10^{-4}$$
$$\phi = 9.5 \times 10^{-4}\,\text{Wb}$$

Question 6 Solution

Let the angle of the magnetic field lines to the normal of the coil be α.

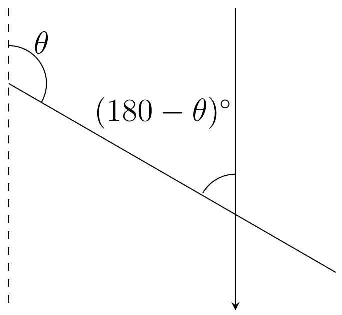
Data List:

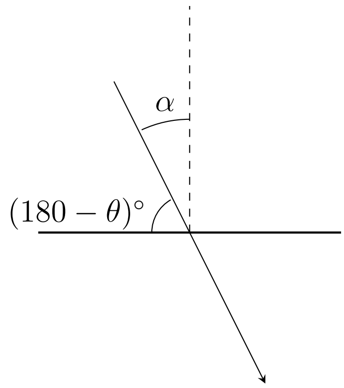
- $\theta = \alpha$
- $B = 0.4\,\text{T}$
- $l = 4.0 \times 10^{-2}\,\text{m}$
- $\phi = 3.2 \times 10^{-4}\,\text{Wb}$

Solution:

$$\begin{aligned}
\phi &= BA\cos\theta \\
\cos\theta &= \frac{\phi}{BA} \\
\cos\theta &= \frac{\phi}{Bl^2} \qquad (A = l^2) \\
\cos\alpha &= \frac{3.2 \times 10^{-4}}{(0.4)(4.0 \times 10^{-2})^2} \\
\cos\alpha &= \frac{1}{2} \\
\alpha &= 60^\circ
\end{aligned}$$

Now find θ in terms of α.





$$\begin{aligned}
\text{Now, } 180^\circ - \theta + \alpha &= 90^\circ \\
\theta &= 90^\circ + \alpha \\
\theta &= 150^\circ
\end{aligned}$$

Faraday's Law and Lenz's Law SOLUTIONS

Question 1 Solution

Imagine that the solenoid will "try to make the movement of the magnet more difficult" in each scenario, due to Lenz's law. Thus, in scenario 1, a magnetic South will be induced towards the right of the solenoid, and thus the current will flow from B to A. In scenario 2, a magnetic North will be induced towards the right, hence current will flow from A to B. In scenario 3, a magnetic North will also be induced towards the right, hence the current will flow from A to B. In scenario 4, a magnetic South will be induced towards the right, hence current will flow form B to A.

Question 2 Solution

Criteria	**Marks**
• State Lenz's law • Applies Lenz's law to the scenario in question • Identifies the current will run from end Y to end X	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
As the wire falls, the wire will experience a change in magnetic flux. By Lenz's law which states that an induced current will be induced in conductors that experience a change in magnetic flux to oppose said change, the wire will experience an induced current so that the induced magnetic field opposes the change in magnetic flux. Hence, the force produced by the induced current of the wire will be upwards (opposite to the direction of motion), meaning the current will run from end X to end Y.

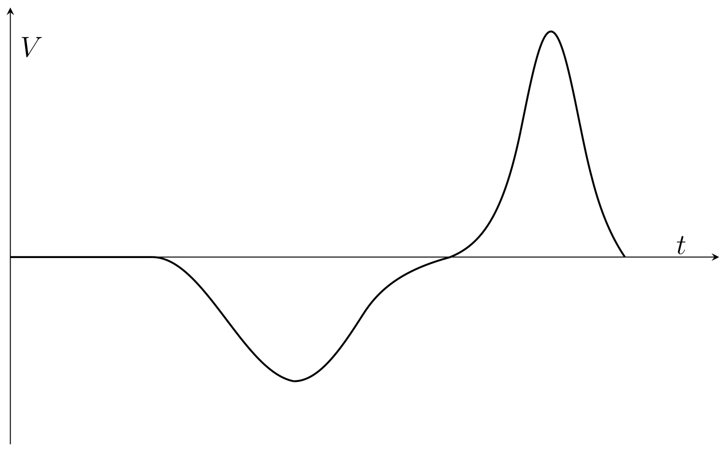
Question 3* Solution

$$A = \int f(x)dx$$
$$A = \int \cos(x^{\cos x})dx$$
$$\frac{dA}{dx} = \cos(x^{\cos x})$$
$$\text{Now } |\varepsilon| = \left|\frac{d\phi}{dt}\right|$$
$$\text{Since } \phi = BA \implies \phi \propto A$$
$$|\varepsilon| \propto \left|\frac{dA}{dt}\right|$$
$$|\varepsilon| = k\left|\frac{dA}{dx} \times \frac{dx}{dt}\right|$$
$$|\varepsilon| = k\left|v\cos(x^{\cos x})\right|$$
$$\therefore |\varepsilon_{\mathrm{f}}| = k\left|v\cos(0^{\cos 0})\right|$$
$$|\varepsilon_{\mathrm{f}}| = k\,|v|$$
$$|\varepsilon_{\mathrm{i}}| = k\left|v\cos(\frac{\pi}{2}^{\cos\frac{\pi}{2}})\right|$$
$$|\varepsilon_{\mathrm{i}}| = k\left|v\cos(\frac{\pi}{2}^{0})\right|$$
$$|\varepsilon_{\mathrm{i}}| = k\,|v\cos(1)|$$
$$\therefore |\varepsilon_{\mathrm{i}}| = |\varepsilon_{\mathrm{f}}|\cos(1)$$

Question 4 Solution

Criteria	Marks
• Correct general shape of the graph • Second peak is greater in magnitude than the first peak • Time interval of the second peak is less than first peak	3
• 2 of the above	2
• 1 of the above	1

The sample answer is given on the following page.



Note: The greater magnitude and the smaller time interval of the second peak are due to the bar magnet exiting the solenoid at a faster velocity than its entering velocity due to gravitational acceleration.

Question 5 Solution

Criteria	**Marks**
• Provides Lenz's law to explain the changes in measured weight • Identifies the increase in measured weight during oscillation downwards • Identifies the decrease in measured weight during oscillation upwards • Refers to the induced magnetic polarities of the solenoid during oscillation	4
• 2-3 of the above	2-3
• 1 of the above	1

Sample answer:
As the solenoid oscillates downwards, by Lenz's law, which states that a change in magnetic flux in a conductor will experience an induced current that will oppose the original change in magnetic flux; the solenoid will have an induced North at the bottom. This results in magnetic repulsion between the solenoid and the magnet, increasing the force acting downwards on the magnet, thus the measured weight increases above 36 g. As the solenoid oscillates upwards, similarly by Lenz's law, the solenoid will have an induced magnetic polarity that opposes the movement upwards. Thus, the bottom of the solenoid will have an induced magnetic South, resulting in magnetic attraction between the solenoid and the magnet. This will apply a force upwards on the magnet, reducing the net force downwards on the magnet, and therefore the measured weight of the magnet decreases below 36 g. Thus, as the solenoid oscillates downwards and upwards, the measured weight of the magnet will increase and decrease, respectively.

Question 6 Solution

Criteria	Marks
• Identifies that motion will slow while approaching the magnet • Identifies conversion from kinetic energy to electrical and heat energy • Uses Lenz's law to support answer	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
As the cart approaches the magnet, by Lenz's law, the solenoid will experience an induced current that opposes the original change in magnetic flux, i.e. the induced current will have an induced magnetic field to oppose the motion of the cart. Thus, a magnetic North will be induced at the left of the solenoid, resulting in magnetic repulsion so that a magnetic force acts to the right on the cart, decelerating the cart's velocity. This results in a reduction of the kinetic energy of the cart, and by the conservation of energy, the loss in kinetic energy must be converted to electrical energy as the induced current in the solenoid. Furthermore, this electrical energy will be converted to heat energy due to the resistance of the solenoid.

Transformers SOLUTIONS

Question 1 Solution

Criteria	Marks
• Identifies current decreases to conserve energy • Provides mathematical reasoning to support the answer	2
• 1 of the above	1

Sample answer:
Although Emmy is true in stating that step up transformers increase the voltage in the secondary current, the conservation of energy is not broken as the current decreases in the secondary current. This is shown by the formula:

$$V_\text{p} I_\text{p} = V_\text{s} I_\text{s}$$
$$\therefore P_\text{p} = P_\text{s} \qquad (P = VI)$$
Let t be an arbitrary time interval.
$$P_\text{p} t = P_\text{s} t$$
$$W_\text{p} = W_\text{s} \qquad (W = Pt)$$

Therefore, the energy of the primary coil is conserved.

Question 2 Solution

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

Let $\varepsilon = V$, as electromotive force is a form of voltage.

$$V = -N\frac{\Delta\phi}{\Delta t}$$
$$\frac{V_\text{p}}{N_\text{p}} = -\frac{\Delta\phi}{\Delta t}$$

Assuming no flux is lost as the transformer is ideal, $-\frac{\Delta\phi}{\Delta t}$ will be equal in both the primary coil and the secondary coil.

The solution continues over the page.

$$\frac{V_\mathrm{p}}{N_\mathrm{p}} = \frac{V_\mathrm{s}}{N_\mathrm{s}}$$

$$\frac{V_\mathrm{p}}{V_\mathrm{s}} = \frac{N_\mathrm{p}}{N_\mathrm{s}}$$

$$\text{Now } V_\mathrm{p}I_\mathrm{p} = V_\mathrm{s}I_\mathrm{s}$$

$$\frac{V_\mathrm{p}}{V_\mathrm{s}} = \frac{I_\mathrm{s}}{I_\mathrm{p}}$$

$$\therefore \frac{I_\mathrm{s}}{I_\mathrm{p}} = \frac{N_\mathrm{p}}{N_\mathrm{s}}$$

Question 3 Solution

Criteria	Marks
• Uses Faraday and Lenz's laws to explain current and voltage in secondary circuit • States Faraday and Lenz's law to explain why the DC voltage supply cannot sustain a constant current and voltage in the secondary circuit	3
• Satisfies 3 mark criteria with insufficient detail	2
• Satisfies 1 of the 3 mark criteria	1

Sample answer:
After the DC voltage supply is turned on, the increase in current in the primary coil will increase the magnetic flux through the solid iron core. This change in magnetic flux will result in an induced electromotive force by Faraday and Lenz's law, $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$. Thus, a current and voltage are produced in the secondary coil, and the measured current and voltage will increase. However, as the voltage supply is DC, the current in the primary coil will be constant, thus the magnetic flux in the iron core will become constant. As Faraday's law $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$ states that the emf is produced in proportion to the change of magnetic flux over time, a constant magnetic flux of no change will produce no emf in the secondary coil. This will lead to the voltage of the secondary coil falling to 0, and thus the current will also fall to 0 after momentarily rising.

Question 4 Solution

The transformer is step down, as the current increases in the secondary coil, implying the voltage decreases to follow the conservation of energy.

Data List:

- $I_s = 60\,\text{A}$
- $I_p = 10\,\text{A}$

Solution:

$$V_p I_p = V_s I_s$$
$$\frac{V_p}{V_s} = \frac{I_s}{I_p}$$
$$\text{Now } \frac{V_p}{V_s} = \frac{N_p}{N_s}$$
$$\therefore \frac{N_p}{N_s} = \frac{I_s}{I_p}$$
$$\frac{N_p}{N_s} = \frac{60}{10}$$
$$\frac{N_p}{N_s} = 6$$

Therefore, the ratio of primary coils to secondary coils in the step down transformer is 6.

Question 5 Solution

Criteria	Marks
• Identifies the toroidal shape as a modification • Explains how the toroidal shape improves efficiency • Identifies lamination as a modification • Explains how lamination improves efficiency	4
• 3 of the above	3
• 2 of the above	2
• 1 of the above	1

Sample answer:

Modification 1 - Toroidal Iron Core

The iron core being shaped as a torus minimises flux leakage by providing a continuous, smooth closed loop path for the flux to remain confined in the core. This results in a higher net electromotive force in the secondary coil.

Modification 2 - Lamination

Lamination improves efficiency by reducing losses to eddy currents. By stacking thin insulated layers of iron, the eddy currents produced are confined to smaller loops, decreasing the resistive heat losses.

Question 6 Solution

Criteria	Marks
• Defines a transformer's function • Briefly explains how transformers work • Refers to the changes in voltage in the distribution of electricity • Refers to power losses and how voltages can be adjusted to minimise power losses • Explains how transformers are implemented to minimise power losses	5
• 3-4 of the above	3-4
• 1-2 of the above	1-2

Sample answer:
Transformers are devices that can change the voltage of an alternating current through electromagnetic induction. Transformers work by the primary alternating current running in a loop around a soft iron core to create changes in the magnetic flux inside the core. This will result in the secondary coil experiencing the said changes in magnetic flux, resulting in an electromotive force by Faraday and Lenz's law, $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$. This electromotive force acts as the voltage of the secondary coil, and can be changed by increasing or decreasing the number of loops N. Now, in the distribution of electricity in our society, the electricity is adjusted in voltage to suit the requirements. When electricity is produced, the voltage is relatively low; however, during transmission, the voltage is extremely high to reduce power losses. Then the voltage is decreased during distribution for household usage for safety. High voltages are used during transmission to reduce power losses as mathematically shown.

$$\begin{aligned} P_{\text{loss}} &= VI \\ \text{Now } V &= IR \\ \therefore P_{\text{loss}} &= I^2R \end{aligned}$$

While resistance is not easily overcome due to the natural resistance of transmission lines, the current can be greatly reduced while keeping the power transmitted constant by increasing the voltage. Thus, in the distribution of electricity, step up transformers can be used to increase the voltage during transmission to minimise energy losses, and step down transformers can be used to decrease the voltage during distribution to households for safety.

Applications of the Motor Effect SOLUTIONS

Back emf SOLUTIONS

Question 1 Solution

Criteria	Marks
• Identifies that a static motor will have no back emf, resulting in overheating	1

Sample answer:
If the window is obstructed by some debris, the motor will not be able to turn. Thus, the rotor will become static, leading to the back emf becoming 0. As $V_{\text{net}} = V_{\text{supplied}} - \varepsilon$, this will lead to an increase in the voltage. As $V = IR \implies I = \frac{V}{R} \implies I \propto V$, the current will also increase, thus resulting in overheating.

Question 2 Solution

During the startup of the motor, the rotation speed will momentarily be 0. This will result in no back emf, meaning the net voltage ($V_{\text{supplied}} - \varepsilon$) will be much higher, resulting in a greater current in the rotor, further resulting in a greater force due to the motor effect, leading to a high torque. As the motor approaches its operating speed, the rotor will experience more back emf, resulting in a smaller net voltage, leading to a lower force due to the motor effect and therefore, a lower torque. Thus, the graph of the magnitude of the torque against rotation speed will appear to start at a high initial value and have a negative gradient; hence, the answer is C.

Question 3 Solution

Criteria	Marks
• Identifies that PWM allows for a gradual increase in voltage • Explains how the lack of back emf may damage static motors • Links a gradual increase of voltage to compensation for back emf	3
• 2 of the above	2
• 1 of the above	1

The sample answer is given on the following page.

As shown in the diagram, PWM allows for a gradual increase in average voltage without altering the magnitude of the voltage being supplied. This allows for the average voltage to be gradually increased from 0% to 100% of the voltage supply. This protects motors as PWM can be applied to initially supply a low voltage to motors during the start up as the back emf will be 0 for a static motor. Thus, if 100% of the voltage was supplied to a static motor, the net voltage would be extremely high due to the lack of back emf to reduce the net voltage, potentially damaging the motor due to reasons such as overheating. Then, the average voltage can be increased safely to supply higher voltages to the motor as the back emf increases as the rotation speed of the motor increases.

Question 4 Solution

Criteria	**Marks**
• Identifies how resistance is decreased as rotation speed increases • Explains how the lack of back emf may damage static motors • Links how the changes in resistance for the motor can protect motors	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
The apparatus shown allows for the resistance of the circuit to be initially at a high value, and further decreased as the rotation speed of the motor increases. The initially high resistance is important as in a static motor, the back emf is 0, meaning the supplied voltage will not be reduced. This can lead to excessively high currents, leading to issues such as overheating in the motor. As $V = IR \implies I = \frac{V}{R} \implies I \propto \frac{1}{R}$, the initially high resistance allows for the current to be decreased due to the inverse relationship between current and resistance. Then as the rotation speed increases, the connection point of the resistor moves upwards by the increased centrifugal force by the increased rotation speed, resulting in a lower resistance as the current travels through less of the resistor. This reduction in resistance is compensated by the increased back emf due to the rotation speed of the motor, reducing the net voltage. Thus, the apparatus allows for continuous compensation for back emf to protect the motor.

Question 5 Solution

Criteria	Marks
• Identifies how the star-delta configuration allows for changes in the current • Explains how the lack of back emf may damage static motors • Links how the changes in current for the motor can protect motors	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
The star and delta configurations have different resistances, meaning the resistance of a motor can be changed by switching between the star and delta configurations. As $V = IR \implies I = \frac{V}{R} \implies I \propto \frac{1}{R}$, changing the resistance will allow for changing the current, as current is inversely proportional to resistance. This protects against the lack of back emf in static motors, as when the rotor is static, the motor will have no back emf meaning the voltage supplied will have no reductions, potentially leading to excessive currents due to the high voltages, potentially resulting in overheating. As the motor starts in the star configuration that has a high resistance, the current will be reduced as the rotor approaches its operating speed, preventing excessive currents. After the motor reaches higher rotation speeds, the back emf will increase, allowing for the low resistance delta configuration to be used as the net voltage will have been reduced by the back emf, allowing for the motor to be used safely.

Question 6 Solution

In an IDEAL motor, a rotor already spinning at a constant rate requires no work as the rotor will naturally continue to spin due to the conservation of angular momentum (a conservation law that states objects that are rotating about an axis will continue to do so until an external force impedes the rotation). Thus, the answer is B.

DC Motors SOLUTIONS

Question 1 Solution

Graph A shows that torque is not correlated to side length, and Graph B shows that torque is proportional to side length ($\tau \propto l$), Graph C shows that torque is proportional to side length to some power ($\tau \propto l^n$), and Graph D shows that torque is proportional to the logarithm of side length ($\tau \propto \log_n l$). To determine which is correct, simply find the relationship between torque and side length.

$$\begin{aligned}\tau &= nIA_{\perp}B \\ \tau &= nIl^2B \qquad (A = l^2 \text{ for square coil}) \\ \therefore \tau &\propto l^2\end{aligned}$$

Therefore, the answer is C.

Question 2 Solution

Criteria	Marks
• Identifies split-ring commutator • Explains how the direction of torque is maintained	2
• 1 of the above	1

Sample answer:
The direction of torque is maintained through the split ring commutator, reversing the direction of the current through the loop every 180 degrees of rotation. By reversing the terminals at each end of the loop are in contact with every half rotation, ensuring the torque is maintained in its direction.

Question 3 Solution

$$\tau = nIA_{\perp}B$$
$$\text{Now } V = IR$$
$$I = \frac{V}{R}$$
$$\therefore \tau = \frac{nVA_{\perp}B}{R}$$
$$\therefore \tau \propto V$$

As a 6.0 V supply results in a torque of 0.1 N m, a 1.5× increase in the voltage to 9.0 V will also result in a 1.5× increase in torque to 0.15 N m because $\tau \propto V$.

Question 4 Solution

Criteria	**Marks**
• Explains how the coil experiences the motor effect • Explains how the membrane moves with respect to the coil • Explains how the membrane's movement results in sound waves	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Initially, an electrical signal (i.e. a current) runs in the coil of wire wrapped around the magnetic north. This current in a magnetic field results in the motor effect, as the coil experiences a force along the axis of the magnetic pole. This is shown in the diagram above, as the coil moves left via the motor effect. The coil is attached to the membrane, and therefore the membrane moves in sync with the coil, meaning that when the coil moves back and forth by the electrical signals, the membrane also moves as the coil does. The movement of the membrane results in fluctuations in the pressure of the atmosphere directly in front of the membrane as the air is pushed back and forth by the movement of the membrane, and such pressure waves are transmitted as sound waves. Thus, the loudspeaker converts electrical signals to sound waves in the air.

Question 5 Solution

Firstly, find the torque produced by the motion in the position shown. Use the right hand rule to determine the direction of torque, and also note that $\theta = 70^\circ$ as when the coil's plane is parallel to the magnetic field lines, $\theta = 90^\circ$.

Data List:

- $n = 500$
- $I = 5.0\,\text{A}$
- $A = 0.24\,\text{m}^2$
- $B = 0.40\,\text{T}$
- $\theta = 70^\circ$
- $\tau = ?\,\text{N m}$

Solution:

$$\tau = nIAB\sin\theta$$
$$\tau = (500)(5)(0.24)(0.4)\sin(70^\circ)$$
$$\tau = 225.526\ldots\ \text{N m anticlockwise from perspective shown}$$

To prevent the motor from turning, an equal torque must be applied in the opposite direction (i.e. clockwise), meaning the mass must be placed at End Y. For the magnitude of the mass, equate the magnitude of the torque of the mass and the coil.

Data List:

- $\tau_{\text{coil}} = 225.526\ldots\,\text{N m}$
- $r = 0.2\,\text{m}$
- $m = ?\,\text{kg}$

Solution:

$$|\tau_{\text{mass}}| = |\tau_{\text{coil}}|$$
$$F_{\text{g}}r = \tau_{\text{coil}}$$
$$mgr = \tau_{\text{coil}}$$
$$m = \frac{\tau_{\text{coil}}}{gr}$$
$$m = \frac{225.526\ldots}{(9.8)(0.2)}$$
$$m = 115.064\ldots$$
$$m = 115\,\text{kg}$$

Question 6 Solution

First, find the torque of the hanging mass. Note that the distance of the force of the mass is half of the diameter of the shaft.

Data List:

- $r = 0.005\,\text{m}$
- $m = 0.5\,\text{kg}$
- $\tau = ?\,\text{N m}$

Solution:

$$\tau = rF_\text{g}$$
$$\tau = rmg$$
$$\tau = (0.005)(0.5)(9.8)$$
$$\tau = 0.0245\,\text{N m}$$

As the mass is lifted at a constant velocity, $F_\text{net} = 0$. Thus, the torque of the mass and the torque of the motor must be equal in magnitude and opposite in direction; however, we can simply focus on the magnitudes to find the current (then the power output) of the motor. Furthermore, for the minimum instantaneous power output, the torque applied at said instant moment must be at a maximum whilst keeping the current the lowest, which is only possible if the plane of the coil is parallel to the magnetic field lines. In other words, the power used is at a minimum when $\theta = 90°$. But first, convert the area to the standard unit of m^2.

$$A = 2.0 \times 10^3\,\text{mm}^2$$
$$A = 2.0 \times 10^3(\times 10^{-3}\text{m})^2$$
$$A = 2.0 \times 10^3 \times 10^{-6}\text{m}^2$$
$$A = 2.0 \times 10^{-3}\,\text{m}^2$$

Data List:

- $\tau_\text{mass} = 0.0245\,\text{N m}$
- $A = 2.0 \times 10^{-3}\,\text{m}^2$
- $B = 0.10\,\text{T}$
- $n = 50$
- $I = ?\,\text{A}$

Solution:

$$|\tau_\text{motor}| = |\tau_\text{mass}|$$
$$nIAB = \tau_\text{mass}$$
$$I = \frac{\tau_\text{mass}}{nAB}$$
$$I = \frac{0.0245}{(50)(2 \times 10^{-3})(0.1)}$$
$$I = 2.45\,\text{A}$$

Now find the power output with this current.

Data List:

- $I = 2.45\,\text{A}$
- $V = 12\,\text{V}$
- $P = ?\,\text{W}$

Solution:

$$P = VI$$
$$P = (12)(2.45)$$
$$P = 29.4$$
$$P = 29\,\text{W}$$

AC Motors SOLUTIONS

Question 1 Solution

A three phase induction motor always has the phase differences of 120°. Thus, the answer is C. Other phase differences only exist in special n phase induction motors ($n \in \mathbb{Z}_{>3}$). !?

!? Special n phase induction motors

Although three phase AC induction motors are by far the most common, other phase induction motors also exist, with 4 phase and 5 phase induction motors being used practically in some fields, and even experimental motors with more than 15 phases. 4 phase induction motors can be used in high end robotic arms, as 4 phases can produce a smoother torque with less vibrations, which are valuable in surgical implications. 5 phase induction motors can be used in stealth marine propulsion in submarines as 5 phases can produce a near-constant torque, reducing acoustic noise and thus, reducing the ease of detection. Even 15 phase induction motors have been experimentally developed for possible use in spacecraft where repairs and maintenance are virtually impossible. The phase offsets of an n phase induction motor can be calculated simply by $\frac{360°}{n}$, for example, a 5 phase induction motor will have a phase offset by $\frac{360°}{5} = 72°$.

Question 2 Solution

Criteria	Marks
• States Lenz's law • Uses Lenz's law to explain why the rotor spins in the same direction	2
• 1 of the above	1

Sample answer:
The rotor rotates in the same direction as the stator's rotating magnetic field due to Lenz's law. When the stator's field rotates, the rotor conductors experience a change in magnetic field, and thus, currents are induced in the rotor conductors, which generate their own magnetic field opposing the relative motion between the rotor and stator field — as stated by Lenz's law. This opposition manifests as torque that accelerates the rotor in the same direction as the stator's field. As the rotor speeds up, the relative speed difference (slip) decreases, reducing the rate of flux change and thus, the induced current. In essence, Lenz's law ensures the rotor "chases" the stator's field to minimise flux change—resulting in rotation in the same direction.

Question 3 Solution

Criteria	Marks
• Identifies magnetic field is created in the stator • Refers to the spatial AND time displacement of 120° • Identifies that each magnetic fields create a net rotating magnetic field	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
The rotating magnetic field in a three-phase AC induction motor is generated by the stator, which consists of three windings (or coils) spatially displaced by 120° from each other. Each winding is supplied with an alternating current that is also 120° out of phase with the others. As the three-phase currents alternate, the magnetic field produced by each winding pulsates in magnitude and reverses direction in a synchronised sequence. The combined effect of these three time-shifted magnetic fields creates a net magnetic field that rotates smoothly in space at a constant speed.

Question 4 Solution

Criteria	Marks
• Describes the role of both components	4
• Describes the role of one component • Identifies the role of the other	3
• Identifies the roles of both components	2
• Identifies the role of one component	1

Sample answer:
The stator serves as the motor's stationary electromagnetic core, specifically designed to generate a rotating magnetic field. This is achieved through three windings that are displaced radially by 120° in spatial orientation around the stator core and electrically powered by three-phase AC currents that are phase-shifted by 120°. The spatial orientation and the phase difference of the windings work to form a resultant magnetic field that rotates smoothly at a constant speed.
The solution continues over the page.

The rotor, typically constructed as a squirrel-cage assembly (comprising conductive bars short-circuited by end rings), interacts dynamically with the rotating magnetic field to rotate. As the stator's magnetic field sweeps past the rotor conductors, it induces electromotive forces according to Faraday's law of electromagnetic induction ($\varepsilon = -\frac{\Delta\phi}{\Delta t}$). These induced currents, whose direction is determined by Lenz's Law to oppose their cause (the relative motion between rotor and stator field), generate their own magnetic fields. The interaction between the magnetic fields of the stator and the conductive bars of the rotor results in a force on the conductive bars due to the motor effect, resulting in torque, and thus, the rotor rotates.

Question 5 Solution

Criteria	**Marks**
• Provides Faraday's law of $\varepsilon = -\frac{\Delta\phi}{\Delta t}$ • Explains why the rotor speed can never be exactly the synchronous speed	3
• Provides Faraday's law of $\varepsilon = -\frac{\Delta\phi}{\Delta t}$ • Poorly explains why the rotor speed can never be exactly the synchronous speed	2
• 1 of the above	1

Sample answer:
The rotor in an AC induction motor spins due to electromagnetic induction, governed by Faraday's law. As the stator's rotating magnetic field sweeps past the rotor conductors, it creates a changing magnetic flux that induces currents in the rotor by Faraday's law of induction $\varepsilon = -\frac{\Delta\phi}{\Delta t}$. These induced currents generate their own magnetic field, which interacts with the stator's field to produce torque through the motor effect, causing the rotor to turn. Crucially, this torque depends on the relative speed difference between the rotating stator field (synchronous speed) and the rotor speed. As the rotor accelerates and approaches the synchronous speed, the rate of change of magnetic flux through the rotor conductors decreases as the differences in their rotational speed decrease, reducing the induced current and resulting torque. At exactly synchronous speed, there would be no relative motion between the rotor and stator field, meaning no flux change ($\frac{\Delta\phi}{\Delta t} = 0$), no induced current ($\varepsilon = 0$), and consequently no torque. This is why the rotor always operates slightly slower than synchronous speed, with this difference in rotational speed being referred to as slip.

Question 6 Solution

Criteria	Marks
• Explains how the stator produces a magnetic field • Explains how the stator's magnetic field leads to rotation in the rotor • Clearly shows how electrical energy is converted to mechanical energy	5
• Explains how the stator produces a magnetic field with limited detail • Explains how the stator's magnetic field leads to rotation in the rotor with limited detail • Shows how electrical energy is converted to mechanical energy	3-4
• Attempts to explain how the stator produces a magnetic field or how the rotor spins • Refers to electrical energy and mechanical energy	1-2

Sample answer:
Initially, electrical energy is supplied to a three phase AC induction motor in the form of a three phase AC power supply. This AC is supplied to the stator's three distinct windings, physically displaced by 120° degrees around the motor's circumference. Furthermore, each winding has its AC phase electrically offset by 120° in time. When powered, the alternating currents in each winding produce continuously reversing magnetic fields. Due to the combined effect of the spatial displacement and phase shift, these individual magnetic fields superimpose to create a resultant magnetic field that rotates at a constant speed. This rotating magnetic field is fundamental to motor operation, establishing the basis for electromagnetic torque production. The rotor, typically constructed as a squirrel cage design with conductive bars short-circuited by end rings, interacts with the stator's rotating field. As this field sweeps past the rotor conductors, it induces electromotive forces (emf) according to Faraday's Law of electromagnetic induction ($\varepsilon = -\frac{\Delta\phi}{\Delta t}$). The relative motion between the rotating field and stationary rotor (at startup) creates a changing magnetic flux linkage, resulting in induced currents in the rotor conductors. These currents generate their own magnetic fields which, following Lenz's Law, oppose the change producing them - in this case, the relative motion between rotor and stator field. The interaction between the stator's rotating magnetic field and the rotor's induced magnetic field results in a force by motor effect on the current carrying rotor conductors. This force manifests as torque about the motor's axis, causing rotor acceleration, and thus, as the rotor rotates, electrical energy is converted to mechanical energy.

DC and AC Generators SOLUTIONS

Question 1 Solution

Criteria	Marks
• Lists 4 changes	2
• Lists at least 2 changes	1

Sample answer:

- Increasing the number of coils ($\varepsilon = -N\frac{\Delta\phi}{\Delta t} \implies \varepsilon \propto N$)
- Increasing rotational speed of coil ($\varepsilon = -N\frac{\Delta\phi}{\Delta t} \implies \varepsilon \propto \frac{\Delta\phi}{\Delta t}$)
- Increase strength of magnetic field ($\varepsilon = -N\frac{\Delta\phi}{\Delta t}$, a stronger magnetic field will lead to a greater change in magnetic flux, leading to a greater change in magnetic flux over time)
- Reduce the air gap between the coil and magnets (Due to the low magnetic permeability of air, reducing the air gap will reduce magnetic flux lost, leading to greater changes in magnetic flux for the coil)

Question 2 Solution

Criteria	Marks
• Explains Faraday's law's significance • Explains Lenz's law's significance • Links both laws to the purpose of a motor for converting mechanical energy to electrical energy	3
• 2 of the above	2
• 1 of the above	1

A sample answer is given on the following page.

In a generator, mechanical energy is used to rotate a coil or conductor within a magnetic field. According to Faraday's law of electromagnetic induction, a change in magnetic flux through the coil induces an electromotive force (emf), expressed mathematically as $\varepsilon = -\frac{\Delta\phi}{\Delta t}$. This induced emf drives a current through the circuit. Lenz's law determines the direction of the induced current, stating that the induced current will produce a magnetic field that opposes the change in magnetic flux that caused it. This opposition creates a resistive force against the rotor's motion, illustrating the conservation of energy. Together, Faraday's and Lenz's laws explain how generators convert mechanical energy into electrical energy.

Question 3 Solution

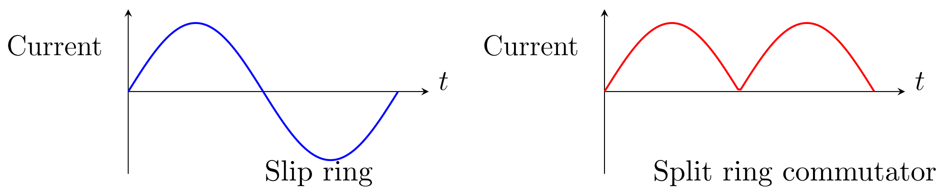
Criteria	**Marks**
• Explains how Faraday's law results in current • Explains how Lenz's law results in AC • Explains how slip rings are utilised	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
An AC generator converts mechanical energy into electrical energy by rotating a coil (or rotor) within a magnetic field. Mechanical energy is supplied to the generator to rotate the coil, which consists of conductors moving through magnetic field lines. According to Faraday's law of electromagnetic induction ($\varepsilon = -N\frac{\Delta\phi}{\Delta t}$), a changing magnetic flux through the rotating coil induces an electromotive force (emf). This induced emf drives a current through the coil. Lenz's law states that the direction of the induced current is such that its magnetic field opposes the change in flux that caused it. As the coil continuously rotates, the orientation of the coil relative to the magnetic field changes, causing the magnetic flux to vary sinusoidally. This results in an alternating emf and therefore an AC. The generated AC is transferred to an external circuit through slip rings and carbon brushes, which maintain continuous electrical contact with the spinning coil. Thus, the generator converts mechanical energy into electrical energy in the form of alternating current.

Question 4 Solution

Criteria	Marks
• Provides functions for both slip ring and split ring commutators • Provides similarity or dissimilarity in their functions • Provides a correct sketch of both currents	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Slip rings are conductive rings connected to the rotating coil of an AC generator, allowing the coil to rotate freely while continuously transmitting the alternating current produced in the coil to the external circuit. In contrast, split-ring commutators are used in DC generators to reverse the connection every 180°, ensuring that the direction of the output current remains constant even though the current in the rotating coil reverses, effectively converting the AC generated in the coil into a DC output. Both slip rings and split ring commutators serve the purpose of transmitting electricity from the coil of a generator to an external circuit, but they differ in the type of current they produce: slip rings transmit AC, while split ring commutators transmit DC. The currents they produce are shown below.

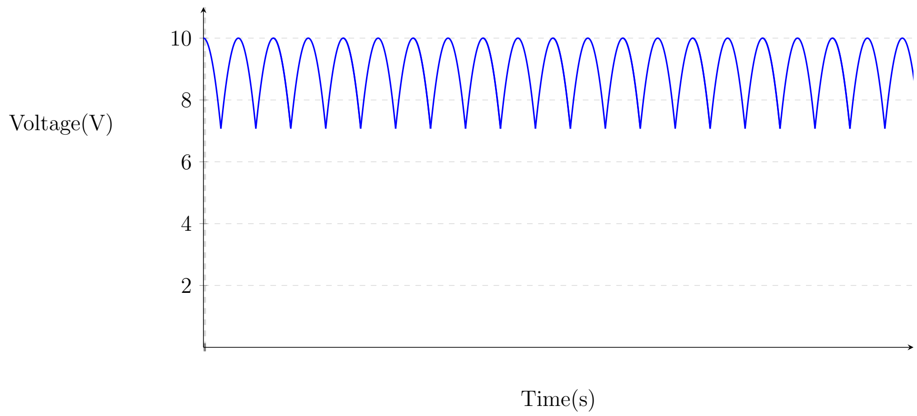


Question 5 Solution

Criteria	Marks
• Identifies a change to the DC generator to reduce noise • Justifies the change to the DC generator to reduce noise • Provides a graph of the current after the change is implemented	3
• 2 of the above	2
• 1 of the above	1

A sample answer is given on the following page.

Another coil can be set at right angles to the single coil of the DC generator. This will reduce the effects of the minimums of change in flux the coil experiences as the improved coil will always be at a higher average of change in flux compared to the original coil, allowing for Faraday's law of electromagnetic induction $\varepsilon = -\frac{\Delta \phi}{\Delta t}$ to induce a more constant voltage, therefore reducing noise. A graph of the DC voltage drawn after this change is shown below.



Question 6 Solution

Criteria	Marks
• Provides 3 advantages • Explains how each advantage makes AC generators more suitable	4
• Provides 1-2 advantages • Explains how each advantage makes AC generators more suitable	2-3
• 1 of the above	1

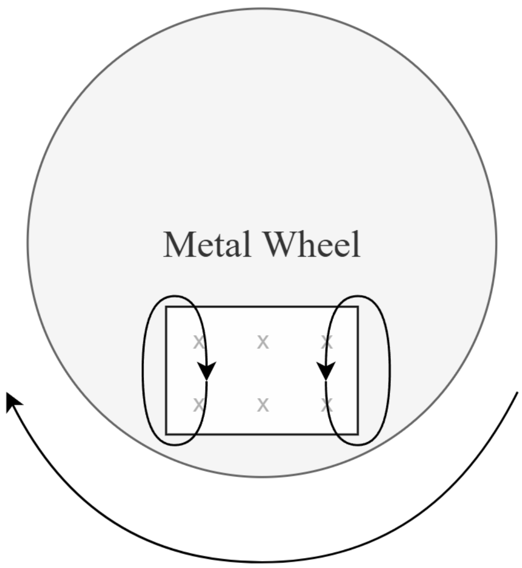
Sample answer:
AC generators are better suited for large-scale applications due to their advantages in lower maintenance requirements, the ability to draw current from the stator, and the transformability of AC. The lower maintenance of AC generators stems from the use of slip rings instead of split ring commutators; split ring commutators experience heavier wear due to the friction from their brushes, whereas slip rings have continuous contact, reducing wear. Additionally, in many AC generators, current can be drawn from the stationary stator, whereas DC generators require current to be drawn from the rotor, making them less efficient and less safe for large scale use. Moreover, AC can be easily stepped up or down using transformers due to its constantly changing nature, which allows for Faraday's law of induction to induce voltage in the transformer. In contrast, DC cannot be used with transformers because it does not change, meaning there is no changing magnetic flux to induce voltage.

Applications of the Conservation of Energy in Electromagnetism SOLUTIONS

Question 1 Solution

Criteria	Marks
• Correctly draws eddy currents	1

Sample answer:



Question 2 Solution

Criteria	Marks
• States Faraday's law of electromagnetic induction • Correctly explains why the magnetic field must be alternating	2
• 1 of the above	1

Sample answer:
The magnetic field must be alternating. This is because for the induction cooktop to induce eddy currents in cookware, it must be by a changing magnetic flux as stated by Faraday's law of electromagnetic induction, $\varepsilon = -\frac{\Delta \Phi}{\Delta t}$, meaning the magnetic field cannot be constant.

Question 3 Solution

Criteria	Marks
• Explains how lamination results in smaller eddy currents • Explains how smaller eddy currents result in a greater efficiency	2
• 1 of the above	1

Sample answer:
Lamination improves the efficiency of a metal core by reducing energy losses due to eddy currents. Laminating the core involves stacking thin, insulated layers of metal rather than using a solid piece. The insulation between the layers prevents a current between the layers, restricting eddy currents to smaller loops. This reduces the magnitude of the currents and therefore the energy lost as heat. As less energy is wasted, the overall efficiency of the device increases through the conservation of energy.

Question 4 Solution

Criteria	Marks
• Identifies changing magnetic flux in regions of the wheel resulting in EM induction • Identifies the conversion of kinetic energy to electrical and thermal energy	2
• 1 of the above	1

Sample answer:
The rotating metal wheel enters a non-uniform magnetic field, with the field strength increasing near the bar magnets. As different regions of the wheel move into and out of this stronger magnetic region, they experience a changing magnetic flux. According to Faraday's Law, this induces eddy currents within the conducting metal. These eddy currents are a form of electrical energy which arises from the conversion of the wheel's kinetic energy. Due to natural resistance in the metal, the eddy currents generate thermal energy as they interact with the atomic metal lattice structure. By the conservation of energy, the wheel's kinetic energy is therefore transformed into both electrical and heat energy, producing a braking effect that slows the wheel's rotation.

Question 5 Solution

Criteria	Marks
• Identifies alternating magnetic field results in a change in magnetic flux • Explains how Faraday's law of induction results in eddy currents • Explains how Lenz's law results in a disturbance in the magnetic field	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
An alternating magnetic field from a metal detector induces a changing magnetic flux in nearby metal objects. According to Faraday's Law, this change in flux generates an electromotive force (emf), which induces eddy currents within the metal. By Lenz's Law, these eddy currents produce their own magnetic field that opposes the original field. This opposition causes a detectable disturbance in the metal detector's alternating magnetic field, indicating the presence of a metal object.

Question 6 Solution

Criteria	Marks
• Identifies the reason to be eddy currents • Explains that eddy currents are larger in apparatus A compared to apparatus B • Links conservation of energy to apparatus A coming to rest sooner	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Apparatus A comes to rest in a shorter time frame because it experiences a greater kinetic energy loss due to larger eddy currents. In contrast, the slits in Apparatus B reduce the continuous conducting area, limiting the formation of eddy currents. As a result, smaller eddy currents are induced, requiring less energy to sustain. This means less kinetic energy is converted into electrical energy, so Apparatus B retains its motion longer. By the conservation of energy, Apparatus A loses kinetic energy at a faster rate, causing it to come to rest sooner.

The Nature of Light
SOLUTIONS

The Electromagnetic Spectrum SOLUTIONS

Maxwell's Theory of Electromagnetism SOLUTIONS

Question 1 Solution

The speed of light is given by the expression:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

where ε_0 is the permittivity of free space and μ_0 is the permeability of free space.

Question 2 Solution

Criteria	Marks
• Links previously experimental value of speed of light to Maxwell's derivation.	1

Sample answer:
After deriving the speed of light from his equations as $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8\,\mathrm{m\,s^{-1}}$, Maxwell compared it to the previously experimentally measured speed of light, which was also approximately $3 \times 10^8\,\mathrm{m\,s^{-1}}$. This led him to conclude that light is an electromagnetic wave.

Question 3 Solution

Criteria	Marks
• Identifies model as Huygens' wave theory of light.	1

Sample answer:
The most widely accepted model of light, previous to Maxwell's theory, was Huygens' wave theory of light.

Question 4* Solution

Criteria	Marks
• Identifies and describes all 4 equations.	2
• Identifies and describes 2 equations.	1

Sample answer:
The 4 equations are:

1. **Gauss's Law for Electricity:** States that the electric flux through a closed surface is proportional to the charge enclosed by the surface.
2. **Gauss's Law for Magnetism:** States that the magnetic flux through a closed surface is a net zero, implying magnetic monopoles do not exist.
3. **Faraday's Law of Induction:** States that a changing magnetic field induces an electromotive force (emf) in a closed loop.
4. **Ampère-Maxwell Law:** States that a changing electric field or current produces a magnetic field.

Question 5 Solution

Criteria	Marks
• Identifies the unification of electricity and magnetism • Identifies the prediction of electromagnetic waves • Identifies the derivation of the speed of light	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Maxwell's theory of electromagnetism unified electricity and magnetism into a single framework, showing that they are not separate forces but rather two aspects of electromagnetic interactions. His addition to Ampère's Law demonstrated that a changing electric field or current produces a magnetic field, predicting the existence of electromagnetic waves. Furthermore, Maxwell identified light as an electromagnetic wave, and mathematically derived the speed of light c to be $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8\,\mathrm{m\,s^{-1}}$. Maxwell's theory has since taken on the role of being the foundation of classical electromagnetism.

Question 6 Solution

Criteria	Marks
• Identifies the unification of electricity and magnetism • Identifies the prediction of electromagnetic waves • Identifies the shift of emphasis from forces to fields	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
Previous to Maxwell's theory, electricity and magnetism were considered to be two separate phenomena. Maxwell's theory unified them into a single framework, showing they were two aspects of electromagnetic interactions. He further showed the close relationship between electricity and magnetism through his addition to Ampère's Law, which shows that a changing electric field or current produces a magnetic field, predicting the existence of self-propagating electromagnetic waves. His theory also marked a shift in the understanding of fields, as the previous Newtonian framework of "forces at a distance" was challenged with the idea of fields being the primary entities of how things interact. Due to the previously mentioned changes in understanding, Maxwell's theory has profoundly impacted the knowledge of electricity & magnetism and fields.

Production and Propagation of EM Waves SOLUTIONS

Question 1 Solution

Electromagnetic waves are transverse waves made of in-phase oscillating electric and magnetic fields that are perpendicular to each other. Hence, B is the answer.

Question 2* Solution

The most relevant of Maxwell's equations in the explanation of the self-propagation of electromagnetic waves are Faraday's Law and the Ampère-Maxwell Law (as shown from top down)

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \text{ or } \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \text{ or } \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi}{dt}$$

To explain the propagation of electromagnetic waves through these two equations, first take a change in the electric field. By the Ampère-Maxwell Law, the change in electric field will result in a change in magnetic field. By Faraday's Law, the change in magnetic field will then result in a change in electric field. This cycle continues, as the electric field and the magnetic field continue to propagate each other, resulting in the self-propagation of an electromagnetic wave. Thus, D is the answer.

Question 3 Solution

Criteria	Marks
• Identifies that the diagram is inaccurate • Identifies that the fields are not in phase	1

Sample answer:
Emmy's diagram is inaccurate. It can be seen on Emmy's diagram that the two fields of the wave are 90° out of phase, while electromagnetic waves have oscillating fields that are in phase.

Question 4 Solution

Criteria	Marks
• Explains self-propagation	2
• Identifies self-propagation	1

Sample answer:
Electromagnetic waves consist of perpendicular in-phase electric and magnetic fields. The changing magnetic field induces a change in the electric field by Faraday's Law, and the changing electric field induces a change in the magnetic field by the Ampère-Maxwell law. This is known as self-propagation, and hence electromagnetic waves are capable of travelling in a vacuum.

Question 5 Solution

Criteria	Marks
• Explains how AC is capable of producing constant EMR • Explains how DC is capable of producing constant EMR	4
• Satisifes the 4 mark criteria with 1-2 error(s)/emission(s)	2-3
• Some relevant information	1

Sample answer:
AC is capable of producing continuous electromagnetic radiation. This is because the electrons of AC current are constantly moving back and forth, and thereby are constantly accelerating. As the electrons are naturally constantly accelerating, AC naturally produces continuous electromagnetic radiation. DC is also capable of producing continuous electromagnetic radiation. Although a straight wire of DC will not produce electromagnetic radiation as the electrons of the DC will be in uniform motion, if the wire has a bend, this will result in the electrons passing through said bend to experience acceleration. Thus, by a change in direction, such as a bend, DC is also capable of producing continuous electromagnetic radiation.

Question 6 Solution

Criteria	Marks
• Links the oscillating electric field to a resulting oscillating magnetic field • Links oscillating fields to the propagation of an electromagnetic radio wave • Links the frequency of electrons' movement to the frequency of radio wave • Links the electron's movement in the receiver antenna to the radio wave	4
• 2-3 of the above	2-3
• 1 of the above	1

Sample answer:
The oscillating electrons of the AC in the transmitter antenna produce an oscillating electric field, which in turn generates an in-phase oscillating magnetic field as described by the Ampère-Maxwell Law. The pair of in-phase oscillating fields produces a self-propagating electromagnetic radio wave. The frequency of the oscillating electric field, and thus the frequency of the radio wave corresponds to the frequency of the oscillation of the electrons in the transmitter antenna. When this wave reaches the receiver antenna, the oscillating electric field exerts a force on the electrons of the receiver antenna, causing them to oscillate at the same frequency as the wave and thus the same frequency as the electrons of the transmitter antenna.

Investigations of the Speed of Light SOLUTIONS

Question 1 Solution

Ole Rømer is the Danish astronomer who utilised the variations of the apparent orbital periods of Io to determine the speed of light in 1676. Thus, the answer is A.

Question 2 Solution

Criteria	Marks
• Identifies all figures correctly	2
• One error	1

Sample answer:
Experiment 1 is the investigation carried out by James Bradley in 1728 that measured the speed of light through stellar aberration. Experiment 2 is the investigation carried out by Léon Foucault in 1850 that measured the speed of light through a rotating mirror. Experiment 3 is the investigation carried out by Hippolyte Fizeau in 1849 that measured the speed of light through a rotating cog wheel.

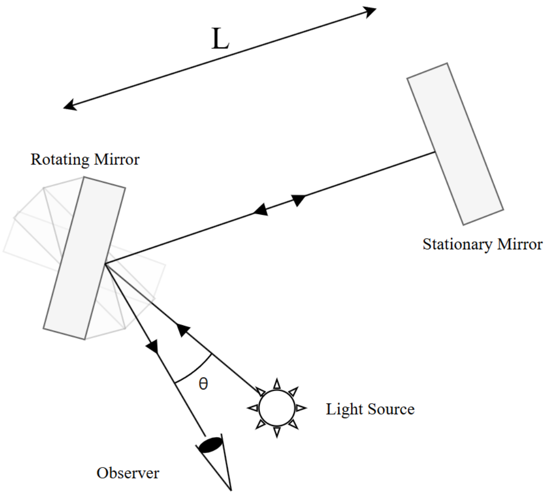
Question 3 Solution

Criteria	Marks
• Identifies the impact of discovering the finite speed of light	1

Sample answer:
Although the value Rømer attained for the speed of light is considered inaccurate by modern standards, Rømer's investigation quantitatively showed that the speed of light was not infinite as believed during his period, but finite, marking an important turning point in scientific history in the understanding of light.

Question 4 Solution

The diagram of the experiment is given below for reference.



Firstly, assume that the time it takes for light to travel from the light source to the mirror, and the time it takes for the light it takes to reach the observer after being reflected is 0. Although it isn't obvious from the diagram, due to the speed of light being very high and the relatively short distance between the observer, the light source and the rotating mirror, it does not considerably affect the accuracy of the measured speed of light to make this assumption.

$$\text{Now } \omega = \frac{\Delta\theta}{\Delta t}$$

Now the angle mirror rotates is half of the angle of deflection

$$\therefore \omega = \frac{\Delta\theta}{2\Delta t}$$

$$t = \frac{\Delta\theta}{2\omega}$$

$$\text{Now } v = \frac{d}{t}$$

$$c = d \div t$$

$$c = 2L \div \frac{\theta}{2\omega}$$

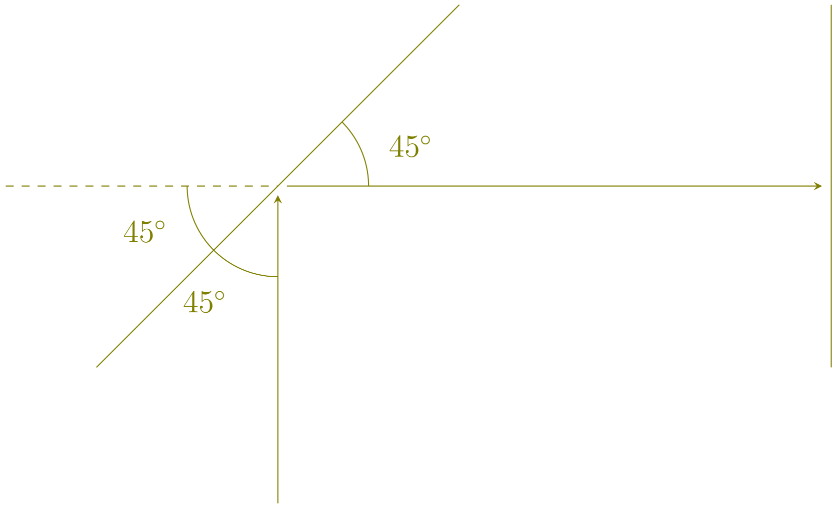
$$c = 2L \times \frac{2\omega}{\theta}$$

$$c = \frac{4L\omega}{\theta}$$

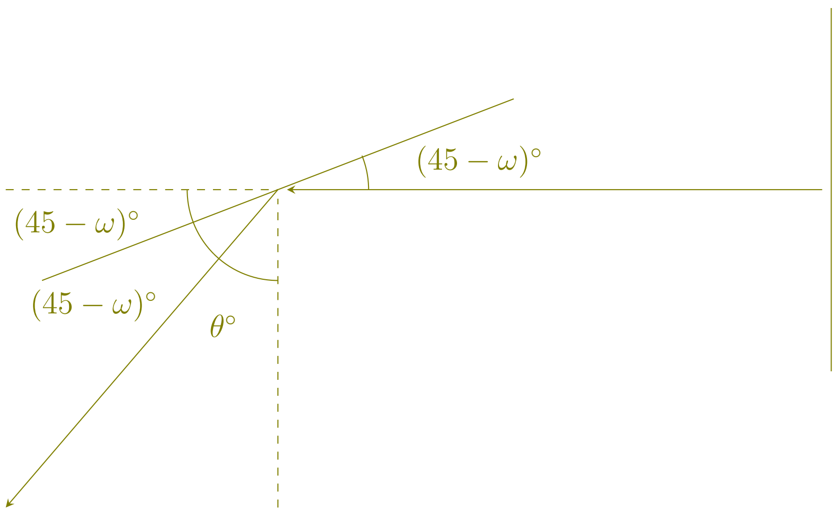
Foucault's Experiment

The reason to why the angle of deflection is twice the angle of rotation of the mirror is because to find the angle of deflection, we must consider both the angle of rotation of the mirror itself, and the angle of incidence of the light ray as it is reflected. This is clarified in the diagrams below.

Before Rotation



After Rotation



$$\begin{aligned} \therefore (45-\omega)^\circ + (45-\omega)^\circ + \theta^\circ &= 90^\circ \\ (90 - 2\omega + \theta)^\circ &= 90^\circ \\ \theta &= 2\omega \end{aligned}$$

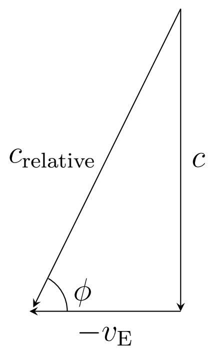
Hence, the angle of deflection is twice the angle of rotation of the mirror.

Question 5 Solution

First, find the orbital velocity of the Earth. This is studied in Module 5, Advanced Mechanics.

$$
\begin{aligned}
v_\mathrm{E} &= \sqrt{\frac{GM}{r}} \\
v_\mathrm{E} &= \sqrt{\frac{(6.67 \times 10^{-11})(1.989 \times 10^{30})}{1.496 \times 10^{11}}} \\
v_\mathrm{E} &= 2.977\ldots \times 10^{4}\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Now, although the light rays are coming directly down, due to the moving frame of the Earth, to find the relative apparent velocity of the light, the velocity of the Earth must be subtracted from the velocity of the light.

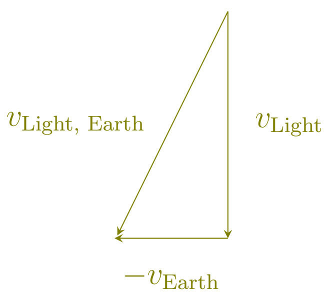


$$
\begin{aligned}
\tan\phi &= \frac{c}{v_\mathrm{E}} \\
\phi &= \tan^{-1}(\frac{c}{v_\mathrm{E}}) \\
\theta &= 90 - \phi \\
\theta &= 90 - \tan^{-1}(\frac{c}{v_\mathrm{E}}) \\
\theta &= 90 - \tan^{-1}(\frac{3 \times 10^{8}}{2.977\ldots \times 10^{4}}) \\
\theta &= 0^\circ\ 0'\ 20.47'' \\
\theta &= 20''
\end{aligned}
$$

Relative Velocities

Relative velocities, further explored later in this module, refer to how fast and in what direction an object is travelling relative to another object. To intuitively understand what this means, imagine a red Mazda and a white Toyota both travelling towards each other at $10\,\mathrm{m\,s^{-1}}$. Now, take away the road, and the Earth, so that the scenario is simplified to the red Mazda and the white Toyota travelling towards each other in free space at $10\,\mathrm{m\,s^{-1}}$. Now, notice that this scenario is the same as the red Mazda being stationary and the white Toyota travelling towards the red Mazda at $20\,\mathrm{m\,s^{-1}}$, or the white Toyota being stationary and the red Mazda travelling towards the white Toyota at $20\,\mathrm{m\,s^{-1}}$. This is because relative to each other, the two objects are approaching at a rate of $20\,\mathrm{m\,s^{-1}}$. Now let's return to the example of the light and the velocity of the Earth. To simplify the scenario, imagine that the light is a particle travelling at $3 \times 10^8\,\mathrm{m\,s^{-1}}$, and that the observer is moving towards the right at the orbital velocity of the Earth. This is visualised in the diagram below.

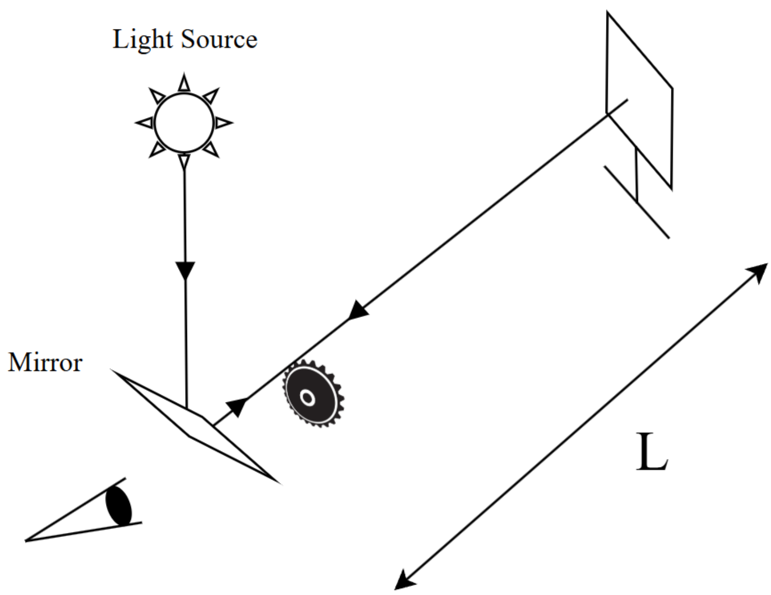
Now, to think of it intuitively, from the perspective of an observer of the Earth it will seem as if the light ray is angled to the left. Another way to intuitively understand this is to think of rain. If you stand still under the rain (without wind), it will appear to you that the rain is dropping directly downwards. However, if you run through the rain, it will appear as if the rain is travelling towards you to hit you. Now, to return to the example of the light and Earth again, the velocity of the light relative to an observer on Earth can be calculated using the relative velocity formula $v_{\mathrm{ab}} = v_{\mathrm{a}} - v_{\mathrm{b}}$, thus $v_{\text{Light, Earth}} = v_{\text{Light}} - v_{\text{Earth}}$. This is visually shown below



If it still isn't clear, it is recommended for you to independently study relative velocities, as it will be further explored later in this textbook.

Question 6 Solution

Firstly, it is known that the number of teeth and gaps sum to $2n$, as there are an equal number of gaps and teeth, and there are n teeth. It is also known that the frequency of the cog is f. Thus, let the period of the cog be $T = \frac{1}{f}$. Now we know that the time for the cog to turn through all of its teeth (which are $2n$ teeth and openings, as there are an equal number of teeth as gaps) is T. Thus, the time for the cog to turn through only 1 tooth or opening is $\frac{T}{2n}$ or $\frac{1}{2nf}$. Thus, let $t = \frac{1}{2nf}$. Now it is known that the light ray travels to and back from the stationary mirror as shown below.



Thus, the distance that the light ray travels is $2L$, and since it is blocked by the subsequent tooth of the cog, the time it takes for the light ray to travel this distance is $t = \frac{1}{2nf}$. Now find the expression for the speed of light as so.

$$v = \frac{d}{t}$$
$$c = d \div t$$
$$c = 2L \div \frac{1}{2nf}$$
$$c = 4Lnf$$

Spectra of Stars SOLUTIONS

Question 1 Solution

Criteria	Marks
• Correctly identifies the similar spectra • Provides justification	2
• Correctly identifies the similar spectra	1

Sample answer:
The Sun and the incandescent filament will have the most similar spectra as they are both black body radiators. The gas discharge tube will have the most unique spectrum as it is not a black body radiator, but will emit electromagnetic radiation only at specific wavelengths.

Question 2 Solution

Criteria	Marks
• Correctly identifies both elements present	2
• Correctly identifies only 1 element present	1

Sample answer:
It can be seen that the discharge tube's spectra lines consist of the Helium spectra lines and the Neon spectra lines. Thus the discharge tube consists of Helium and Neon.

Question 3 Solution

Criteria	Marks
• Identifies radial velocity away from Earth • Identifies rotational velocity • Provides justification	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
It can be seen that the spectral lines of Zeta have been redshifted and broadened. The red shift of the spectra lines allows for the determination that Zeta has a radial velocity away from the Earth to cause the red shift by the Doppler effect, and the broadening of the spectra lines has been caused by the rotational velocity of Zeta. Thus, Zeta has a radial velocity away from Earth and has a rotational velocity.

Question 4 Solution

Criteria	Marks
• Describes the importance of spectroscopy in the identification of elements • Utilises an example	2
• Describes the importance of spectroscopy in the identification of elements	1

Sample answer:
Electromagnetic spectroscopy can be utilised to identify elements due to the unique spectral lines of each element. For example, helium was discovered in 1868 by French astronomer Jules Janssen. As the spectra lines of helium had not yet been discovered, and did not match the spectra lines of any other element, it could be identified that the then unknown element was a new element, now named helium.

Question 5 Solution

It can be seen that elements A and B have relatively equally spaced spectral lines. Although Zeta's spectral lines are between both elements, due to the information that Zeta is travelling towards Earth, it can be known that the spectral lines of Zeta have been blue shifted. Thus, the true spectra lines of Zeta can be found by red shifting the apparent (blue shifted) spectral lines. Since shifting the Zeta spectra lines to the red side results in the spectra lines of element B, it can be determined that Zeta is composed of element B.

Question 6 Solution

Criteria	**Marks**
• Describes the determination of surface temperature • Describes the determination of radial velocity • Describes the determination of rotational velocity • Describes the determination of density • Describes the determination of chemical composition	7
• Satisfies all of the above, however, to less detail OR • Fails to address 1 of the 7 mark criteria	5-6
• Fails to address 2 of the 7 mark criteria	3-4
• Provides some relevant information	1-2

Sample answer is given on the following page:

The electromagnetic spectrum of a star can allow for the determination of its surface temperature, radial and rotational velocity, density and chemical composition.

Surface Temperature
By examining the intensities of the electromagnetic wavelengths emitted by the star, the surface temperature of the star can be calculated by Wien's displacement law $\lambda_{\text{max}} = \frac{b}{T} \implies T = \frac{b}{\lambda_{\text{max}}}$, where λ_{max} is the most intense wavelength of the spectrum, b is Wien's displacement constant, and T is the surface temperature of the star.

Velocity
By examining the spectral lines of the star's spectrum, both the direction and magnitude of the radial velocity of the star can be determined using the Doppler shift of such lines. This can be achieved by analysing the displacement of the spectral lines through Doppler's formula $f' = f\frac{v_{\text{wave}}+v_{\text{observer}}}{v_{\text{wave}}-v_{\text{source}}}$. The rotational velocity of the star can be inferred from the broadening of the spectral lines. This is because if a star has a rotational velocity, one half of the star will be redshifted (rotating away from the observer) and the other half of the star will be blue shifted (rotating towards the observer). The extent of this broadening allows for the determination of the magnitude and direction of the star's angular velocity, also by utilising Doppler's formula.

Density
By examining the spectral lines of the star's spectrum, the density of the star can be determined. This is because the pressure caused by the star's density affects the electron clouds of atoms, causing interactions between neighbouring atoms' electron clouds. This then changes the energy transitions of the atoms by Bohr's postulate $\Delta E = E_{\text{initial}} - E_{\text{final}}$, thus changing the energy of the photons produced from these transitions. These energy changes correspond to a change in the apparent wavelength of the photons by $E = hf$, and thus, the density of a star can be determined through its spectral lines.

Chemical Composition
By examining the specific wavelengths of absorption and emission lines in a star's spectrum, the chemical composition of the star can be identified. Each element produces a unique set of spectral lines at characteristic wavelengths due to electron transitions between energy levels. By matching these spectral lines to known atomic signature spectra lines, the presence and relative abundance of different elements within the star can be determined. Thus, a star's chemical composition can be determined through its spectral lines.

In summary, the analysis of a star's electromagnetic spectrum provides a powerful tool to determine key stellar properties such as surface temperature, velocity, density, and chemical composition.

Light: Wave Model SOLUTIONS

Diffraction and Interference of Light SOLUTIONS

Question 1 Solution

The phenomenon shown is diffraction. As the wavefront approaches the slit (or aperture), it bends and spreads out radially after passing through, forming a diffraction pattern. Hence, the answer is A.

Question 2 Solution

The phenomenon that produces the pattern of bands is interference. As two diffraction patterns overlap, they interfere constructively and destructively, forming an interference pattern consisting of bright regions (constructive interference) and dark regions (destructive interference). Hence, the answer is D.

Question 3 Solution

Criteria	**Marks**
• Links the interference pattern shown by the experiment as support for the wave model of light	2
• Some relevant information	1

Sample answer:
Young's double-slit experiment formed an interference pattern by directing a coherent, monochromatic source of light at two slits. This could only be produced by diffraction occurring at both slits, and the two wavefronts interfering with each other constructively and destructively. This contradicted the predictions made by Newton's corpuscular theory of light, as it could not account for diffraction or interference. However, this aligned with Huygens' wave model of light, which was capable of both. Thus, Young's experiment provided support for Huygens' model of light.

Question 4 Solution

Criteria	Marks
• Identifies coherence • Identifies monochromaticity	1

Sample answer:
The two aspects required for a light source to form an interference pattern in Young's experiment are coherence and monochromaticity.

Note: Coherence means the light waves are in phase, and monochromaticity means the light consists of only one wavelength.

Question 5 Solution

Criteria	Marks
• Identifies the longer wavelength of red light • Clearly supports why the longer wavelength leads to a greater deviation	2
• 1 of the above	1

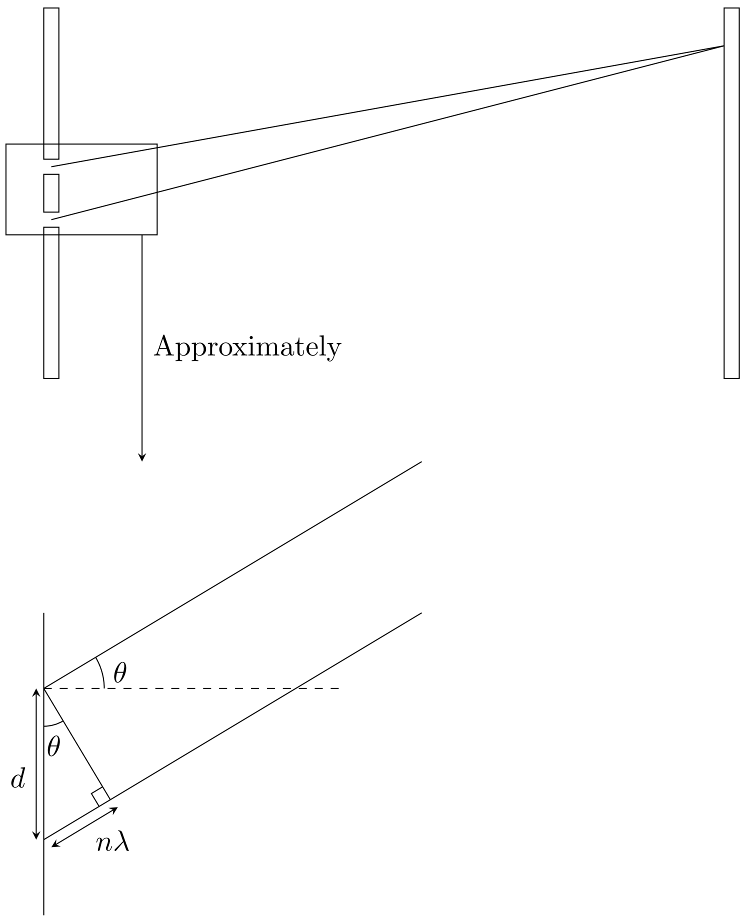
Sample answer:
Red light has a longer wavelength than violet light. The angle of deviation can be derived from the diffraction grating equation as follows.

$$
\begin{aligned}
m\lambda &= d\sin\theta \\
\sin\theta &= \frac{m\lambda}{d} \\
\theta &= \sin^{-1}\left(\frac{m\lambda}{d}\right) \\
\theta &= \sin^{-1}(k\lambda) \text{ where } k = \frac{m}{d}
\end{aligned}
$$

And as $\sin^{-1}(\phi)$ increases as ϕ increases, a greater wavelength λ results in a greater angle of deviation θ. Thus, the red light is deviated at a greater angle than the violet light.

Question 6 Solution

First, note that the spacing between the two slits must be comparable to the wavelength of light. Since the wavelength of visible light is so small, it can be assumed that as two lights rays from each slit approach the same point, the rays are parallel.



Now the side of the right angled triangle must be $n\lambda$ where $n \in \mathbb{N}$. This is because for constructive interference to occur, the two light rays must be in phase, meaning that the extra distance the bottom ray travels must be an integer multiple of its wavelength to result in constructive interference for the bright band to occur. Thus, using the triangle,

$$\sin\theta = \frac{n\lambda}{d}$$
$$n\lambda = d\sin\theta \text{ as required.}$$

Newton and Huygens' Model of Light SOLUTIONS

Question 1 Solution

Huygens' principle states

> All points of a wave front of light in a vacuum or transparent medium may be regarded as new sources of wavelets that expand in every direction at a rate depending on their velocities.

Which can be simplified to

> Each point of a wavefront may be considered as a source of secondary wavelets.

Thus, the answer is C.

Question 2 Solution

A - Corpuscle

B - Wavefront

C - Secondary wavelet

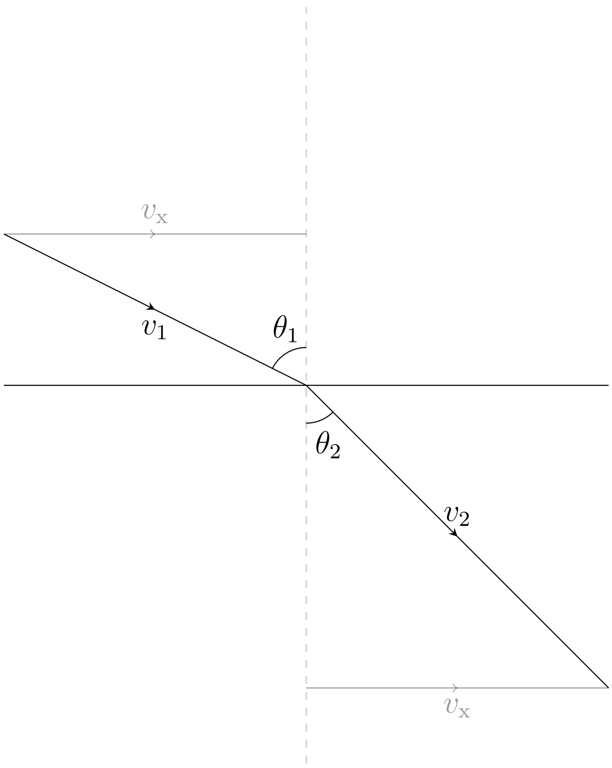
Question 3 Solution

Criteria	**Marks**
• Identifies the slower speed of light in denser mediums • Links slower speed to Huygens' model of light	2
• 1 of the above	1

Sample answer:
Foucault's investigation of the speed of light in various mediums identified that light travelled at a slower speed in denser mediums. This directly contradicted Newton's model that stated light's velocity (perpendicular to the surface) was increased upon entering a denser medium. Instead, the result aligned with Huygens' model of light, which stated light's velocity was decreased upon entering a denser medium.

Question 4 Solution

Newton's model of light predicted that light's velocity (perpendicular to the surface) was increased to account for the lesser angle of incidence, whilst the velocity parallel to the surface was unchanged. Thus, the following diagram can be utilised to compare the velocities of light in the two mediums (Note the sides parallel to the surface are the same across both triangles to signify the velocity parallel to the surface has not changed).



$$\sin\theta_1 = \frac{v_x}{v_1} \text{ and } \sin\theta_2 = \frac{v_x}{v_2}$$
$$v_x = v_1 \sin\theta_1 \text{ and } v_x = v_2 \sin\theta_2$$
$$\therefore v_1 \sin\theta_1 = v_2 \sin\theta_2 \text{ as required.}$$

Note: Even though Newton's model is physically inaccurate, Snell's law can still be derived from its assumptions, showing that an accurate result does not always imply an accurate theory.

Question 5 Solution

Criteria	Marks
• Explains in terms of Newton's model • Explains in terms of Huygens' model • Includes a relevant diagram	3
• 2 of the above	2
• 1 of the above	1

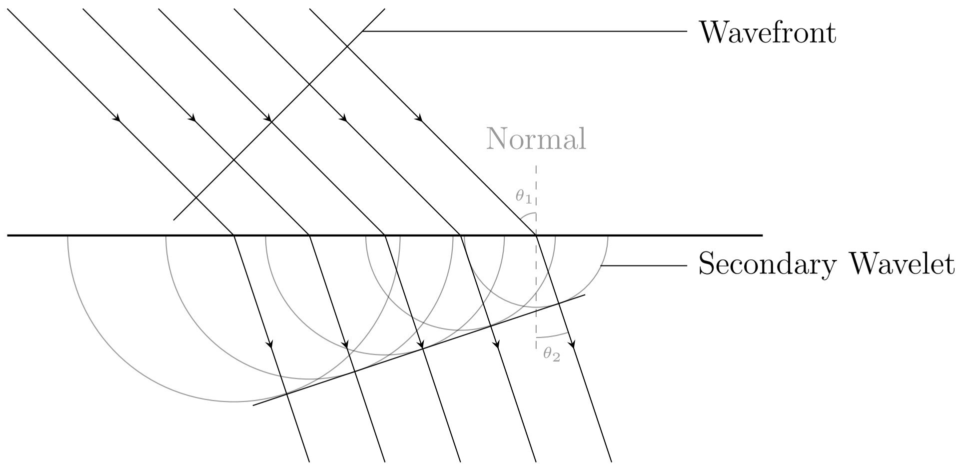
Sample answer:

Newton's Model

In Newton's corpuscular model of light, the corpuscle (representing light) that enters the denser medium at a certain angle of incidence is said to have its velocity that is perpendicular to the surface increased, decreasing the angle of incidence in the denser medium.

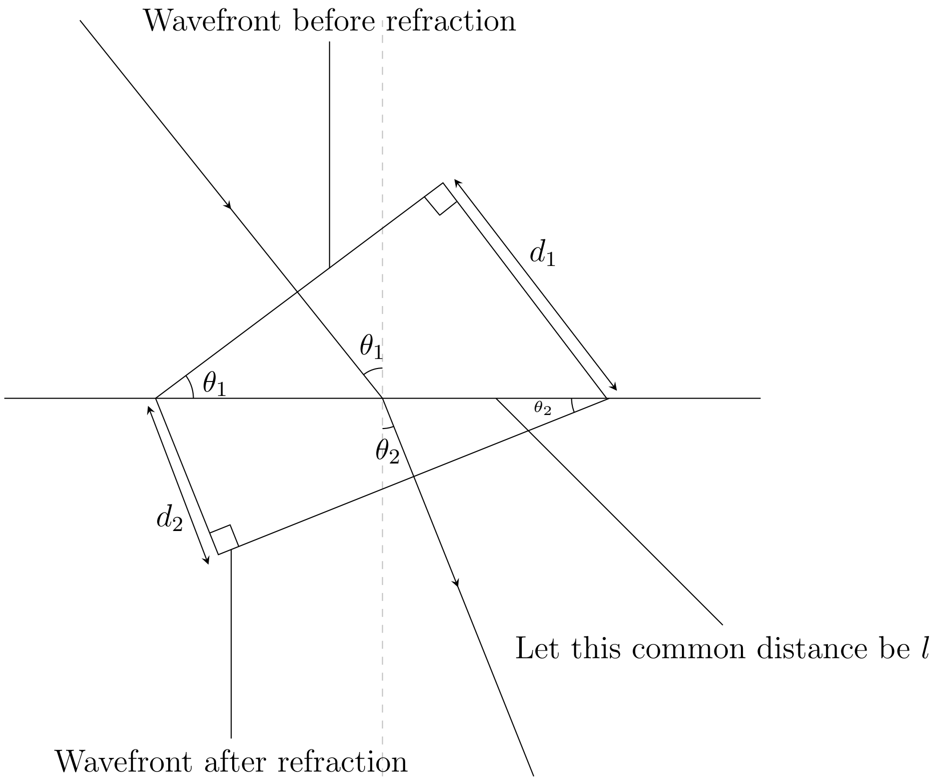
Huygens' Model

In Huygens' wave model of light, each point of the wavefront that comes to the surface of the denser medium creates secondary wavelets. As the secondary wavelets travel more slowly in the denser medium, the angle of incidence of the wavefront in the denser medium is decreased.



Question 6 Solution

Huygens' model of light predicted that light's speed of propagation would be slower in a denser medium. Thus, the following diagram can be utilised to compare the velocities of light in the two mediums.



$$\begin{aligned}
\text{Now } d_1 &= v_1\Delta t \text{ and } d_2 = v_2\Delta t \\
\sin\theta_1 &= \frac{d_1}{l} \text{ and } \sin\theta_2 = \frac{d_2}{l} \\
l &= d_1\sin\theta_1 \text{ and } l = d_2\sin\theta_2 \\
\therefore d_1\sin\theta_1 &= d_2\sin\theta_2 \\
v_1\Delta t\sin\theta_1 &= v_2\Delta t\sin\theta_2 \\
v_1\sin\theta_1 &= v_2\sin\theta_2 \text{ as required.}
\end{aligned}$$

Note: As stated in Solution 4, a correct result does not imply a correct theory. However, a correct theory must consistently produce correct results, as Huygens' model does here. This is not to say Huygens' model is the most accurate model of light we have now, as there will be inconsistencies in Huygens' model of light explored later in this book.

Malus' Law SOLUTIONS

Question 1 Solution

Criteria	Marks
• Identifies that polarisation is only possible in transverse waves	1

Sample answer:
Polarisation can only occur in transverse waves. Hence, the polarisation of light supported the idea that light is a transverse wave rather than a longitudinal wave, or a stream of particles, as they cannot be polarised.

Note: While light is more accurately described as a stream of photons, photon polarisation is a quantum phenomenon and involves concepts beyond the HSC physics syllabus.

Question 2 Solution

Criteria	Marks
• Identifies that unpolarised light consists of polarisations of all directions • Identifies that half of the intensity of light is aligned with any given axis	2
• 1 of the above	1

Sample answer:
An unpolarised beam of light consists of electric field vectors oscillating in all directions perpendicular to the direction of propagation. When this beam passes through a polarising filter, only the component of the electric field aligned with the filter's transmission axis is allowed through. Since the light is equally distributed across all polarisation directions, exactly half of its intensity is aligned with any given axis, while the other half is blocked. As a result, the filter transmits 50% of the incident intensity, regardless of its orientation.

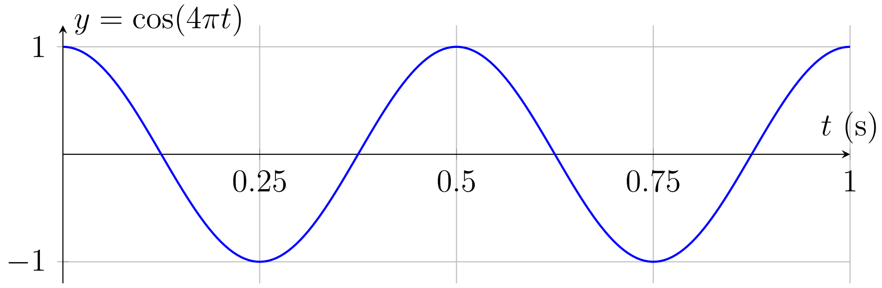
Question 3 Solution

Let the original intensity be I_0. After the first filter, the light would be at an intensity of 50%. Let this intensity be I_1. After the second filter, the light is at an intensity of 46.65%. Now,

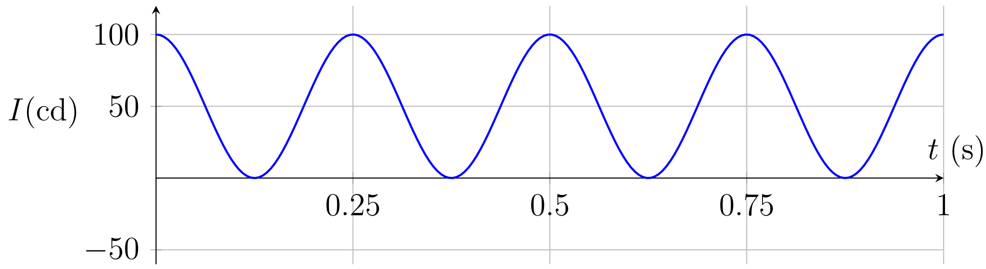
$$\begin{aligned} I_2 &= I_1 \cos^2\theta \\ 0.4665 I_0 &= 0.5 I_0 \cos^2\theta \\ \cos^2\theta &= 0.933 \\ \cos\theta &= \sqrt{0.933} \\ \theta &= \cos^{-1}(\sqrt{0.933}) \\ \theta &= 15.001\ldots^\circ \\ \therefore \theta &= 15^\circ. \end{aligned}$$

Question 4* Solution

Using Malus' law, the intensity of the resultant light can be determined by $I = I_0 \cos^2\theta$ where θ is the angle between the light's polarisation direction and the filter axis. Assuming the filter rotates at 4π radians per second, $\theta = 4\pi t$. Firstly, graph $y = \cos 4\pi t$ from $t = 0\,\text{s}$ to $t = 1\,\text{s}$.



Now utilising graphical manipulation skills from NESA's mathematics extension 1 course, graph the square of $\cos(4\pi t)$ to sketch $\cos^2(4\pi t)$. Then multiply the function by 100 to sketch the intensity function of $I = 100\cos^2(4\pi t)$.



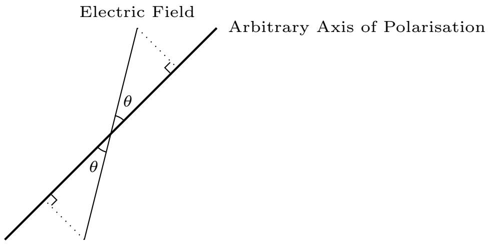
Question 5 Solution

Criteria	Marks
• Identifies a maximum and a minimum of resultant light • Identifies that there will be 2 maxima and minima • Explains the observations	2
• 2 of the above	1

Sample answer:
As Emmy rotates the polarising filter, she will observe two maxima and two minima in the transmitted light intensity over a full rotation. This occurs because there exists one orientation in which the filter's transmission axis aligns with the plane of partial polarisation, allowing maximum light to pass through. Conversely, when the filter is perpendicular to this direction, it blocks the most light, resulting in a minimum. Due to the symmetrical nature of angular alignment, both the maximum and minimum intensities occur twice per revolution. Therefore, as the filter completes one full 360 degree rotation, the intensity follows a squared cosine pattern with two full oscillations, consistent with the function $I = I_0 \cos^2\theta$, explaining the two maxima and minima in intensities.

Question 6 Solution

Firstly, take an electric field polarised in any direction, then resolve the electric field's component aligned with an arbitrary axis of polarisation that is at an angle of θ to the polarisation of the electric field.



Now it can be seen that the amplitude/magnitude of the electric field aligned with the arbitrary axis of polarisation E can be expressed in terms of the amplitude/magnitude of the original electric field E_0 as $E = E_0\cos\theta$. Now utilising $I \propto E^2 \implies I = kE^2$,

$$\begin{aligned} I &= kE^2 \\ E &= E_0\cos\theta \\ \therefore I &= k(E_0\cos\theta)^2 \\ I &= kE_0^2\cos^2\theta \\ I &= I_0\cos^2\theta \text{ as required.} \end{aligned}$$

Light: Quantum Model SOLUTIONS

Black Body Radiation SOLUTIONS

Question 1 Solution

Planck resolved the ultraviolet catastrophe by proposing that energy is quantised, introducing the relation $E = hf$ through his analysis of black body radiation. Hence, the correct answer is D.

Question 2 Solution

Criteria	Marks
• Identifies 2 features of a black body • Provides a judgment of whether the Earth is a black body through a set of criteria	3
• Identifies a feature of a black body • Provides a judgment of whether the Earth is a black body through a set of criteria	2
• 1 of the above	1

Sample answer:
A black body is an idealised object that perfectly absorbs all incident electromagnetic radiation, regardless of wavelength or angle. It emits thermal radiation solely based on its temperature, described by Planck's law (giving it its black body radiation curve). As the Earth reflects a considerable portion of the electromagnetic radiation from the Sun, it does not satisfy the condition of being a perfect absorber; hence, the Earth is not a black body.

Question 3 Solution

Wein's displacement law states that for a star, $\lambda_{\text{max}} = \frac{b}{T}$ where b is Wien's displacement constant. This means the product of the wavelength of maximum intensity λ_{max} and the star's surface temperature T is a constant $\lambda_{\text{max}}T = b$ Thus, any value of $\lambda_{\text{max}}T$ will equal the same constant (Wien's displacement constant) for all stars, hence the answer is D.

Question 4 Solution

Criteria	Marks
• Links all components of the diagram to Planck's contribution	3
• 1 error in response	2
• 2 errors in response	1

Sample answer:

Diagram Component	Planck's Contribution
Contradiction	The ultraviolet catastrophe was a clear contradiction from classical physics regarding black body radiation, as the energy radiated by black bodies was clearly finite.
Proposal of Hypothesis	Planck aimed to create a hypothesis that could explain this contradictory experimental observation. Hence, he proposed the idea that energy was quantised by his relation $E = hf$.
Investigation	A black body radiator was experimentally approximated by a hot cavity, and the radiation emitted by the experimental black body was recorded.
Observation	The recorded results were then written graphically to find the experimental black body curve.
Analysis	This experimental black body curve was then compared to the predicted black body curve according to Planck's new hypothesis.
Support	The experimental black body curve aligned with the predictions made through Planck's hypothesis, and hence, Planck's hypothesis was later accepted, forming the foundation for quantum physics.

Question 5 Solution

Criteria	Marks
• Describes the ultraviolet catastrophe • Explains how Planck resolved the catastrophe	3
• 1 error in response	2
• 2 errors in response	1

The sample answer is given on the following page:

The ultraviolet catastrophe was a contradiction in classical physics, predicting infinite radiation intensity that at very short wavelengths from a black body. Planck resolved this by hypothesising that energy is quantised as $E = hf = \frac{hc}{\lambda}$, making energy inversely proportional to wavelength. This means at very small wavelengths, insufficient energy exists to produce radiation, causing intensity to approach zero as wavelength approaches 0, thus eliminating the ultraviolet catastrophe.

Question 6 Solution

Criteria	Marks
• References contradictions in classical physics • References Planck's introduction of the relation $E = hf$ • References the fundamental conceptual shifts from classical physics to quantum physics	3
• 2 of the above	2
• Some relevant information	1

Sample answer:
Classical physics failed to explain the shape of the black body radiation curve, predicting infinite energy at short wavelengths, a contradiction known as the ultraviolet catastrophe. To resolve this, Planck proposed that energy is quantised, introducing the relation $E = hf$, where energy is proportional to frequency. This shows that observations were made that couldn't be explained by the then current model, and thus was replaced by a new model that quantised energy. This new idea marked a significant departure from classical physics. Planck's work laid the foundation for quantum physics, which introduced the concept that energy and other physical quantities are discrete, not continuous; a fundamental shift in our understanding of the universe.

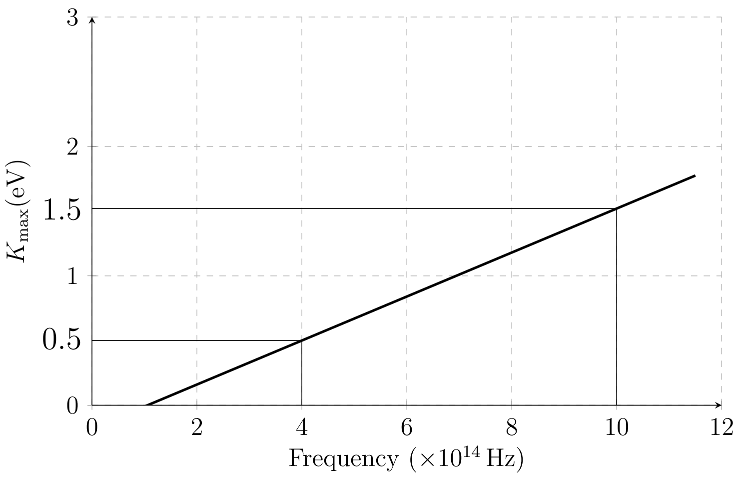
Photoelectric Effect SOLUTIONS

Question 1 Solution

Emmy can determine the work function of the metal by finding the negative of the y-intercept of the graph of maximum kinetic energy of photoelectrons versus the energy of incident photons. According to the photoelectric equation $K_{\text{max}} = hf - \phi$, this graph has a y intercept of $-\phi$, the negative of the work function. Since both axes are already in units of energy, no further unit conversions are necessary.

Question 2 Solution

First, find the gradient of the graph Emmy has plotted.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{1.5 - 0.5}{10 - 4} \; \frac{\text{eV}}{10^{14}\,\text{Hz}}$$
$$m = 0.166\ldots \; \frac{1.602 \times 10^{-19}\,\text{J}}{\times 10^{14}\,\text{s}^{-1}}$$
$$m = 2.67 \times 10^{-34}\,\text{J s}$$

Now, note that since this graph is plotted as the maximum kinetic energy of photoelectrons versus the frequency of incident photons, the gradient must be Planck's constant. However Planck's constant is 6.626×10^{-34} J s, more than two times the experimental value. Thus, as the percentage error of Emmy's data exceeds 50%, her data is not accurate.

Question 3 Solution

Criteria	Marks
• Correctly explains the effect for all 3 metals	3
• Correctly explains the effect for 2 metals	2
• Correctly explains the effect for 1 metal	1

Sample answer:
Now to find the frequency of a photon of wavelength 750 nm,

Data List:

- $\lambda = 750 \times 10^{-9}$ m

Solution:

$$f\lambda = c$$
$$f = \frac{c}{\lambda}$$
$$f = \frac{3 \times 10^{8}}{750 \times 10^{-9}}$$
$$f = 4 \times 10^{14}\,\text{Hz}.$$

According to the graph,

- In metal A, the photon frequency is greater than the threshold frequency, and photoelectrons of a maximum kinetic energy 1 eV are emitted.
- In metal B, the photon frequency equals the threshold frequency. The photons have just enough energy to overcome the work function, resulting in photoelectrons with zero kinetic energy (i.e., $K_{\text{max}} = 0$). Hence, photoelectrons may be created, but they will have no kinetic energy.
- In metal C, the photon frequency is below the threshold frequency; thus, the photons will not have sufficient energy to overcome the work function, and thus no photoelectrons will be emitted.

Hence, doubling the intensity of the light source (or doubling the number of incident photons):

- In metal A and metal B will ideally double the number of photoelectrons emitted or created.
- In metal C, it will have no effect as no photoelectrons can be produced due to the insufficient energy of the incident photons.

Note: Since the maximum kinetic energy of the photoelectrons in metal B is zero, it can be argued that these electrons may be reabsorbed immediately after creation. For this question, the key aspect is your line of reasoning rather than the physical correctness of this argument.

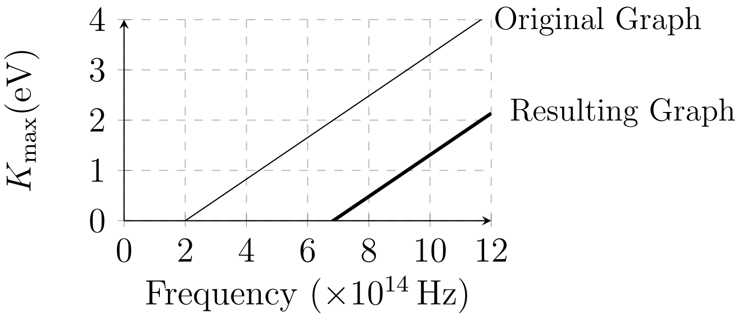
Question 4 Solution

Criteria	Marks
• Identifies that wave theory could not explain the photoelectric effect • Explains how the quantum model of light resolved the unexplained observations	3
• 1 error	2
• Some relevant information	1

Sample answer:
According to the classical wave theory of light, energy is delivered continuously across the wavefront. Thus, even if no photocurrent is initially observed, increasing the intensity of the light source should eventually provide enough energy to liberate electrons from the metal surface, resulting in a photocurrent. However, experimental observations showed that if the frequency of the incident light is below a certain threshold, no photocurrent is produced, regardless of the intensity. This contradicted classical expectations. To resolve this, the quantum model of light was introduced. In this model, light is composed of discrete energy packets called photons, each with energy $E = hf$, where h is Planck's constant and f is the frequency of the light. Increasing the intensity only increases the number of photons, not their energy. Therefore, if each photon lacks sufficient energy to overcome the work function of the metal, no photoelectrons will be emitted, no matter how many photons strike the surface. Thus, the quantum model of light successfully explains the frequency-dependent threshold observed in the photoelectric effect, a phenomenon that the classical wave theory could not account for.

Question 5 Solution

When the potential difference between the plates of a photocell is removed, the electric field that would normally accelerate the photoelectrons is removed. As a result, photoelectrons gain less kinetic energy in the absence of the field. In this case, the graph of maximum kinetic energy versus frequency will be vertically shifted downward by 2.0 eV, indicating that each photoelectron now has 2.0 eV less kinetic energy than before. This shift reflects the energy they no longer gain from the electric potential.



Question 6 Solution

First, convert each wavelength of light into frequency using the relation $f = \frac{c}{\lambda}$, to determine whether each metal is capable of producing a photocurrent.

Wavelength (nm)	770	520	350
Frequency ($\times 10^{14}$ Hz)	3.896...	5.769...	8.571...

Using this information and the supplied graph, it can be noted that the longest wavelength that will result in photoelectrons in metal A is 770 nm, in metal B is 520 nm and in metal C is 350 nm. Now, using the information that each wavelength setting of the light source has a power of 1 W, the number of photons transmitted per second for each setting can be found. This is because the power represents the product of the energy of each photon and the number of photons transmitted per unit time. Letting the time interval be 1 s, the relation can be found:

Data List:

- $P = 1\,\text{W}$
- $\Delta t = 1\,\text{s}$

Solution:

$$\text{Power} = \frac{\text{Number of Photons} \times \text{Energy of Photon}}{\text{Time Interval}}$$

$$P = \frac{nE}{\Delta t}$$

$$P = \frac{nhf}{\Delta t} \qquad (E = hf)$$

$$n = \frac{\Delta t P}{hf}$$

$$n = \frac{1}{hf}$$

Using this relation, the number of photons transmitted per second can be found.

Wavelength (nm)	770	520	350
Frequency ($\times 10^{14}$ Hz)	3.896...	5.769...	8.571...
n_{photons} (s^{-1})	$3.873\ldots \times 10^{18}$	$2.615\ldots \times 10^{18}$	$1.760\ldots \times 10^{18}$

Now, assume that in the context of the photoelectric effect, each of these photons will produce one photoelectron that produces a photocurrent. Thus the number of photons will represent the number of photoelectrons, and similarly to how a current can be found in a wire by the number of electrons flowing per second, the maximum photocurrent of each light setting can be found by the relation $I = \frac{nq_e}{t}$, or $I = nq_e$ as $t = 1\,\text{s}$. The answers have been given to 2 significant figures.

Wavelength (nm)	770	520	350
Frequency ($\times 10^{14}$ Hz)	3.896...	5.769...	8.571...
n_{photons} (s^{-1})	$3.873\ldots \times 10^{18}$	$2.615\ldots \times 10^{18}$	$1.760\ldots \times 10^{18}$
Maximum Photocurrent (A)	0.62	0.42	0.28

Thus the maximum photocurrent of metal A is 0.62 A, metal B is 0.42 A, and metal C is 0.28 A.

Light and Special Relativity SOLUTIONS

Einstein's Postulates SOLUTIONS

Question 1 Solution

Criteria	Marks
• States light's relative speed does not change • States the sound wave will appear to be stationary	2
• 1 of the above	1

Sample answer:
According to Einstein's second postulate that states the speed of light is invariant for all inertial frames of reference, the speed of light for Emmy will not change.

The sound wave, however, travels at $340\,\mathrm{m\,s^{-1}}$ through air, and since Emmy is moving at that same speed in the same direction, the sound wave will appear stationary relative to him.

Question 2 Solution

Criteria	Marks
• States Einstein's first postulate • Links postulate to Emmy's incorrect statement	2
• 1 of the above	1

Sample answer:
Einstein's first postulate states that "the laws of physics are the same in all inertial frames of reference." Since the spaceship is travelling at a uniform velocity, it is considered an inertial frame. Therefore, any experiment performed inside the spaceship will produce accurate results. Hence, Emmy's prediction is incorrect.

Question 3 Solution

Criteria	Marks
• Identifies the exception as light • Explains with Einstein's second postulate	2
• 1 of the above	1

Sample answer:
Emmy cannot change the relative speed of light. According to Einstein's second postulate, the speed of light in a vacuum is constant and the same for all observers, regardless of their motion. Therefore, light always moves at speed c (in a vacuum) relative to Emmy and cannot be made to move slower or faster relative to him.

Question 4 Solution

Criteria	Marks
• Identifies the exception as light • Explains with Einstein's second postulate	2
• 1 of the above	1

Sample answer:
This thought experiment supports Einstein's first postulate, which states that the laws of physics are the same in all inertial frames. Since the insects' motion inside the cabin is unchanged whether the ship is moving at a uniform velocity or stationary, it supports that the laws of physics are identical in all inertial frames of reference.

Question 5 Solution

Criteria	Marks
• Provides a suitable observation • Explains how the observation supports Einstein's second postulate	3
• 1 error	2
• Some relevant information	1

The sample answer is given on the following page:

An observation that supports Einstein's postulate of the absolute velocity of light is the Michelson-Morley experiment. In this experiment, an interferometer was used to detect differences in the speed of light in various directions as Earth moved through the hypothetical luminiferous aether. The expectation was that if light travelled through an aether, then its speed would vary depending on the direction of Earth's motion relative to the aether. However, the experiment consistently produced a null result (no change in the interference pattern), indicating no change in the speed of light regardless of the direction of travel. This supports Einstein's postulate that the speed of light in a vacuum is constant for all observers, regardless of their motion relative to the source of the light.

Additional sample answer:
An observation that supports Einstein's postulate of the absolute velocity of light is the apparent Keplerian orbit of binary star systems. According to classical physics, if light obeyed Galilean velocity addition, the light from the star moving towards Earth should arrive faster than light from the star moving away. This would result in the binary stars appearing to be in a non-Keplerian orbit due to the distortion. However, observations show all binary stars appear to follow Keplerian orbits, indicating the speed of light is not affected by the orbital velocity of the stars. This supports Einstein's postulate that the speed of light in a vacuum is constant for all observers, regardless of their motion relative to the source of the light.

Question 6 Solution

Criteria	**Marks**
• Correctly states Emmy's description	1

Sample answer:
Emmy will describe Amanda's wave to also be in slow motion, identical to Amanda's description.

!?

!? Galilean Relativity

This question is best understood through the logic of Einstein's first postulate, which is why it appears in the **Einstein's Postulates** section rather than the **Time Dilation and Length Contraction** section. Note that Amanda observing Emmy pass at $0.8c$ is physically equivalent to Emmy observing Amanda pass at $0.8c$. If either observer saw the other's wave behave differently in a way that violated symmetry, it would contradict Einstein's first postulate: that the laws of physics are the same in all inertial frames of reference. This reflects the same principle explored in Galileo Galilei's thought experiment in **Question 4**: no experiment or observation made within an inertial frame can determine its absolute velocity. The relative motion is all that is relevant.

Question 1 Solution

Data List:

- $L_0 = 11\,\text{m}$
- $L = 10\,\text{m}$

Solution:

$$l = l_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\frac{l}{l_0} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\left(\frac{l}{l_0}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{l}{l_0}\right)^2$$

$$v^2 = c^2\left(1 - \left(\frac{l}{l_0}\right)\right)^2$$

$$v = \sqrt{c^2\left(1 - \left(\frac{l}{l_0}\right)\right)^2}$$

$$v = \sqrt{(3 \times 10^8)^2\left(1 - \left(\frac{10}{11}\right)^2\right)}$$

$$v = 124\ 979\ 337.1\ldots$$

$$v = 0.42c$$

Question 2 Solution

Data List:

- $d = 5\ 994\,\text{m}$
- $t_0 = 2.2\,\mu\text{m}$
  $= 2.2 \times 10^{-6}\,\text{s}$

Solution:

$$t = \frac{d}{v}$$

$$\frac{t_0}{\sqrt{(1-\frac{v^2}{c^2})}} = \frac{d}{v}$$

$$t_0 v = d\left(\sqrt{\left(1-\frac{v^2}{c^2}\right)}\right)$$

$$t_0^2 v^2 = d^2\left(1-\frac{v^2}{c^2}\right)$$

$$t_0^2 v^2 = d^2 - \frac{d^2 v^2}{c^2}$$

$$t_0^2 v^2 + \frac{d^2 v^2}{c^2} = d^2$$

$$v^2\left(t_0^2 + \frac{d^2}{c^2}\right) = d^2$$

$$v^2 = \frac{d^2}{t_0^2 + \frac{d^2}{c^2}}$$

$$v = \sqrt{\frac{d^2}{t_0^2 + \frac{d^2}{c^2}}}$$

$$v = \sqrt{\frac{(5\ 994)^2}{(2.2\times 10^{-6})^2 + \frac{(5\ 994)^2}{(299\ 792\ 458)^2}}}$$

$$v = 297\ 993\ 909.9$$

$$v = 0.994c$$

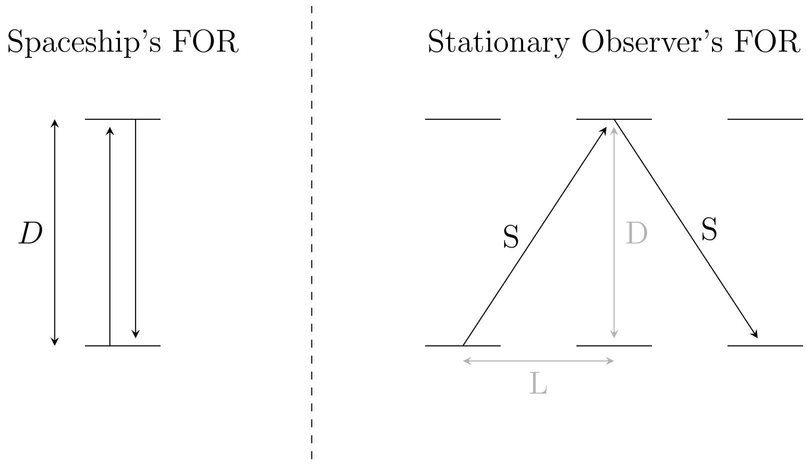
Question 3 Solution

Criteria	Marks
• Correctly states length contraction	1

Sample answer:
While Emmy maintains a distance of x m between the spaceships, the string contracts due to length contraction at relativistic speeds, causing the string to undergo tension and then snap.

Question 4 Solution

Imagine a light clock in a spaceship travelling at v to the right. Now see the light clock from both the frame of reference (FOR) of the spaceship and a stationary observer.



Now t_0 represents the time interval in the frame of reference of the moving object, t represents the time interval in the frame of reference of the stationary observer. Thus, using time $= \frac{\text{distance}}{\text{velocity}}$ the following can be determined.

$$t_0 = \frac{2D}{c} \quad t = \frac{2S}{c} \quad t = \frac{2L}{v}$$

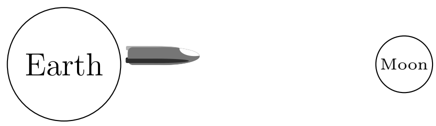
$$D = \frac{ct_0}{2} \quad S = \frac{ct}{2} \quad L = \frac{vt}{2}$$

And now using the diagram and Pythagoras' theorem,

$$\begin{aligned}
S^2 &= D^2 + L^2 \\
\left(\frac{ct}{2}\right)^2 &= \left(\frac{ct_0}{2}\right)^2 + \left(\frac{vt}{2}\right)^2 \\
(ct)^2 &= (ct_0)^2 + (vt)^2 \\
c^2t^2 &= c^2t_0^2 + v^2t^2 \\
c^2t^2 - v^2t^2 &= c^2t_0^2 \\
t^2(c^2 - v^2) &= c^2t_0^2 \\
t^2 &= \frac{c^2t_0^2}{c^2\left(1 - \frac{v^2}{c^2}\right)} \\
t^2 &= \frac{t_0^2}{\left(1 - \frac{v^2}{c^2}\right)} \\
t &= \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \text{ as required.}
\end{aligned}$$

Question 5 Solution

Imagine a spaceship travelling at relativistic speeds towards another celestial body (for example, the moon), and an observer on Earth.



Now let l_0 be the length of the trip and t be the time it took to complete the trip from Earth's frame of reference, and let l be the length of the trip and t_0 be the time it took to complete the trip from the spaceship's frame of reference. Using velocity $= \frac{\text{distance}}{\text{time}}$,

$$v = \frac{l}{t_0} \quad v = \frac{l_0}{t}$$

$$\frac{l}{t_0} = \frac{l_0}{t}$$

$$l = l_0\frac{t_0}{t}$$

$$\text{Now } t = \frac{t_0}{\sqrt{(1 - \frac{v^2}{c^2})}} \qquad \text{(This has been derived in \textbf{Solution 4})}$$

$$\frac{t_0}{t} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\therefore l = l_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)} \text{ as required.}$$

Question 6 Solution

Criteria	Marks
• Correctly identifies the general of Belvaria • Gives reasoning	2
• Correctly identifies the general of Belvaria	1

Sample answer:
The general of Belvaria receives the phone call. This is because from the spy's frame of reference outside of the train, the light appears to takes longer to travel to the general of San Cordoba as the general is moving away from the light, while it takes a shorter amount of time to travel to the general of Belvaria as she is travelling towards the light. Thus, the light reaches the general of Belvaria sooner from the spy's perspective, resulting in the general of Belvaria signing before the general of San Cordoba.

Question 1 Solution

Data List:

- $m = 1.673 \times 10^{-27}\,\text{kg}$
- $p = 6 \times 10^{-19}\,\text{kg m s}^{-1}$
- $v = ?\,\text{m s}^{-1}$

Solution:

$$p = \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$p\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = mv$$

$$p^2\left(1 - \frac{v^2}{c^2}\right) = m^2v^2$$

$$p^2 - \frac{p^2v^2}{c^2} = m^2v^2$$

$$m^2v^2 + \frac{p^2v^2}{c^2} = p^2$$

$$v^2 = \frac{p^2}{m^2 + \frac{p^2}{c^2}}$$

$$v = \sqrt{\frac{p^2}{m^2 + \frac{p^2}{c^2}}} \qquad (v \geq 0)$$

$$v = \sqrt{\frac{(6 \times 10^{-19})^2}{(1.673 \times 10^{-27})^2 + \frac{(6\times10^{-19})^2}{(3\times10^{8})^2}}}$$

$$v = 111\ 053\ 312.5\ldots$$

$$v = 0.37c$$

Question 2 Solution

Data List:

- $m = 1.673 \times 10^{-27}\,\text{kg}$
- $v = 0.999c\,\text{m s}^{-1}$
- $r = 4000\,\text{m}$
- $q = 1.602 \times 10^{-19}\,\text{C}$
- $B = ?\,\text{T}$

Solution:

$$F_\text{c} = \frac{\gamma mv^2}{r}$$

$$\text{Now } |F_\text{B}| = |F_\text{c}|$$

$$\therefore |F_\text{B}| = \frac{\gamma mv^2}{r}$$

$$qvB = \frac{\gamma mv^2}{r}$$

The solution continues over the page.

Data List:

- $m = 1.673 \times 10^{-27}\,\text{kg}$
- $v = 0.999c\,\text{m}\,\text{s}^{-1}$
- $r = 4000\,\text{m}$
- $q = 1.602 \times 10^{-19}\,\text{C}$
- $B = ?\,\text{T}$

Solution:

$$qvB = \frac{\gamma m v^2}{r}$$
$$B = \frac{\gamma m v}{rq}$$
$$B = \frac{(1.673 \times 10^{-27})(0.999c)}{(4000)(1.602 \times 10^{-19})\sqrt{\left(1 - \frac{(0.999c)^2}{c^2}\right)}}$$
$$B = 1.750\ldots \times 10^{-2}$$
$$B = 1.75 \times 10^{-2}\,\text{T}$$

Question 3 Solution

Criteria	Marks
• Mathematically uses Parker Solar Probe as an example • Clearly explains the small difference in the two formulae for momentum in low-speed scenarios	2
• 1 of the above	1

Sample answer:
The classical formula for momentum, $p = mv$, differs from the relativistic formula of momentum, $p = \gamma mv$, due to the inclusion of the Lorentz factor γ, which accounts for time dilation and length contraction at relativistic speeds. In the example of the fastest man-made object, NASA's Parker Solar Probe, the Lorentz factor can be determined as so.

Data List:

- v = $700\,000\,\text{km}\,\text{h}^{-1}$
  $=194444.4\ldots\text{m}\,\text{s}^{-1}$

Solution:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$
$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{(194444.4\ldots)^2}{(3\times10^8)^2}\right)}}$$
$$\gamma = 1.0000002\ldots$$

The resulting Lorentz factor is extremely close to 1, meaning the relativistic momentum differs from the classical momentum by only about 0.00002%. Therefore, even at extremely high velocities, such as those achieved by the Parker Solar Probe, the relativistic correction is negligible. This explains why the classical formula $p = mv$ provides an accurate model for momentum in everyday contexts where velocities are much less than the speed of light.

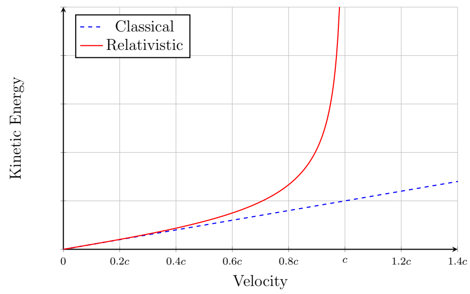
Question 4 Solution

Criteria	Marks
• Mathematically supports the answer • Identifies that objects with c velocity have infinite energy or similar nonsensical quantities • Contrasts relativistic predictions with classical predictions • Utilises supporting diagram	4
• 2-3 of the above	2-3
• 1 of the above	1

Sample answer:
In classical physics, the kinetic energy of an object is given by $K = \frac{1}{2}mv^2$. Thus, if an object were travelling at c, the kinetic energy of the object would simply be $K = \frac{1}{2}mc^2$, implying masses can travel at light speed. However, special relativity provides a more accurate expression for kinetic energy at high velocities.

$$
\begin{aligned}
E_{\text{Total}} &= E_{\text{Rest}} + K \\
K &= E_{\text{Total}} - E_{\text{Rest}} \\
K &= \gamma mc^2 - mc^2 \\
K &= mc^2 \left(\frac{1}{\sqrt{(1 - \frac{v^2}{c^2})}} - 1 \right) \\
\therefore \lim_{v \to c} K &= +\infty
\end{aligned}
$$

As velocity v approaches c, the Lorentz factor γ increases without bound. This implies that an infinite amount of energy would be required for an object with mass to reach the speed of light. The diagram below illustrates this difference between classical and relativistic models of kinetic energy.



Thus, special relativity imposes a strict upper limit of c on the velocity of any object with mass, in contrast to classical mechanics, which predicts no such limit.

Question 5* Solution

Data List:

- $K = 2 \times 10^{-14}\,\text{J}$
- $m = 9.109 \times 10^{-31}\,\text{kg}$

Solution:

$$\begin{aligned}
E_{\text{Total}} &= E_{\text{Rest}} + K \\
E_{\text{Total}} - E_{\text{Rest}} &= K \\
\gamma mc^2 - mc^2 &= K \\
mc^2(\gamma - 1) &= K \\
\gamma - 1 &= \frac{K}{mc^2} \\
\gamma &= 1 + \frac{K}{mc^2} \\
\gamma &= 1 + \frac{2 \times 10^{-14}}{(9.109 \times 10^{-31})(3 \times 10^8)^2} \\
\gamma &= 1.243\ldots
\end{aligned}$$

$$\begin{aligned}
\text{Now } \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} &= \gamma \\
\sqrt{\left(1 - \frac{v^2}{c^2}\right)} &= \frac{1}{\gamma} \\
1 - \frac{v^2}{c^2} &= \frac{1}{\gamma^2} \\
\frac{v^2}{c^2} &= 1 - \frac{1}{\gamma^2} \\
v^2 &= c^2\left(1 - \frac{1}{\gamma^2}\right) \\
v &= c\sqrt{\left(1 - \frac{1}{\gamma^2}\right)} \qquad (v \geq 0) \\
v &= (3 \times 10^8)\sqrt{\left(1 - \frac{1}{(1.243\ldots)^2}\right)} \\
v &= 178\ 435\ 411.8\ldots \\
v &= 0.59c
\end{aligned}$$

Question 6* Solution

$$\begin{aligned}
p &= m\frac{dx}{dt_0} \\
p &= m\frac{dx}{dt}\frac{dt}{dt_0} \qquad \text{(Chain Rule)}
\end{aligned}$$

$$\begin{aligned}
p &= \gamma m\frac{dx}{dt} \\
p &= \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}
\end{aligned}$$

Note: This derivation is extremely elementary to suit the HSC level.

Question 1 Solution

$$m = \frac{125 \text{ GeV}}{c^2}$$
$$m = \frac{125 \times 10^9 \text{ eV}}{c^2}$$
$$m = \frac{125 \times 10^9 \times 1.602 \times 10^{-19}\,\text{J}}{(3 \times 10^8)^2\,\text{m}^2\,\text{s}^{-2}}$$
$$m = 2.225 \times 10^{-25}\,\text{kg}$$

Question 2 Solution

If Sirius' power output is 9.77×10^{27} W, the energy it outputs in one second is 9.77×10^{27} J (remember W is equivalent to $\text{J}\,\text{s}^{-1}$). Thus,

Data List:

- $E = 9.77 \times 10^{27}\,\text{J}$
- $m = ?\,\text{kg}$

Solution:

$$E = mc^2$$
$$m = \frac{E}{c^2}$$
$$m = \frac{9.77 \times 10^{27}}{(3 \times 10^8)^2}$$
$$m = 1.085\ldots \times 10^{11}$$
$$m = 1.09 \times 10^{11}\,\text{kg}$$

Question 3 Solution

Note that due to the conservation of energy, the energy input must be equal to the energy output. Since no mass is created in the reaction (γ rays have no mass), the reaction must produce pure energy. Thus, the energy of the reaction can be determined:

Data List:

- $m_0 = 9.109 \times 10^{-31}\,\text{kg}$
- $v = 2 \times 10^8\,\text{m}\,\text{s}^{-1}$

Solution:

$$E_{\text{Total}} = 2\gamma mc^2 \qquad \text{(Relativistic Total Mass)}$$
$$E_{\text{Total}} = 2\left(\frac{(9.109 \times 10^{-31})(3 \times 10^8)^2}{\sqrt{\left(1 - \frac{(2\times10^8)^2}{(3\times10^8)^2}\right)}}\right)$$
$$E_{\text{Total}} = 2.199\ldots \times 10^{-13}$$
$$E_{\text{Total}} = 2.2 \times 10^{-13}\,\text{J}$$

Question 4 Solution

First, find the solar power that reaches the Earth before atmospheric losses as $1000 \times \frac{1}{0.75} = 1333.33\ldots$ W. Now we know that the solar power at the surface of the Earth can be measured as $1333.33\ldots\,\mathrm{W\,m^{-2}}$. But before we continue, we must first find the orbital radius of the Earth around the Sun. To be as precise as possible, the orbital period of the Earth will be taken as 365.2425 d to account for leap years. Now use Kepler's third law to find the orbital radius.

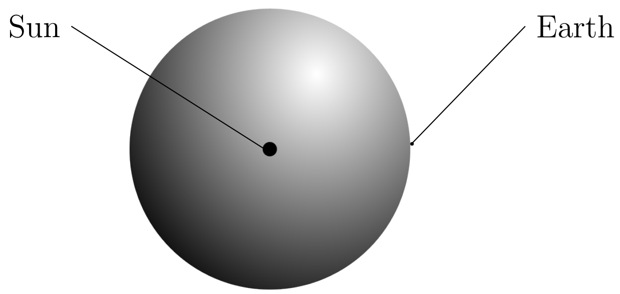
Data List:

- $T = 365.2425\,\mathrm{d}$ $= 3.15\ldots \times 10^{7}\,\mathrm{s}$
- $M = 1.989 \times 10^{30}\,\mathrm{kg}$
- $r = ?\,\mathrm{m}$

Solution:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$
$$r^3 = \frac{GMT^2}{4\pi^2}$$
$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$
$$r = \sqrt[3]{\frac{(6.67 \times 10^{-11})(1.989 \times 10^{30})(3.15\ldots \times 10^{7})^2}{4\pi^2}}$$
$$r = 1.49\ldots \times 10^{11}\,\mathrm{m}$$

Now, imagine the solar radiance as a sphere touching the Earth's surface.



As we have found the orbital radius, we can now find the surface area of this sphere (To ensure correctness, the radius of the Earth has been subtracted from this radius).

Data List:

- $r = 1.49\ldots \times 10^{11}\,\mathrm{m}$
- $A = ?\,\mathrm{m^2}$

Solution:

$$A = 4\pi r^2$$
$$A = 4\pi(1.49\ldots \times 10^{11})^2$$
$$A = 2.811\ldots \times 10^{23}\,\mathrm{m^2}$$

The solution continues over the page.

As we know from the observation on the surface of the Earth, this sphere has a power per area of $1333.33\ldots\,\mathrm{W\,m^{-2}}$. Hence, the total power can be determined as,

Data List:

- $A = 2.811\ldots \times 10^{23}\,\mathrm{m^2}$
- $r = 1333.33\ldots\,\mathrm{W\,m^{-2}}$
- $P = ?\,\mathrm{W}$

Solution:

$$P = A \times \text{Power Per Area}$$
$$P = Ar$$
$$P = (2.811\ldots \times 10^{23})(1333.33\ldots)$$
$$P = 3.74\ldots \times 10^{26}\,\mathrm{W}.$$

Thus, every second, the Sun outputs $3.74\ldots \times 10^{26}\,\mathrm{J}$ of energy. Now convert this energy to mass.

Data List:

- $E = 3.74\ldots \times 10^{26}\,\mathrm{W}$
- $m = ?\,\mathrm{kg}$

Solution:

$$E = mc^2$$
$$m = \frac{E}{c^2}$$
$$m = \frac{3.74\ldots \times 10^{26}}{(3 \times 10^8)^2}$$
$$m = 4.164\ldots \times 10^9$$
$$m = 4.16 \times 10^9\,\mathrm{kg}$$

Question 5 Solution

The angle θ can be determined by first applying the conservation of energy, then utilising de Broglie's wavelength formula to find the energy of the photons produced, and finally applying the conservation of momentum to find θ.

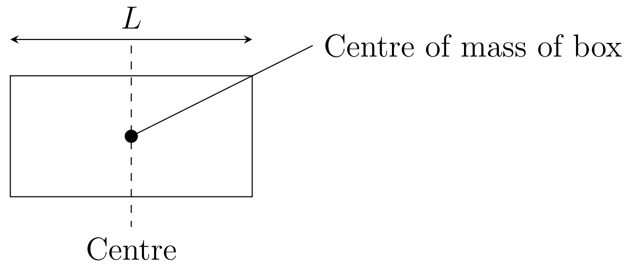
$$E_\mathrm{f} = E_\mathrm{i}$$
$$2E_\mathrm{photon} = E_\mathrm{muon} + E_\mathrm{antimuon}$$
$$2E_\mathrm{photon} = \gamma mc^2 + mc^2 \text{ (Where } m \text{ is the mass of a muon)}$$
$$2E_\mathrm{photon} = (\gamma + 1)mc^2$$
$$\text{Now } \lambda = \frac{h}{p}$$
$$\lambda f = \frac{hf}{p}$$
$$c = \frac{E}{p}$$
$$p = \frac{E}{c}$$

The solution continues over the page.

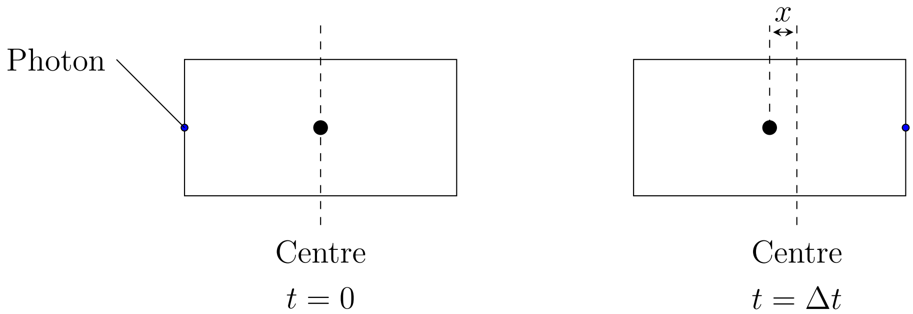
$$
\begin{aligned}
\text{Now } p_{\text{i}} &= p_{\text{f}} \\
p_{\text{muon}} &= 2p_{\text{photon}} \cos\theta \text{ (Horizontal Component)} \\
\gamma m v &= \frac{2E}{c} \cos\theta \\
0.8\gamma m c &= \frac{(\gamma+1)mc^2}{c} \cos\theta \\
\cos\theta &= \frac{4\gamma}{5(\gamma+1)} \\
\text{Now } \gamma &= \frac{1}{\sqrt{1-(\frac{v^2}{c^2})}} \\
\gamma &= \frac{1}{\sqrt{1-(\frac{(0.8c)^2}{c^2})}} \\
\gamma &= \frac{5}{3} \\
\therefore \cos\theta &= \frac{4(\frac{5}{3})}{5(\frac{5}{3}+1)} \\
\cos\theta &= \frac{1}{2} \\
\therefore \theta &= 60^\circ
\end{aligned}
$$

Question 6 Solution

To derive this formula, imagine a box of length L with its centre of mass in the middle.



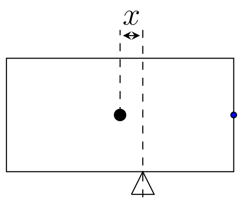
Now, imagine that a photon shoots off from the left side of the box, causing the box to recoil to the left until the photon collides with the right side of the box. Let the distance the box travels in this time interval be x.



By the conservation of momentum, we know the box must move to the left with the same momentum as the photon to the right. Hence,

$$\begin{aligned}
p_{\text{box}} &= p_{\text{photon}} \\
m_{\text{box}} v_{\text{box}} &= \frac{E}{c} \text{ (Refer to Page 309)} \\
v_{\text{box}} &= \frac{E}{m_{\text{box}} c} \\
\text{Now time} &= \frac{\text{distance}}{\text{velocity}} \\
\therefore \Delta t &= \frac{x}{v_{\text{box}}} \\
\Delta t &= \frac{m_{\text{box}} x c}{E}
\end{aligned}$$

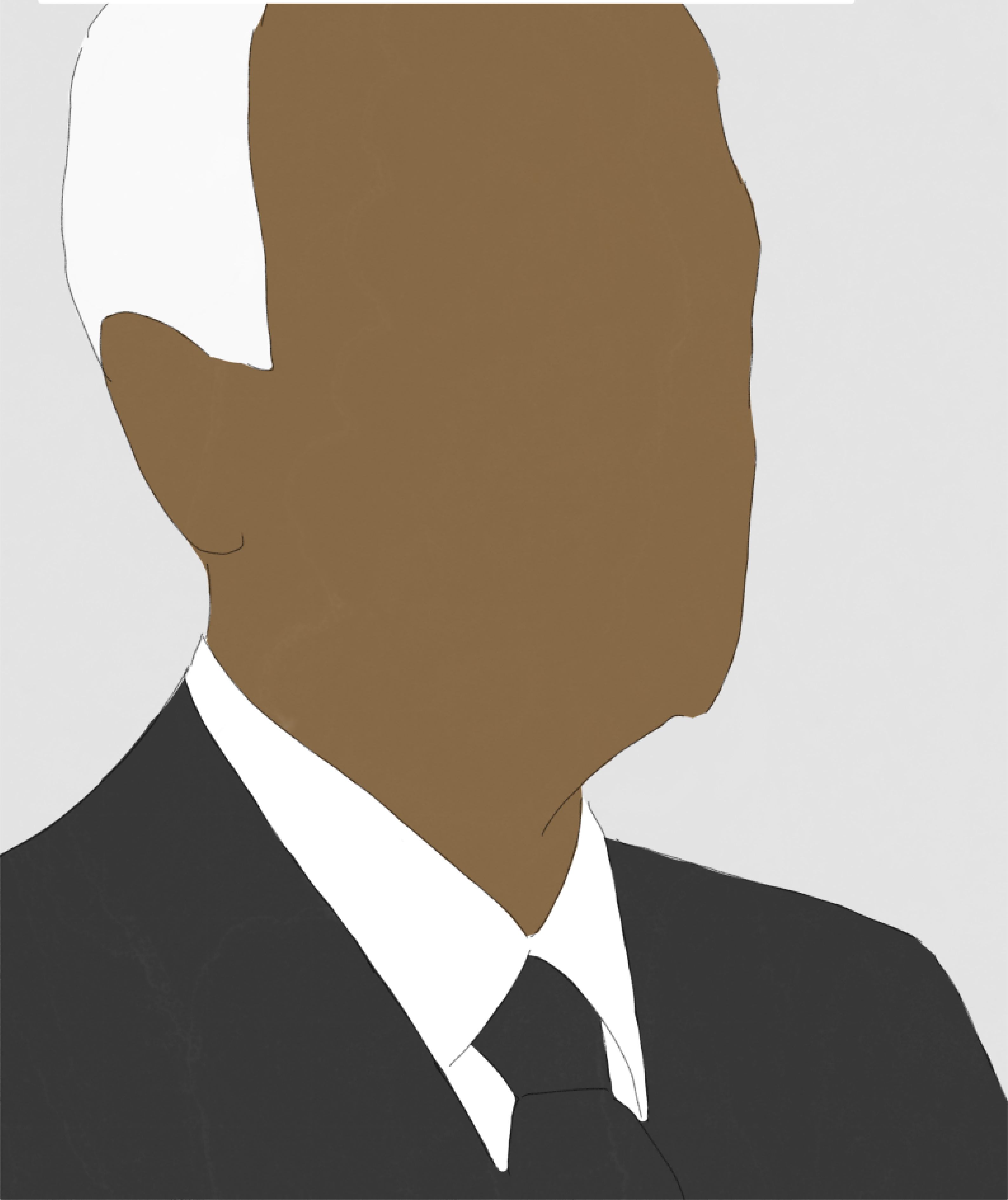
Now, note that the centre of mass of the entire system cannot have moved (as the box itself is a closed system). This means we can imagine placing a fulcrum in the original centre of the box, and the box still be balanced as depicted.



Since the system is balanced, we can equate the torques applied by the centre of mass of the box and the photon itself. However, we must consider the fact that as the photon originated from the left and the photon has ended up on the right, we must apply both the torques of the photon being on the left side of the box and the torque of the photon being on the right side of the box (Although technically the photon travels travels $L - x$ and thus the distances used in the torque is incorrect, x is a negligable value and is ignored in the derivation).

$$\begin{aligned}
\tau_{\text{box}} &= \tau_{\text{left}} + \tau_{\text{right}} \\
m_{\text{box}} g x &= mg(\frac{L}{2}) + mg(\frac{L}{2}) \\
m_{\text{box}} x &= mL \\
\text{Now distance} &= \text{velocity} \times \text{time} \\
\therefore L &= c\Delta t \\
L &= \frac{m_{\text{box}} x c^2}{E} \qquad (\Delta t = \frac{m_{\text{box}} x c}{E}) \\
\therefore m_{\text{box}} x &= \frac{m m_{\text{box}} x c^2}{E} \\
1 &= \frac{mc^2}{E} \\
E &= mc^2 \text{ as required.}
\end{aligned}$$

From the Universe to the Atom SOLUTIONS



Origin of the Elements SOLUTIONS

Big Bang Theory & Expansion of the Universe SOLUTIONS

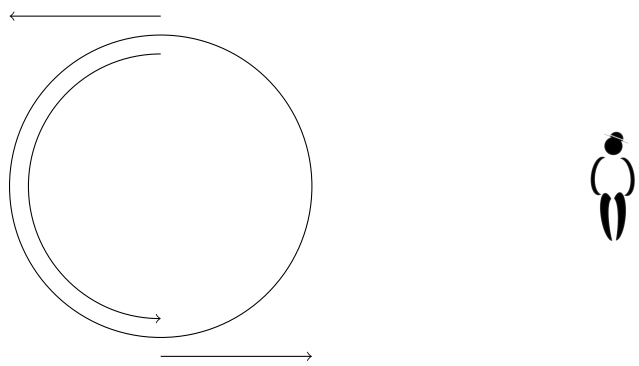
Question 1 Solution

Hubble's law, $v = H_0 D$ shows that the recessional velocity, v, is proportional to the distance, D. However, all answers display that relationship. The only way the answers differ is in the units of the distance. In Hubble's law, distance is given in megaparsecs, or Mpc. Therefore, the answer must be D, as all other answers use a unit of measurement other than parsecs for the distance.

Question 2 Solution

Criteria	Marks
• Correctly identifies that both red and blue shifts occur • Provides reasoning for the shiftings	2
• One of the above	1

Sample Answer:
As the star spins, at the top of the star, the linear velocity is away from the observer, while at the bottom of the star, the linear velocity is towards the observer, as the diagram below shows: !?



This will result in the light emitted from the top of the star being red shifted, while oppositely at the bottom of the star, the light emitted will be blue shited.

Diagrams in Worded Responses

Diagrams are often appreciated in long responses to communicate more complex ideas in not only physics at the HSC level, but also other subjects such as HSC biology or HSC chemistry. This also includes tables and flow charts.

Question 3 Solution

Criteria	Marks
• Identifies a suitable time period after the Big Bang for nucleons to form as between 10^{-6} s to 1 s after the Big Bang.	1

Refer to the timeline given, as the hadron epoch where hadrons (nucleons) were formed is stated to have spanned from 10^{-6} s to 1 s after the Big Bang.

Question 4 Solution

Criteria	Marks
• Identifies Leavitt's discovery of the relationship between luminosity and period of Cepheid variable stars • Explains how the said relationship allows for the calculation of distance • Makes a judgment of the significance of Leavitt's discovery to Hubble's discovery of the expansion of the universe.	3
• Response satisfies 2 of the 3 mark criteria items	2
• Response satisfies 1 of the 3 mark criteria items	1

Sample answer:
Leavitt discovered that the larger the period of a Cepheid variable star in which it increases and decreases in brightness, the greater its luminosity, meaning the luminosity can be calculated regardless of the distance, as the period of the star does not get affected by the distance. By comparing the apparent brightness and luminosity of a Cepheid variable star, the distance to the star and their residing galaxies could be calculated, allowing for Hubble's discovery of the expansion of the universe as he was required to find the distance of galaxies, along with their recessional velocity to establish his law, $v = H_0D$.

Question 5 Solution

Criteria	Marks
• Draws out elements of the model and links elements to the expansion of the universe • Provides advantages and/or disadvantages of the model • Provides a logical judgement	5
• Draws out elements of the model and links elements to the expansion of the universe • Provides a logical judgement	4
• Draws out elements of the model • Shows some understanding of the expansion of the universe • Provides a judgement	3
• Draws out elements of the model • Shows some understanding of the expansion of the universe	2
• Provides some relevant information	1

Sample answer:
The investigation models the expansion of the universe by the expanding fabric of the balloon (as it is being inflated), representing the inflation of the universe. This effectively demonstrates the universe's inflation and the outward velocity of galaxies in all three spatial dimensions. This model also successfully models the outward velocity of galaxies, the increasing distance between galaxies, and the inflation of the universe in all 3 spatial dimensions. However, the model is inaccurate in its representation of the spots, representing galaxies, getting stretched as well. This representation is flawed, as galaxies themselves do not enlarge due to the inflation of the universe. The model also inaccurately implies a clear central point of expansion of the universe; however, the universe's geometric form is highly complicated with no definite centre. Therefore, this investigation models the overarching concepts of the expansion of the universe accurately; however, fails in precisely representing certain details.

Question 6 Solution

Criteria	Marks
• Identifies 3 remnants of the Big Bang. • Provides a thorough explanation of how each remnant supports the retrospective discovery of the Big Bang. • Critically evaluates the significance of each remnant in its contribution to the understanding of the Big Bang.	9
• Identifies 3 remnants of the Big Bang. • Provides an explanation of how each remnant supports the retrospective discovery of the Big Bang. • Critically evaluates the significance of at least 2 remnants.	7-8
• Identifies 2-3 remnants of the Big Bang. • Provides a basic explanation of how each remnant supports the retrospective discovery of the Big Bang. • Identifies the significance of the given remnants.	5-6
• Identifies 1-2 remnants of the Big Bang. • Provides a limited explanation of how each remnant supports the retrospective discovery of the Big Bang. • Identifies the significance of the given remnants.	3-4
• Identifies 1 remnant of the Big Bang. • Provides a vague explanation of how each remnant supports the retrospective discovery of the Big Bang.	1-2

Ensure that you understand each HSC verb. Critically evaluate calls for a thorough description of the elements involved, and a judgment of the value of the elements. A sample answer is given on the next page.

Sample answer:
By the Big Bang theory, the Photon epoch will have allowed for photons to freely travel, unimpeded by Compton scattering due to the reduction in density of free electrons. Theoretical calculations predicted that such photons from the epoch will have cosmically redshifted to the microwave range, which has been experimentally discovered by the presence of CMBR, a constant microwave radiation that permeates the universe. The presence of CMBR has provided strong evidence for the Big Bang theory, as it is a remnant of the Photon epoch as stated by the Big Bang theory.

The theory also states the universe has been constantly expanding from a point of singularity, implying galaxies will also be travelling away from the point of singularity. Hubble experimentally confirmed that surrounding galaxies have a recessional velocity proportional to the distance to the Milky Way galaxy. Hubble's law serves as strong evidence for the Big Bang theory as a remnant of the inflation of the universe that has caused galaxies to travel away from each other.

Furthermore, calculations that pertain to the Big Bang nucleosynthesis that occurred during the Lepton epoch predicted a number ratio of protons to neutrons of 7:1, implying a number ratio of hydrogen to helium of 12:1. This then further implies a mass ratio of hydrogen to helium of 3:1 as helium is approximately 4 times heavier than hydrogen. Observations of the universe show an approximate mass ratio of hydrogen to helium of 3:1, aligning with theoretical predictions based on the Big Bang theory. Therefore, the present mass ratio of hydrogen to helium serves as strong evidence of a remnant of the nucleosynthesis of the Big Bang.

Spectra of Stars SOLUTIONS

Question 1 Solution

As a star's density increases, the spectral lines of the star broaden as collisions between photons and the particles of the star become more frequent, leading to distortion. As a star's rotational velocity increases, the light emitted by the star can be both redshifted and blue shifted due to the Doppler effect (as seen in the sample answer on page 10), also leading to distortion. As density and rotational velocity both distort the spectra of stars, a high density and a high rotational velocity will lead to the largest distortion, meaning the answer is A.

Question 2 Solution

As galaxies' recessional velocity increases, the red shift of the emitted light increases. This is mathematically shown through the Doppler effect: $f' = f\frac{(v_{\text{wave}}+v_{\text{observer}})}{(v_{\text{wave}}-v_{\text{source}})}$ (Although the red shift of light emitted form a galaxy is from the expansion of space rather than the Doppler effect, this is beyond the syllabus). Therefore, the spectra that show the greatest red shift (i.e. the greatest shift to the red side of the spectrum of the absorption lines) show the greatest recessional velocity, which is D.

Question 3 Solution

The spectrum shows 3 distinct emission lines, at wavelengths of approximately 430 nm, 485 nm and 655 nm. These wavelengths align with the Balmer series: 434 nm, 486 nm, 656 nm. Therefore, the spectrum likely shows the Balmer series, meaning the electrons will have transitioned to a final energy level of n = 2, meaning the answer is B.

Question 4 Solution

The graph shown is a spectrum, and the dips shown in the spectrum must be absorption lines, as emission lines are an increase in intensity, not a decrease. Therefore, the answer must be B.

Question 5 Solution

Criteria	Marks
• Identifies a difference • Provides reasoning for the difference	2
• Identifies a difference	1

Sample answer:
The spectral lines emitted by the hydrogen in the Sun are broadened as a result of the frequent interactions of the photons and the particles of the Sun due to the high density, resulting in pressure broadening. In contrast, the spectral lines emitted by the hydrogen gas discharge tube will be sharp as a result of the infrequent interactions of photons and atoms due to the relatively low density of the hydrogen gas in the tube.

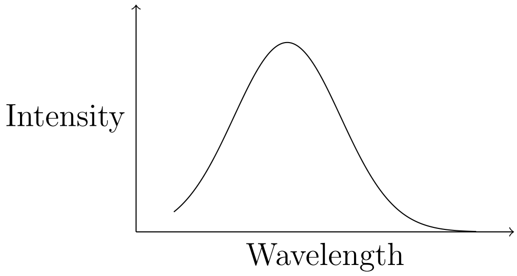
Question 6 Solution

Criteria	Marks
• Describes continuous, emission, and absorption spectra with specific examples for each spectrum • Evaluates the significance of each spectrum type in modern scientific understanding • Uses appropriate scientific terminology and demonstrates logical flow throughout	7
• Describes continuous, emission, and absorption spectra with some examples • Evaluates the significance of each spectrum type in modern scientific understanding	5-6
• Describes continuous, emission, and absorption spectra • Identifies the significance of each spectrum type in modern scientific understanding	3-4
• Provides a limited description of continuous, emission, and absorption spectra	1-2

The sample answer is given on the next page.

Sample answer:
Continuous spectra consist of a gradient of wavelengths emitted by blackbody radiators, such as the Sun or incandescent light sources.

Example of continuous spectrum



Continuous spectra allow astronomers to calculate the surface temperature of stars via mathematical models such as Wien's displacement law $\lambda_{\text{max}} = \frac{b}{T}$, and therefore are integral to astronomy as they allow for stellar classifications for a deeper understanding of stars.

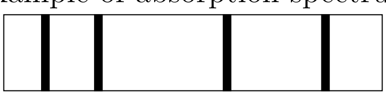
Emission spectra are the spectrum of discrete wavelengths emitted in the form of electromagnetic radiation by the transition of electrons.

Example of emission spectrum

Emission spectra play an essential role in identifying the composition of stars, as each element has a unique emission spectrum due to its unique nucleus, and therefore its unique electron transitions. Emission spectra are essential for identifying the composition of stars, as their unique emissions reveal the elements present, deepening our understanding of stellar and galactic chemistry.

Absorption spectra are the spectra of discrete wavelengths absorbed by electrons transitioning to a more energetic state.

Example of absorption spectrum



Hubble used absorption spectra to measure the redshift of surrounding galaxies, enabling the calculation of their recessional velocities and the formulation of Hubble's law, $v = H_0 D$. Therefore, absorption spectra have played a crucial role in our understanding of cosmology, serving as the foundation of one of the supporting pillars to the most widely accepted theory for the creation of the universe, the Big Bang.

The Hertzsprung-Russell Diagram SOLUTIONS

Question 1 Solution

Criteria	Marks
• Provides 3 points of comparison	3
• Provides 2 points of comparison	2
• Provides 1 point of comparison	1

Sample answer:
Stellar bodies A and B differ in their spectral classes. A is classified as O, while B is classified as K, meaning that A's surface temperature must be much hotter than B's. They also differ in their luminosity. A is measured to have an absolute magnitude of approximately -5, while B is measured to be approximately +7. This indicates that A is also much more luminous than B due to the absolute magnitude scale being inversely logarithmic. However, A and B are similar as their stellar type are both main sequence stars.

Question 2 Solution

Criteria	Marks
• Identifies 2 components • Explains how both components affect the luminosity	4
• Identifies 2 components • Explains how one of the components affects the luminosity	3
• Identifies 2 components	2
• Identifies a component	1

Sample answer:
The mass and radius of a star are major components of a star's luminosity. The mass determines the rate of nuclear fusion in its core, which is the main source of the luminosity in the form of electromagnetic radiation, meaning the greater the mass, the greater the luminosity. The square of the radius of a star is proportional to its surface area, meaning the radius also affects the luminosity. Luminosity refers to the magnitude of the

electromagnetic radiation a star emits in a certain time frame across all wavelengths, and therefore, with a larger surface area, a star can emit a larger amount of electromagnetic radiation and therefore have a greater luminosity. Therefore, as the radius of a star increases, its luminosity increases.

Question 3 Solution

Criteria	Marks
• Identifies bodies A, B and C • Correctly orders bodies A, B and C in their evolutionary stages	2
• Identifies bodies A, B and C OR • Correctly orders bodies A, B and C in their evolutionary stages	1

Sample answer:
A is a red giant, B is a white dwarf, and C is a main sequence star. Main sequence stars transition to red giants after the hydrogen in the core is exhausted, and then red giants transition to white dwarfs via the outer layers being shed due to stellar winds. Therefore, C is the earliest in its lifespan, then A, then B.

Question 4 Solution

Criteria	Marks
• Identifies both A and B • Compares temperature and luminosity • Provides reasoning for the difference and similarity	4
• Identifies both A and B • Compares temperature and luminosity	3
• Identifies both A and B • Compares temperature or luminosity	2
• Identifies both A and B	1

A sample answer is given on the following page.

Sample answer:
A is a white dwarf, and B is a low-mass main sequence star.

A, being a white dwarf, has an extreme surface temperature of approximately 17000 K due to its density. This is more than 6 times hotter than the surface of star B, with a surface temperature of approximately 2500 K, due to its much lower density and mass. Although A is more than 6 times hotter than B, they both have approximately the same luminosity of $3.2 \times 10^{-4}\,L_\odot$. This is because the radius, and therefore surface area of A is much smaller than B (white dwarfs typically have a radius of approximately 7000 km, or $0.01\,R_\odot$), resulting in its luminosity being similar to B, despite its extreme surface temperature. Furthermore, white dwarves do not undergo nuclear fusion, and hence radiate less energy.

Reading Logarithmic Scales

Reading off a non-linear scale can be confusing. However, there is an easy way to read off logarithmic scales. This is because although the values themselves are not increasing linearly, the orders are as shown below.

That means a value, say a half of an increment to the right from the 10^1 can be read as so.

$$10^{1.5} = 3.162\ldots \times 10^1$$

Question 5 Solution

Criteria	Marks
• Proposes a scenario of two class M bodies differing highly in luminosities • Provides an explanation of how the two bodies differ in luminosities	2
• Proposes a scenario of two class M bodies differing highly in luminosities	1

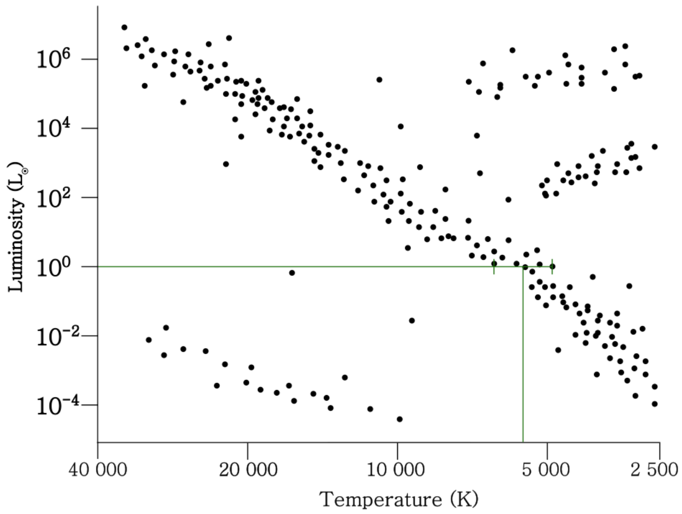
Sample answer:
It is possible for 2 stellar class M bodies to differ in their luminosities. For example, a red supergiant and a low mass main sequence star can both be classified as M; however, the red supergiant will be many orders above the main sequence star in terms of luminosity, despite a similar surface temperature. This is because red supergiants are one of the largest stellar bodies known to man, meaning their surface area is so great that even a low surface temperature will result in an extreme luminosity.

Question 6 Solution

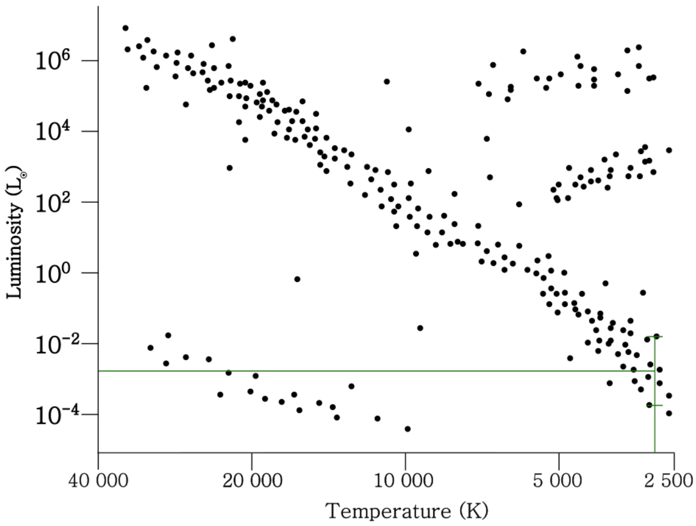
Criteria	Marks
• Utilisese the diagram to estimate the surface temperature of the Sun • Utilisese the diagram to estimate solar luminosity • Correctly estimates values from the graph	3
• Utilisese the diagram to estimate the surface temperature of the Sun • Utilisese the diagram to estimate solar luminosity	2
• Utilisese the diagram to estimate the surface temperature of the Sun	1

A sample answer is given on the following page.

Sample answer:
It is known that the Sun will have a luminosity of 1 $L_\odot$. That value can be used to find the estimate of the Sun's temperature as shown below.



From the graph, the Sun's surface temperature is estimated to be approximately 5800 K. Now this value can be halved, and then used to find the expected luminosity of a main sequence star that has a surface temperature of 2900 K as shown below.



Therefore, by the diagram given, it can be estimated that a main sequence star that has half the surface temperature of the Sun will have a luminosity of approximately $1.6e - 3\,L_\odot$.

Energy Sources of Stars SOLUTIONS

Question 1 Solution

Criteria	Marks
• Identifies the main process of energy production in the Sun • Accounts for all steps correctly and gives the correct energy produced.	3
• Identifies the main process of energy production in the Sun • Accounts for some steps correctly	2
• Identifies the main process of energy production in the Sun	1

Sample answer:
The main process that produces energy in the Sun is the proton-proton chain. The proton-proton chain begins with 2 instances of the following reaction occurring.

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + {}^{\;\,0}_{+1}\mathrm{e} + \nu $$

Then, 2 instances of the following reaction occur.

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + \gamma $$

And finally, 1 instance of the following reaction occurs to finish the proton-proton chain.

$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H} $$

This produces a total energy of 26.73 MeV. Thus, the proton-proton chain can be summarised as:

$$ 4\,{}^{1}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{\;\,0}_{+1}\mathrm{e} + 2\,\nu + 2\gamma + \text{Energy}(26.73\,\text{MeV}) $$

Question 2 Solution

Criteria	Marks
• Identifies the main process of energy production in Rigel • Accounts for all steps correctly and gives the correct energy produced.	3
• Identifies the main process of energy production in Rigel • Accounts for some steps correctly	2
• Identifies the main process of energy production in Rigel	1

Sample answer:
The main process that produces energy in Rigel is the CNO cycle. The CNO cycle begins with the following reaction occurring.

$$^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

Then the following reaction occurs:

$$^{13}_{7}\mathrm{N} \longrightarrow {}^{13}_{6}\mathrm{C} + {}^{0}_{+1}\mathrm{e} + \nu$$

Then the following reaction occurs:

$$^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{14}_{7}\mathrm{N} + \gamma$$

Then the following reaction occurs:

$$^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{15}_{8}\mathrm{O} + \gamma$$

Then the following reaction occurs:

$$^{15}_{8}\mathrm{O} \longrightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{+1}\mathrm{e} + \nu$$

Then the following reaction occurs:

$$^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$$

This produces a total energy of 26.7 MeV. Thus, the proton-proton chain can be summarised as:

$$4\,{}^{1}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{+1}\mathrm{e} + 2\,\nu + 3\gamma + \mathrm{Energy}(26.73\,\mathrm{MeV})$$

Question 3 Solution

Criteria	Marks
• Provides 2 similarities between the proton-proton chain and the CNO cycle • Provides 2 differences between the proton-proton chain and the CNO cycle	4
• Provides 2-3 similarities and/or differences between the proton-proton chain and the CNO cycle	2-3
• Makes some comparison between the proton-proton chain and the CNO cycle	1

Sample answer:

Comparison of proton-proton chain and the CNO cycle

Feature	Proton-Proton Chain	CNO Cycle
Type of Energy	Fusion Energy	
Reactants	4 Protons ($4\,^1_1\mathrm{H}$)	
Main Occurrence	Low Mass ($< 1.3\,\mathrm{M}_\odot$) Main Sequence Stars	High Mass ($> 1.3\,\mathrm{M}_\odot$) Main Sequence Stars
Presence of Catalyst	None	Presence of Carbon-12 as Catalyst

Question 4 Solution

Criteria	Marks
• Assesses Emmy's statement as partly true • Gives reasons for the statement's true and false elements	3
• Assesses Emmy's statement as partly true • Gives a reason for the statement's true or false element	2
• Assesses Emmy's statement as partly true	1

The sample answer is given on the following page.

Sample answer:
Emmy's statement is only partly true. The element that is true in his statement is that the products of both processes are very similar. The products of the proton-proton chain are as follows:

$$^{4}_{2}\text{He} + 2\,^{0}_{+1}\text{e} + 2\,\nu + 2\gamma + \text{Energy}(26.73\,\text{MeV})$$

And the products of the CNO cycle are as follows:

$$^{4}_{2}\text{He} + 2\,^{0}_{+1}\text{e} + 2\,\nu + 3\gamma + \text{Energy}(26.73\,\text{MeV})$$

As can be seen, the only difference in the products is that the CNO cycle produces one additional gamma ray. However, the sub-processes that exist within the proton-proton chain and the CNO cycle are very different. The proton-proton chain does not have the presence of a catalyst; however, in the CNO cycle, a catalyst of a Carbon-12 nucleus is present, allowing for the reaction to take place. Thus, although the products of both processes are similar, the processes themselves are not similar, making Emmy's statement only partly true.

Question 5 Solution

Criteria	**Marks**
• Provides 2 reasons • Clearly links reasons why CNO cycles occur in heavier stars and why proton-proton chains occur in lighter stars	4
• Provides 2 reasons • Links reasons why CNO cycles occur in heavier stars and why proton-proton chains occur in lighter stars	3
• Provides 1 reason • Links reasons why CNO cycles occur in heavier stars and why proton-proton chains occur in lighter stars	2
• Provides relevant information	1

The sample answer is given on the following page.

Sample answer:
Higher Core Temperature
The CNO cycle requires a higher temperature to overcome the stronger Coulombic repulsive forces due to the larger charge of the catalyst's nucleus. This higher temperature is achieved through the gravitational pressure exerted by the star's mass. In more massive stars, the greater gravitational pressure results in a higher core temperature. This elevated temperature allows the CNO cycle to proceed efficiently. In contrast, the cores of less massive stars lack the necessary pressure and temperature to initiate the CNO cycle, explaining why it predominantly occurs in more massive stars.

Catalyst Availability
The CNO cycle also requires the presence of Carbon-12 as a catalyst. Heavy elements like Carbon-12 are generally less abundant than hydrogen and helium. However, they are more commonly found in massive stars due to their typical heavy element-rich formation environments and nucleosynthesis processes. Since more massive stars have a greater abundance of heavy elements like Carbon-12, they are more capable of sustaining the CNO cycle. In contrast, lighter stars lack the necessary abundance of these heavy elements, limiting the occurrence of the CNO cycle.

Question 6 Solution

Firstly, let the number of CNO cycle reactions that occur every second be x, and the number of proton-proton chain reactions that occur every second be y. Since the CNO cycle produces 4 fifths of the energy deposited into the star, it can be stated that the CNO cycle reactions deposits 4 times the energy that the proton-proton chain reactions deposts. Therefore:

$$\begin{aligned} E_{\text{CNO}} &= 4(E_{\text{PP}}) \\ 24.73x &= 4(26.13)y \\ y &= \frac{2473}{10452}x \end{aligned}$$

Therefore, we can express the total energy produced per second as so:

$$\begin{aligned} E_{\text{produced}} &= 24.73x + 26.13y \\ &= 24.73x + 26.13\left(\frac{2473}{10452}\right)x \\ &= \frac{2473}{80}x \,\text{MeV} \\ &= \frac{2473}{80}x \times 10^{6}\,\text{eV} \\ &= \frac{2473}{80}x \times 10^{6} \times 1.602 \times 10^{-19}\,\text{J} \\ &= 4.952\ldots \times 10^{-12}x\,\text{J} \end{aligned}$$

The solution continues over the page.

And because we know the power deposited into the star is 4.35×10^{27} W, we know the energy deposited per second is 4.35×10^{27} J. (W=J s^{-1})

$$
\begin{aligned}
E_{\text{produced}} &= 4.35 \times 10^{27} \\
4.952\ldots \times 10^{-12} x &= 4.35 \times 10^{27} \\
x &= 4.35 \times 10^{27} \div (4.952\ldots \times 10^{-12}) \\
x &= 8.784\ldots \times 10^{38} \\
\therefore y &= \frac{2473}{10452}(8.784\ldots \times 10^{38}) \\
y &= 2.078\ldots \times 10^{38}
\end{aligned}
$$

And because both the CNO cycle and the proton-proton chain use 4 protons ($4\,{}^{1}_{1}\text{H}$), it can be stated:

$$
\begin{aligned}
\text{mass fused per second} &= 4m_{\text{p}} \times n_{\text{CNO}} + 4m_{\text{p}} \times n_{\text{PP}} \\
\text{mass fused per second} &= 4m_{\text{p}} \times (n_{\text{CNO}} + n_{\text{PP}}) \\
\text{mass fused per second} &= 4(1.673 \times 10^{-27}) \times (8.784\ldots \times 10^{38} + 2.078\ldots \times 10^{38}) \\
\text{mass fused per second} &= 7.26908\ldots \times 10^{12} \\
\text{mass fused per second} &= 7.269 \times 10^{12}\,\text{kg}
\end{aligned}
$$

Therefore, the star fuses hydrogen at a rate of 7.269×10^{12} kg s^{-1}.

Alternatively, we may find the mass required to produce such power, then find the average energy deposited by each fusion reaction to find the number of reactions that take place per second.

$$
\begin{aligned}
E &= mc^2 \\
m &= \frac{E}{c^2} \\
m &= \frac{4.35 \times 10^{27}}{(3 \times 10^8)^2} \\
m_{\text{required}} &= 4.833\ldots \times 10^{10}\,\text{kg}
\end{aligned}
$$

As an exact average, each reaction releases 24.997... MeV. Reaching this exact value is left as an exercise to the reader, as it is a more complicated process than using an approximate average of 25.01 MeV, which will still allow for finding the approximate answer.

$$
\begin{aligned}
m &= \frac{E}{c^2} \\
m &= \frac{24.997\ldots \times 10^6 \times 1.602 \times 10^{-19}}{(3 \times 10^8)^2} \\
m_{\text{per reaction}} &= 4.449\ldots \times 10^{-29}\,\text{kg}
\end{aligned}
$$

The solution continues over the page.

$$
\begin{aligned}
n_{\text{reactions}} &= \frac{m_{\text{required}}}{m_{\text{per reaction}}} \\
n_{\text{reactions}} &= \frac{4.833\ldots \times 10^{10}}{4.449\ldots \times 10^{-29}} \\
n_{\text{reactions}} &= 1.086\ldots \times 10^{39}
\end{aligned}
$$

$$
\begin{aligned}
\therefore m_{\text{fused}} &= n_{\text{reactions}} m_{\text{reactants}} \\
m_{\text{fused}} &= 4 n_{\text{reactions}} m_{\text{proton}} \\
m_{\text{fused}} &= 4(1.086\ldots \times 10^{39})(1.673 \times 10^{-27}) \\
m_{\text{fused}} &= 7.26908\ldots \times 10^{12} \\
\text{mass fused per second} &= 7.269 \times 10^{12}\,\text{kg}
\end{aligned}
$$

!? The Value of Approximations

The concept of approximations is often overlooked at the high school level, as it isn't emphasised that almost the entirety of the NESA Physics course is made up of models that make approximations. To go further, it can be argued that the study of physics itself is an approximation that humans have constructed only somewhat accurately to explain and predict the world around them. However, the compromise of accuracy must not deter students from learning about approximations, as the benefits they bring often (strongly) outweigh the losses in accuracy. For example, we approximate that the effect of gravity in subatomic particles is negligible or that the relativistic effects of everyday phenomena are negligible. If we wanted a completely accurate prediction of such scenarios, we must account for gravity or relativity; however, we must always consider the accuracy that is required of our calculations, and accordingly make approximations to ensure we are able to both produce an answer and a good one.

A good example of making approximations can be found in Question 32 of the 2023 HSC Physics examination.

Structure of the Atom SOLUTIONS

Discovery of the Electron SOLUTIONS

Question 1 Solution

Thomson first found the charge-to-mass ratio in 1897, and subsequently Millikan found the charge of a single electron in 1909, allowing for the mass of the electron to be found. Therefore, the answer is C.

Question 2 Solution

Criteria	Marks
• Utilises at least 2 distinct cathode ray experiments to support the answer • Clearly creates logical inferences from the observations of the experiments	3
• Utilises 1 distinct cathode ray experiment to support the answer • Creates logical inferences from the observation of the experiment	2
• Some relevant information	1

Sample answer:
Cathode rays are observed to be emitted by cathodes made of various metals, suggesting that their constituents are present in all atoms. The Maltese cross experiment demonstrates that cathode rays cast a distinct shadow, indicating they are composed of particles rather than waves. The application of an electric field shows that cathode rays are deflected toward the positively charged anode, meaning the constituent particles of cathode rays are negatively charged. Therefore, it can be logically inferred that these constituent particles of the cathode rays are negatively charged fundamental particles that exist in atoms.

Question 3 Solution

Criteria	Marks
• Identifies the equalisation of the forces of the magnetic and the electric fields • Mathematically defines of velocity in terms of field strengths • Identifies the force of the magnetic field as the centripetal force • Mathematically finds a definition of the charge-to-mass ratio in measurable quantities	4
• Satisfies 3 items of the criteria for 4 marks	3
• Satisfies 2 items of the criteria for 4 marks	2
• Satisfies 1 item of the criteria for 4 marks	1

Sample answer:
First, adjust the electric or magnetic field strength so that the cathode ray follows a straight path to the screen without deflection. Since the electric and magnetic forces act in opposite directions, the condition for no deflection implies that their magnitudes must be equal:

$$\begin{aligned} F_{\mathrm{E}} &= F_{\mathrm{B}} \\ qE &= qvB \\ E &= vB \\ v &= \frac{E}{B} \end{aligned}$$

Now, deactivate the electric field so the magnetic field deflects the cathode ray into a circular path. Therefore, the magnetic field's force is applying a centripetal force to the particles of the cathode ray, meaning:

$$\begin{aligned} F_{\mathrm{B}} &= F_{\mathrm{c}} \\ qvB &= \frac{mv^2}{r} \\ qB &= \frac{mv}{r} \\ \frac{q}{m} &= \frac{v}{Br} \\ \frac{q}{m} &= \frac{E}{B^2 r} && (v = \frac{E}{B}) \end{aligned}$$

The sample answer continues over the page.

Since the electric field strength is measurable as as $E = \frac{V}{d}$, and both B and the radius of curvature r can simply be measured as well, the charge-to-mass ratio $(\frac{q}{m})$ is expressed purely by measurable quantities, and hence, the charge-to-mass ratio of the electron can be found.

Question 4 Solution

Criteria	**Marks**
• Identifies the creation and ionisation of oil droplets • Mathematically shows when the oil droplets are experiencing no net force • Mathematically defines charge in measurable quantities • Links finding the total charges of oil droplets and finding the charge of a single electron	4
• Satisfies 3 items of the criteria for 4 marks	3
• Satisfies 2 items of the criteria for 4 marks	2
• Satisfies 1 item of the criteria for 4 marks	1

Sample answer:
First, spray oil droplets into the compartment above. Some of these droplets will fall through the hole into the lower compartment. As they fall, ionising X-rays impart a negative charge to the droplets, causing them to acquire a net negative charge. In the lower compartment, the electric field exerts an upward force on the droplets because the upper plate is positively charged. This results in some droplets experiencing a net force of zero, as the downward force due to gravity is exactly balanced by the upward electrostatic force. Consequently, these droplets will either remain stationary or travel at a constant velocity, as there is no net acceleration. This scenario can be expressed mathematically as:

$$F_{\mathrm{E}} = F_{\mathrm{g}}$$
$$qE = mg$$
$$q = \frac{mg}{E}$$

The solution continues over the page.

Now, since the radius of the oil droplets can be measured through a telescopic eyepiece and the density of the oil is known, the mass of each droplet can be determined.

$$m = V \times \rho$$
$$m = \frac{4\pi r^3}{3} \times \rho$$
$$m = \frac{4\pi r^3 \rho}{3}$$

Now, substituting this expression for mass into the expression for charge:

$$q = \frac{4\pi r^3 \rho}{3} \times \frac{g}{E}$$
$$q = \frac{4\pi r^3 \rho g}{3E}$$

And since the droplet's radius r, the gravitational field strength g, the electric field strength E, and the density of the oil ρ are all known values, the overall charge of each oil droplet q can be found. Then, the largest common factor of the discrete differences between the total charges of the droplets can be found, which will be the likely charge of the electron.

Question 5 Solution

It can be seen that this scenario models Millikan's oil drop experiment, as the mystery boxes represent the oil droplets, and the dice's mass represents the individual charges of electrons. Therefore, to find the mass of a single die, find the largest common factor of the discrete differences between the boxes' masses.

Mass of Mystery Box (g)	Difference in Mass to Next Box (g)
343	46
389	69
458	207
665	

Therefore, the largest common factor of the discrete differences between the boxes is 23 g, meaning a die's likely mass is 23 g.

Question 6 Solution

If the electric field and the magnetic field's applied forces were adjusted so they do not deflect the cathode ray's path, the magnitude of their forces must have been equal. Hence,

$$\begin{aligned} F_{\mathrm{B}} &= F_{\mathrm{E}} \\ qvB &= qE \\ vB &= E \\ v &= \frac{E}{B} \end{aligned}$$

Now, if the electric field is deactivated, the magnetic force applies a centripetal force. Hence, the following expression for the electric field strength E used previously can be found like so.

$$\begin{aligned} F_{\mathrm{B}} &= F_{\mathrm{c}} \\ qvB &= \frac{mv^2}{r} \\ qB &= \frac{mv}{r} \\ qB &= \frac{Em}{Br} \\ E &= \frac{B^2rq}{m} \\ E &= \frac{q}{m} \times B^2r \end{aligned}$$

Now, looking at the graph, the gradient is rise over run, or $\frac{B^2}{r^{-1}} = B^2r$. Hence, the gradient of the graph represents B^2r. For the following calculation concerning the gradient, the points $(1.0 \times 10^1, 2.5 \times 10^{-7})$ and $(8.75 \times 10^1, 22.5 \times 10^{-7})$ have been taken.

$$\begin{aligned} \text{gradient} &= \frac{\Delta y}{\Delta x} \\ B^2r &= \frac{22.5 \times 10^{-7} - 2.5 \times 10^{-7}}{8.75 \times 10^1 - 1.0 \times 10^1} \\ B^2r &= 2.580\ldots \times 10^{-8} \end{aligned}$$

Now substituting B^2r into the expression for E,

$$\begin{aligned} E &= \frac{q}{m} \times B^2r \\ E &= \frac{1.602 \times 10^{-19}}{9.109 \times 10^{-31}} \times 2.580\ldots \times 10^{-8} \\ E &= 4538.581\ldots \\ E &= 4.5 \times 10^3\,\mathrm{V\,m^{-1}} \end{aligned}$$

Rutherford and Chadwick's Nuclear Discoveries SOLUTIONS

Question 1 Solution

Previous to the Rutherford model of the atom, the Thomson model of the atom was widely accepted. The Thomson model of the atom envisioned that an atom was a spherical positive charge with embedded electrons. By this model, the alpha particles emitted by the radiation source (e.g. radium) would travel through the gold atoms with minimum deflection. Therefore, the answer is A.

Question 2 Solution

Criteria	Marks
• Correctly identifies the discrepancy of energies • Clearly describes the discrepancy of energies	3
• Correctly identifies the discrepancy of energies • Gives a limited description of the discrepancy of energies	2
• Correctly identifies the discrepancy of energies	1

Sample answer:
In Chadwick's experiment, an alpha particle source bombarded beryllium, producing an unknown neutral radiation. This radiation was confirmed to be neutral as it was not deflected by an electric field and was initially believed to be gamma rays. However, when the neutral radiation was aimed towards paraffin wax, it ejected protons that implied the gamma rays' kinetic energy was 50 MeV. However, the alpha particles used to produce the radiation had a kinetic energy of 5 MeV, meaning that, under the law of conservation of energy, any emitted gamma rays should have had at most 5 MeV. Since gamma rays could not transfer more energy than their own, this energy discrepancy suggested the radiation was not gamma rays but rather a neutral particle with a mass similar to a proton. This led to the discovery of the neutron.

Question 3 Solution

Criteria	Marks
• Describes all components of the Geiger-Marsden experiment • Explains how observations led to a change in the model of the atom	5
• Describes most components of the Geiger-Marsden experiment • Describes the observations that led to a change in the model of the atom	3-4
• Describes few components of the Geiger-Marsden experiment • Identifies the observations that led to a change in the model of the atom	1-2

Sample answer:
In the Geiger-Marsden experiment, an alpha source, such as Polonium, emits alpha radiation towards a thin gold foil. A curved fluorescent screen is placed around the gold foil so that scattered alpha particles can be detected, allowing for their trajectories to be observed. Initially, the widely accepted model of the atom was Thomson's model of the atom, and because the atom was thought to be a spherical diffuse positive charge with embedded electrons, it was thought that the alpha particles would be minimally deflected by the gold foil. However, when the experiment was carried out, it was observed that although most alpha particles were minimally deflected, a small portion of the alpha particles were deflected at extreme angles (greater than 90°), with some alpha particles being deflected almost directly backwards by the gold foil. The Thomson model of the atom failed to explain such a phenomenon as the model lacked a concentrated region of positive charge capable of exerting a strong repulsive force. Thus, Rutherford proposed the nuclear model of the atom, which stated nearly all of the atom's mass is concentrated in a small, dense positive nucleus, while electrons orbit in a very large radius around this nucleus, essentially making an atom mostly empty space. This model was able to successively explain why most alpha particles passed through undeflected, as the atom was mostly empty space; however, a few experienced extreme deflections due to the interactions with the dense, positively charged nucleus.

Question 4 Solution

Criteria	Marks
• Describes all components of Chadwick's experiment • Explains how observations led to the discovery of the neutron • Inclusion of a nuclear equation	5
• Describes most components of Chadwick's experiment • Explains how observations led to the discovery of the neutron	3-4
• Describes few components of Chadwick's experiment • Identifies that the neutron was discovered	1-2

Sample answer:
In Chadwick's experiment, an alpha source such as Polonium was used to bombard Beryllium, causing the following nuclear reaction:

$$^{9}_{4}\mathrm{Be} + ^{4}_{2}\mathrm{He} \longrightarrow ^{12}_{6}\mathrm{C} + ^{1}_{0}\mathrm{n}$$

Therefore, when Beryllium was bombarded with alpha radiation, a neutral radiation was emitted. The neutrality was confirmed by the lack of deflection in an electric field. However, at the time of the experiment, neutrons were not yet confirmed to exist, leading to physicists believing that the neutral radiation was gamma rays. When Chadwick then bombarded Paraffin wax, a rich source of protons, the neutral radiation caused protons to be ejected at very high kinetic energies of around 5 MeV. This led to calculations that stated if the neutral radiation were gamma rays, the gamma rays would require a kinetic energy of approximately 50 MeV for such an interaction. However, this led to a discrepancy as the initial alpha particles that would cause the neutral radiation to be emitted were only of kinetic energies of approximately 5 MeV, thereby the gamma rays having a kinetic energy of approximately 50 MeV would be a violation of the conservation of energy. For this interaction to be possible, it was known that the neutral radiation must have consisted of particles that had a mass similar to protons; however, they were of neutral charge. These particles were later named neutrons, and thus, the neutron was discovered.

Question 5 Solution

Criteria	Marks
• Links all components of the model to the experiment • Describes an advantage of the model • Describes a disadvantage of the model • Provides a judgment on the criteria of the effectiveness of the model	5
• Links most components of the model to the experiment • Describes an advantage of the model • Describes a disadvantage of the model • Provides a judgment of the model	3-4
• Links few components of the model to the experiment • Provides a judgment of the model	1-2

Sample answer:
Emmy's model uses bullets to represent alpha particles and different obstructions to model the expected and actual results of the Geiger-Marsden Experiment. When Emmy shoots at the target with the obstruction of a water balloon, Emmy models the expected outcomes of the experiment. This is because the bullet's trajectories are mostly undeflected, representing the minimal expected deflection of alpha particles in the presence of a diffuse positive charge, as stated in the Thomson model of the atom. When Emmy shoots at the target with the obstruction of a durable thin pole, Emmy models the actual experimental outcomes of the experiment. This is because the dense pole is thin, meaning most bullets miss the pole and hit the target, however, when the bullets do hit the pole they are deflected at unexpected angles, representing the alpha particles being mostly undeflected, but some being drastically deflected by the extremely dense positive nucleus of the atom by Rutherford's model. A major strength of Emmy's model is the accurate distinction between the expected outcomes by Thomson's model and the observed outcomes that were explained by Rutherford's model. This effectively contrasts the Thomson model's diffuse large charge with Rutherford's extremely dense positive nucleus. However, a limitation of Emmy's model is that when she shoots bullets that barely miss the pole, to accurately represent the electrostatic repulsion between alpha particles and positive nuclei, the bullet should be deflected by the pole even when the bullet misses it. Overall, Emmy's model provides a mostly accurate representation of the Geiger-Marsden experiment; however, it has a limitation in failing to capture the electrostatic repulsion between nuclei and alpha particles accurately.

Question 6 Solution

Criteria	Marks
• Uses examples of shift to the Rutherford model and the Chadwick model • Explains that when theories fail to explain experimental phenomena, the theories change • Makes reference to the statement	4
• Uses examples of shift to the Rutherford model and the Chadwick model • Identifies that when theories fail to explain experimental phenomena, the theories change	2-3
• Some relevant information	1

Sample answer:
In physics, models are created to provide a framework for understanding certain concepts or phenomena. However, for models to hold value, they must accurately explain experimental results. As Feynman stated, "If it doesn't agree with experiment, it's wrong." When a model fails to align with experimental observations, it must be revised or replaced accordingly. This principle is clear in the shift to the Rutherford model of the atom and the shift to the Chadwick model of the atom. In the Geiger-Marsden experiment, some alpha particles were shown to be deflected at extreme angles by a gold foil; however, the Thomson model of the atom's diffuse positive charge was incapable of producing such a great electrostatic repulsion. To account for this observation, Rutherford introduced the nuclear model of the atom, which stated that an extremely small but dense positive nucleus resided in the centre of an atom. A similar occurrence is seen after Chadwick's experiment. In Chadwick's experiment, the neutral radiation emitted by beryllium when bombarded with alpha particles was able to eject protons from paraffin wax at extremely high kinetic energies. These high kinetic energies could not be explained by any known particle at the time, leading to the discovery of neutrons. Therefore, the model of the atom was revised again to account for experimental results. From the previous examples, it can be seen that when observations challenge existing theories, the models must be adapted or replaced to maintain their explanatory value.

Quantum Mechanical Nature of the Atom SOLUTIONS

Bohr's Atomic Model and Hydrogen Spectra SOLUTIONS

Question 1 Solution

The transitions of a Bohr model hydrogen atom that emits visible light (such as red light) must belong to the Balmer series. Therefore, the energy transition must end at $n = 2$. Therefore, the answer is C or D. However, the red side of visible light is the longer wavelength, which corresponds to a smaller energy. Since C is the smallest energy transition, C is the answer.

Question 2 Solution

Criteria	**Marks**
• Identifies the truth of the statement that the Bohr model accounts for the discrete spectral lines • Identifies the falsity of the statement that absorption causes a decrease in energy level • Includes a mathematical model	3
• 2 of the above	2
• 1 of the above	1

Sample answer:
The first part of the statement, "The Bohr model of the atom accounts for the discrete spectral lines of hydrogen", is true. In his model, Bohr stated that there were discrete energy levels electrons could occupy, and a transition from one energy level to another would cause the absorption or emission of a photon, with its energy being linked to the transition as $E = E_{\mathrm{i}} - E_{\mathrm{f}} = hf$. Thus, such discrete transitions led to discrete spectral lines of hydrogen. However, the second part of the statement, "when the hydrogen atom absorbs a photon, an electron transitions to a lower energy level to represent the reduction in energy due to absorption" is false, as when a hydrogen atom absorbs a photon an electron must transition to a higher energy level as the atom has gained energy. This is also true for the contrary, as if a hydrogen atom emits a photon, an electron must transition to a lower energy level as the atom has lost energy.

Question 3 Solution

Data List:

- $n_{\mathrm{f}} = 6$
- $n_{\mathrm{i}} = 2$
- $\lambda = ?\,\mathrm{m}$

Solution:

$$\frac{1}{\lambda} = R(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2})$$

$$\lambda = [R(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2})]^{-1}$$

$$\lambda = [(1.097 \times 10^7)(\frac{1}{(6)^2} - \frac{1}{(2)^2})]^{-1}$$

$$\lambda = -4.102\ldots \times 10^{-7}$$

However λ cannot be negative.

$$\therefore \lambda = 4.10 \times 10^7\,\mathrm{m}$$

Because the electron has transitioned to a higher energy level, a photon must have been absorbed.
Thus, a photon of wavelength $4.10 \times 10^7\,\mathrm{m}$ has been absorbed.

Question 4 Solution

Criteria	Marks
• Identifies the continuous spectrum of light used in producing absorption spectra • Identifies absorption of specific discrete frequencies of photons • Links specific frequencies that are absorbed to energy level transitions • Identifies the external energy source used in producing emission spectra • Identifies the emission of specific discrete frequencies of photons • Links specific frequencies that are emitted to energy level transitions	6
• Mostly explains the processes listed above	4-5
• Somewhat explains the processes listed above	2-3
• Provides some relevant information	1

Sample answer is provided on the following page.

To produce an absorption spectrum, a continuous spectrum of light is directed at a sample of hydrogen gas. Electrons in hydrogen atoms can absorb photons only if the photon energy matches the energy difference between two allowed electron energy levels. This follows the relation: $hf = E_\mathrm{f} - E_\mathrm{i}$. As a result, specific frequencies are removed from the continuous spectrum, appearing as dark lines in the observed hydrogen absorption spectrum. To produce an emission spectrum, hydrogen gas must first be excited by an external energy source, such as an electrical discharge. This energy excites electrons to higher energy levels. These excited electrons then return to lower energy states, emitting photons whose frequencies correspond to the energy difference between the two levels, also given by: $hf = E\mathrm{i} - E_\mathrm{f}$. Because electron transitions are discrete, the emitted photons form distinct spectral lines rather than a continuous distribution, creating a hydrogen emission spectrum.

Question 5 Solution

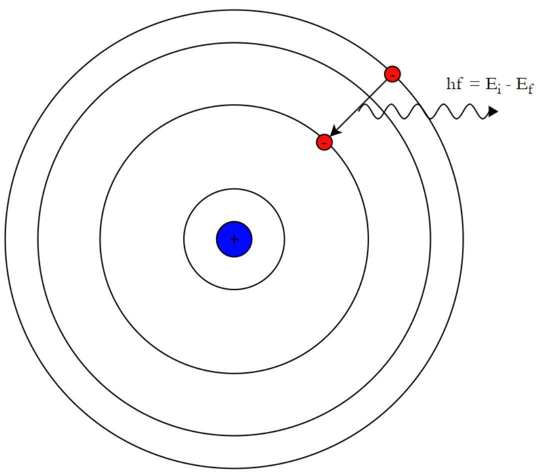
$$\frac{1}{\lambda} = R(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2})$$

$$\begin{aligned}
\lambda f &= c \\
f &= c \times \frac{1}{\lambda} \\
f &= cR(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}) \\
f &= cR(\frac{1}{2^2} - \frac{1}{n_\mathrm{i}^2}) \\
f &= cR(\frac{1}{4} - \frac{1}{n_\mathrm{i}^2}) \\
f &= \frac{cR}{4} - \frac{cR}{n_\mathrm{i}^2} \\
\therefore f &\propto -\frac{1}{n_\mathrm{i}^2}
\end{aligned}$$

Question 6 Solution

Criteria	Marks
• Describes the Bohr model of the atom • Links electron energy level transitions to discrete wavelength emissions • Includes mathematical model $E_{\text{i}} - E_{\text{i}} = hf$ • Includes a diagram that shows an energy level transition • Identifies observation as the Balmer series • Includes sample calculation	6
• 4-5 of the above	4-5
• 2-3 of the above	2-3
• Provides some relevant information	1

Sample answer:
The Bohr model of the atom introduced the idea of discrete energy levels that electrons could occupy, and the idea that when electrons transition between these energy levels, a photon will be absorbed or emitted according to the relation $hf = E_{\text{i}} - E_{\text{f}}$. Due to the discrete nature of the energy levels, transitions between them were also discrete, meaning the emitted photons also had discrete frequencies. This process is shown in the diagram provided.



The observation shown is the Balmer series, which was explained as the emitted visible light photons from the electrons of a Bohr model hydrogen atom transitioning to the second energy level from higher energy levels. For example, the largest frequency in the Balmer series can be calculated using $n_{\text{i}} = 3$, and $n_{\text{f}} = 2$ as so:

$$\frac{1}{\lambda} = R(\frac{1}{n_{\text{f}}^2} - \frac{1}{n_{\text{i}}^2})$$

$$\lambda = [R(\frac{1}{n_{\text{f}}^2} - \frac{1}{n_{\text{i}}^2})]^{-1}$$

$$\lambda = [(1.097 \times 10^7)(\frac{1}{(2)^2} - \frac{1}{(3)^2})]^{-1}$$

$$\lambda = 6.563\ldots \times 10^{-7}$$

$$\lambda = 656\,\text{nm}$$

Louis de Broglie's Matter Waves SOLUTIONS

Question 1 Solution

By de Broglie's matter wave formula $\lambda = \frac{h}{mv}$, it can be seen that $\lambda \propto \frac{1}{m}$, meaning the wavelength decreases as the mass increases. As the magnitude of the velocity is kept constant by Emmy, while her mass will increase due to her consumption of the chips, her de Broglie wavelength will decrease, hence the answer is C.

Question 2 Solution

Data List:

- $V = 25\,\text{V}$

Solution:

$$W = qV$$
$$E_{\text{kinetic}} = qV$$
$$\frac{1}{2}mv^2 = qV$$
$$v^2 = \frac{2qV}{m}$$
$$v = \sqrt{\frac{2qV}{m}}$$

$$\lambda = \frac{h}{mv}$$
$$\lambda = \frac{h}{m(\sqrt{\frac{2qV}{m}})}$$
$$\lambda = \frac{(6.626 \times 10^{-31})}{(9.109 \times 10^{-31})(\sqrt{\frac{2(1.602\times10^{-19})(25)}{(9.109\times10^{-31})}})}$$
$$\lambda = 2.453\ldots \times 10^{-10}$$
$$\lambda = 2.5 \times 10^{-10}\,\text{m}$$

Question 3 Solution

Criteria	Marks
• Identifies that de Broglie wavelengths are too small • Provides a sample calculation, or mathematically supports the answer otherwise	2
• Identifies that de Broglie wavelengths are too small	1

Sample answer:
The matter wavelengths of classical objects do not manifest at the observable level due to the de Broglie wavelength being extremely small for massive macroscopic objects. This is due to the small scale of Planck's constant h of 6.626×10^{-34} J s. For example, a baseball of mass 0.015 kg travelling at $40.0\,\text{m}\,\text{s}^{-1}$ would have a de Broglie wavelength:

$$\lambda = \frac{h}{mv}$$
$$\lambda = \frac{(6.626 \times 10^{-34})}{(0.015)(40.0)}$$
$$\lambda = 1.104\ldots \times 10^{-33}$$
$$\lambda = 1.10 \times 10^{-33}\,\text{m}$$

Such a small wavelength would have no perceptible impacts at the observable level, like all other macroscopic massive objects that have too high a mass for the de Broglie wavelength to have apparent effects.

Question 4 Solution

$$\lambda_{\text{B}} = \frac{h}{m_{\text{B}} v_{\text{B}}} \qquad \lambda_{\text{A}} = \frac{h}{m_{\text{A}} v_{\text{A}}}$$
$$\lambda_{\text{B}} = \frac{h}{3m_{\text{A}} v_{\text{B}}} \qquad \lambda_{\text{A}} = \frac{h}{4m_{\text{A}} v_{\text{B}}}$$

$$\lambda_{\text{B}} = x\lambda_{\text{A}}$$
$$\left(\frac{h}{3m_{\text{A}} v_{\text{B}}}\right) = x\left(\frac{h}{4m_{\text{A}} v_{\text{B}}}\right)$$
$$x = \frac{4}{3}$$
$$\therefore \lambda_{\text{B}} = \frac{4}{3}\lambda_{\text{A}}$$

Question 5 Solution

$$\lambda = 2d\sin\theta$$

$$\begin{aligned}
\lambda &= \frac{h}{mv} \\
\therefore \frac{h}{mv} &= 2d\sin\theta \\
v &= \frac{h}{2md\sin\theta} \\
v &= \frac{(6.626\times10^{-34})}{2(1.675\times10^{-27})(0.282\times10^{-9})\sin(30.0^\circ)} \\
v &= 1.4027\ldots\times10^{3} \\
v &= 1.40\times10^{3}\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Question 6 Solution

Criteria	Marks
• Identifies limitations of the Bohr model of the atom • Explains how de Broglie's proposal addressed such limitations	4
• Identifies one limitation of the Bohr model of the atom • Explains how de Broglie's proposal addressed the limitation	2-3
• Some relevant information	1

Sample answer:
Bohr's model of the atom had some notable limitations, for example, Bohr was unable to explain why electrons were only able to occupy discrete energy levels, and Bohr was also unable to explain why the electrons, although they were undergoing circular motion, did not emit electromagnetic radiation and therefore lose energy and plummet into the nucleus. De Broglie's proposal resolved both of these limitations, as de Broglie stated that electrons were capable of exhibiting wave features; therefore, the electrons were capable of existing around the nucleus in the form of a standing wave. This would then imply that the orbitals could only be of a length that was an integer multiple of the de Broglie wavelength of the electrons to form a standing wave ($C = n\lambda, n \in \mathbb{Z}^+$), explaining why electrons were only able to occupy discrete energy levels. Furthermore, since standing waves are stationary, it also resolved why electrons could have orbited the nucleus without losing energy.

Question 1 Solution

Schrödinger's wave equation, $\hat{H}\left|\Psi\right\rangle = E\left|\Psi\right\rangle$ (far outside the syllabus), is known to be one of the most influential equations in the entirety of physics. Thus, the answer is C. Deep knowledge of his equation is not required; however, the NESA syllabus dot point specifies that the contribution of Schrödinger's to the current model of the atom must be known. The answer cannot be A as Schrödinger's contribution began during the 1910s, with his wave equation being proposed in 1926. While the "pioneer of quantum physics" is a very debatable topic, it can be said that Max Planck is a considerable nominee for his quantum hypothesis in 1900, which proposed that energy is emitted and absorbed in discrete packets called quanta. The answer cannot be B as it's more accurate to say Bohr heavily influenced Schrödinger due to his earlier quantum contributions, and similarly, statement D is inaccurate as it's more accurate to say de Broglie influenced Schrödinger in his work.

!?

!? Schrödinger's contribution

The NESA curriculum dot point only states that students must "analyse the contribution of Schrödinger to the current model of the atom", and due to Schrödinger's work being very complicated in its nature, only very brief explanations are given at the HSC level. To understand his equation, you'll need to take undergraduate physics at a university, but an extremely simplified explanation is given below.

Schrödinger was initially inspired by reading a paper from Einstein with a footnote referencing de Broglie's proposal of matter waves. Then, there were classical wave equations that explained how waves move through time, such as the equation given below.

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

At the time of Schrödinger, no wave equation explained quantum waves, due to their probabilistic nature. Thus, Schrodinger proposed his wave equation, which modelled how quantum waves behave. (The equation provided is a specific one dimensional time independent form of his more general wave equation.)

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Phi}{\partial x^2} + V\Phi = E\Phi$$

You can almost think of this as Isaac Newton inventing the mathematical branch of calculus to help with his investigation in mechanics, as Schrödinger essentially created an equation that is now pivotal to understanding quantum systems with accuracy. With this equation, essentially, a new model of the atom was born. Schrödinger's model of the atom consists of a nucleus with electron clouds, which dictates only the probabilities of the electrons' positions at a given time through his wave equation.

Question 2 Solution

Criteria	Marks
• Identifies the true component of the statement • Identifies the false component of the statement	2
• Identifies the true component of the statement OR • Identifies the false component of the statement	1

Sample answer:
The student is true in saying the Schrödinger equation has revolutionised quantum mechanics, specifically quantum wave mechanics. However, the student is wrong to say that the equation was the first mathematical model to calculate definite paths of quantum objects, as the equation calculates the probabilities of quantum objects' behaviour.

Question 3 Solution

The most accurate description is A. B is a description of the Bohr atomic model, C is a description of de Broglie's matter wavelength, and D is a description of Einstein's special relativity.

Question 4 Solution

Criteria	**Marks**
• Identifies the creator of the model as Schrödinger • Identifies diagrams as electron orbitals/clouds and links the density to the probability of the electron's position	2
• Identifies the creator of the model as Schrödinger OR • Identifies diagrams as electron orbitals/clouds and links the density to the probability of the electron's position	1

Sample answer:
The diagrams above are electron probability density diagrams, with the "clouds" representing the likelihood of an electron being at the said location. Such electron probability density diagrams have been proposed by Erwin Schrödinger, following his work on quantum wave mechanics and his wave equation.

Question 5 Solution

Criteria	**Marks**
• Identifies that electrons do not travel in definite paths • Identifies that the density of the electron clouds refers to the likelihood of the presence of an electron	2
• Identifies that electrons do not travel in definite paths OR • Identifies that the density of the electron clouds refers to the likelihood of the presence of an electron	1

Sample answer:
The electron orbitals are not called electron paths, as that would imply electrons travel in a definite path, which would be inaccurate due to their probabilistic nature. Instead, the orbitals may be referred to as electron clouds due to the more accurate way of displaying electron orbitals being a 3D map of varying densities that represent the likelihood of the presence of an electron, resembling a cloud.

Question 6 Solution

Criteria	Marks
• Identifies the quantised energy levels of electrons in the Bohr model • Identifies the matter wavelength of electrons in the de Broglie model • Identifies the probable nature of electrons in the Schrödinger model • Makes a clear distinction between all 3 models rather than listing components	4
• 3 of the above	3
• 2 of the above	2
• Talks about some capacity of the models' representations of electrons	1

Sample answer:
Bohr's atomic model depicted electrons of an atom to exist in discrete quantised energy levels, with the following (ad hoc) postulate that an electron transitioning between these energy levels will absorb or emit an equal amount of the energy that they have gained or lost respectively ($\Delta E = E_{\text{i}} - E_{\text{f}}$). De Broglie then proposed that electrons (previously seen as particles) also exhibit wave-like behaviour, with a wavelength tied to their momentum called the matter wavelength given by the formula $\lambda = \frac{h}{mv}$. Schrödinger, through his wave equation, introduced the idea that electrons exist not in a definite manner but in a probable state, only collapsing through observation.

Properties of the Nucleus SOLUTIONS

Nuclear Decay SOLUTIONS

Question 1 Solution

Criteria	Marks
• Provides 3 suitable methods of shielding	3
• Provides 2 suitable methods of shielding	2
• Provides 1 suitable method of shielding	1

Sample answer:
Americium-241
As Am-241 emits alpha particles, which can be stopped by a sheet of paper or a few centimetres of air, no shielding is required. However, paper or aluminium foil will ensure that the alpha radiation is properly shielded.

Strontium-90
As Sr-90 emits beta-negative particles, the radiation can travel several metres through the air. A shielding of about 5 cm of plastic or glass will shield the beta negative radiation.

Cobalt-60
As Co-60 emits gamma radiation, the radiation has a very high penetrating power. Thick shielding of lead or cement, several centimetres thick, is required to ensure the gamma radiation is shielded.

Question 2 Solution

1. ${}^{232}_{90}\text{Th} \longrightarrow {}^{228}_{88}\text{Ra} + {}^{4}_{2}\text{He}$

2. ${}^{228}_{88}\text{Ra} \longrightarrow {}^{228}_{89}\text{Ac} + {}^{0}_{-1}\text{e}^{-} + \nu_e$

3. ${}^{228}_{89}\text{Ac} \longrightarrow {}^{228}_{90}\text{Th} + {}^{0}_{-1}\text{e}^{-} + \nu_e$

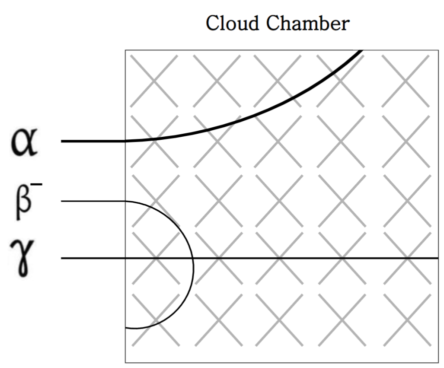
4. ${}^{229}_{90}\text{Th} \longrightarrow {}^{224}_{88}\text{Ra} + {}^{4}_{2}\text{He}$

5. ${}^{224}_{88}\text{Ra} \longrightarrow {}^{220}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

Question 3 Solution

Criteria	Marks
• Identifies and describes 3 types of radiation • Correctly explains the emitted particles' motion in the cloud chamber	4
• Identifies and describes 3 types of radiation	3
• Identifies and poorly describes 3 types of radiation	2
• Identifies 3 types of radiation	1

Sample answer:
Alpha decay is the emission of a Helium-4 nucleus from a parent nucleus, and has very high ionising power due to its high mass and charge. However, it has low penetrating power, due to its large size.
Beta (negative) decay is the emission of an electron and an electron neutrino as a result of a neutron within the parent nucleus decaying to a proton. Beta negative particles have less ionising power, but can travel further, up to several metres in the air.
Gamma decay is the emission of electromagnetic radiation from an excited nucleus to become more stable. Gamma rays have the weakest ionisation power of the three identified, but have the highest penetrating power.



From the cloud chamber shown, the alpha particles will travel in a circular path in an anticlockwise direction at a large radius. This is due to its positive charge causing the alpha particle to experience a force rightwards of its velocity, and its large mass causing it to travel at a large radius as shown by $F_c = \frac{mv^2}{r}$. Beta negative particles will travel in a circular path in a clockwise direction at a small radius. This is due to its negative charge causing the beta negative particle to experience a force leftwards of its velocity, and its small mass causing it to travel at a small radius, also shown by $F_c = \frac{mv^2}{r}$. Finally, gamma rays will travel straight, as gamma rays are uncharged.

Question 4 Solution

Criteria	Marks
• Correctly identifies type as alpha radiation • Provides 2 supporting reasons relevant to cancer therapy	3
• Correctly identifies type as alpha radiation • Provides 1 supporting reason relevant to cancer therapy	2
• Correctly identifies type as alpha radiation	1

Sample answer:
Alpha radiation is suitable for the required mode of radiation. Firstly, it fulfils the necessary ionising power for severe damage to cancer cells due to their high mass and strong positive charge. Secondly, alpha particles cannot penetrate through tissue deeply due to their low penetrating power, which in turn minimises collateral damage. Due to alpha radiation allowing for precise destruction, alpha radiation is a suitable candidate for cancer treatment.

Question 5 Solution

If Emmy wants to stop an alpha particle, all of the kinetic energy of the alpha particle must be depleted. Therefore, $E_{\text{Required}} = E_{\text{Kinetic}}$.

Data List:

- $E_{\text{Required}} = 4.78\,\text{MeV}$
  $= 7.657\ldots \times 10^{-13}\,\text{J}$
- $q = 2(1.602 \times 10^{-19})\,\text{C}$
  $= 3.204 \times 10^{-19}\,\text{C}$
- $V = ?\,\text{V}$

Solution:

$$W = qV$$
$$V = \frac{W}{q}$$
$$V = \frac{E_{\text{Required}}}{q}$$
$$V = \frac{7.657\ldots \times 10^{-13}}{3.204 \times 10^{-19}}$$
$$V = 2.39 \times 10^{6}\,\text{V}$$

Question 6 Solution

Criteria	Marks
• Correctly identifies Emmy's statement as partly valid • Clearly assesses why the statement is partly valid with mathematical models	4
• Correctly identifies Emmy's statement as partly valid • Clearly assesses why the statement is partly valid	3
• Correctly identifies Emmy's statement as partly valid • Assesses why the statement is partly valid	2
• Correctly identifies Emmy's statement as partly valid	1

Sample answer:
The first part of Emmy's statement, "fewer beta negative particles will be emitted", is true. By the model, $N_t = N_0 e^{-\lambda t}$, it is shown that as time passes, the rate of radioactivity and therefore the amount of beta negative particles being emitted will decrease. However, the second part of Emmy's statement, "I can reduce the voltage and keep the beta particles from reaching the electrode", is false. This is because although the amount of beta negative particles being emitted decreases, the kinetic energy the emitted beta negative particles individually hold do not change, and as the voltage required to stop the beta negative particles are solely dependent on their charge and energy as shown in $W = qV$, the amount of beta negative particles being emitted do not change the required voltage to stop the particles from reaching the electrode. Hence, Emmy's statement is only partly valid.

Question 1 Solution

Firstly, determine the decay constant.

Data List:

- $t_{\frac{1}{2}} = 1.415e5\,\text{s}$
- $\lambda = ?\,\text{s}^{-1}$

Solution:

$$\lambda = \frac{ln2}{t_{\frac{1}{2}}}$$
$$\lambda = \frac{ln2}{(1.451e5))}$$
$$\lambda = 4.898\ldots \times 10^{-6}\ \text{s}^{-1}$$

Now use the decay constant to find how much is left, keeping in mind that the fortnight must be converted to seconds.

$$1 \text{ fortnight} = 1 \text{ fortnight} \times \frac{2 \text{ weeks}}{1 \text{ fortnight}} \times \frac{7\,\text{d}}{1 \text{ week}} \times \frac{24\,\text{h}}{1\,\text{d}} \times \frac{60\,\text{min}}{1\,\text{h}} \times \frac{60\,\text{s}}{1\,\text{min}}$$
$$= 1.2096e6\,\text{s}$$

Data List:

- $N_0 = 2.36 \times 10^{21}$ atoms
- $\lambda = 4.898\ldots \times 10^{-6}\,\text{s}^{-1}$
- $t = 1.2096e6\,\text{s}$
- $N_\text{t} = ?$ atoms

Solution:

$$N_\text{t} = N_0 e^{-\lambda t}$$
$$N_\text{t} = (2.36e21)e^{(4.898\ldots^{-6})(1.2096e6)}$$
$$N_\text{t} = 6.303\ldots \times 10^{18}$$
$$N_\text{t} = 6.30 \times 10^{18} \text{ atoms}$$

!?

> !?
>
> # An Easier Way
>
> There is a more efficient form of $N_\text{t} = N_0 e^{-\lambda t}$ when the half-life is known. The more efficient form is:
>
> $$\frac{1}{2^{\frac{t}{t_{\frac{1}{2}}}}} = \frac{N_\text{t}}{N_0}$$
>
> The derivation of this expression is shown in Question 6 Solution; however, it is recommended to attempt the question before looking at the solution.

Question 2 Solution

First, find the decay constant.

Data List:

- $N_t = 13.36\,\text{mCi}$
- $N_0 = 15\,\text{mCi}$
- $t = 1\,\text{h}$
- $\lambda = ?\,\text{h}^{-1}$

Solution:

$$N_t = N_0 e^{-\lambda t}$$
$$\frac{N_t}{N_0} = e^{-\lambda t}$$
$$-\lambda t = \ln\left(\frac{N_t}{N_0}\right)$$
$$\lambda = -\frac{\ln\left(\frac{N_t}{N_0}\right)}{t}$$
$$\lambda = -\frac{\ln\left(\frac{(13.36)}{(15)}\right)}{(1)}$$
$$\lambda = 0.115\ldots\ \text{h}^{-1}$$

Now use the decay constant to find the half-life.

Data List:

- $\lambda = 0.115\ldots\text{h}^{-1}$
- $t_{\frac{1}{2}} = ?\,\text{h}$

Solution:

$$\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}}$$
$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda}$$
$$t_{\frac{1}{2}} = \frac{\ln(2)}{0.115\ldots}$$
$$t_{\frac{1}{2}} = 5.986\ldots$$
$$t_{\frac{1}{2}} = 6.0\,\text{h}$$

Question 3 Solution

From the information provided, simultaneous equations can be used to find the equation.

At $t = 1\,\text{s}$

$$N_t = N_0 e^{-\lambda t}$$
$$400 = N_0 e^{-\lambda}$$
$$\therefore e^{-\lambda} = \frac{400}{N_0}$$

At $t = 2\,\text{s}$

$$N_t = N_0 e^{-\lambda t}$$
$$250 = N_0 e^{-2\lambda}$$
$$250 = N_0 (e^{-\lambda})^2$$
$$250 = N_0 \left(\frac{400}{N_0}\right)^2$$
$$250 = \frac{160000}{N_0}$$
$$N_0 = 640$$

The solution continues over the page.

Now using the initial counts per second (cps), the equation can be found, and therefore allow for finding the cps at $t = 3\,\text{s}$.

$$
\begin{aligned}
N_\text{t} &= 640e^{-\lambda t} \\
\text{At t} = 1,\ N_\text{t} &= 400 \\
\therefore 400 &= 640e^{-\lambda} \\
\frac{5}{8} &= e^{-\lambda} \\
\lambda &= -\ln\left(\frac{5}{8}\right) \\
\lambda &= 0.470\ldots \\
\therefore N_\text{t} &= 640e^{-0.470\ldots t}
\end{aligned}
$$

$$
\begin{aligned}
\text{At } t = 3\,\text{s},\ N_\text{t} &= 640e^{-0.470\ldots(3)} \\
N_\text{t} &= 156.25 \\
N_\text{t} &= 156 \text{ clicks per second}
\end{aligned}
$$

Question 4 Solution

Using both the equation for radioactive decay and the equation for the half-life, the age of the sample can be determined.

Data List:

- $\frac{N_\text{t}}{N_0} = 2025^{-2025}$
- $t_{\frac{1}{2}} = 1.349 \times 10^{-4}\,\text{s}$
- $t = ?\,\text{s}$

Solution:

$$
\begin{aligned}
N_\text{t} &= N_0 e^{-\lambda t} \\
\lambda &= \frac{ln(2)}{t_{\frac{1}{2}}} \\
\therefore N_\text{t} &= N_0 e^{-\frac{ln(2)\times t}{t_{\frac{1}{2}}}} \\
\frac{N_\text{t}}{N_0} &= e^{-\frac{ln(2)\times t}{t_{\frac{1}{2}}}} \\
-\frac{ln(2)\times t}{t_{\frac{1}{2}}} &= \ln\left(\frac{N_\text{t}}{N_0}\right) \\
t &= -\frac{\ln\left(\frac{N_\text{t}}{N_0}\right) \times t_{\frac{1}{2}}}{\ln(2)} \\
t &= -\frac{\ln(2025^{-2025}) \times (1.349 \times 10^{-4})}{\ln(2)} \\
t &= \frac{2025\ln(2025) \times (1.349 \times 10^{-4})}{\ln(2)} \\
t &= 3.000\ldots \\
t &= 3.0\,\text{s}
\end{aligned}
$$

!?

Calculation Ranges

If the expression $-\frac{\ln(2025^{-2025})\times(1.349\times10^{-4})}{\ln(2)}$ is inputted into your calculator, a MATH ERROR will be displayed. However, if the expression is rearranged, although still equivalent, instead of a MATH ERROR, a value close to 3 is returned. This is because of the number 2025^{-2025} being such a small number the calculator treats this number as 0, and since $\ln(0)$ will return a MATH ERROR, $\ln(2025^{-2025})$ will also return a MATH ERROR. Thus, $\ln(2025^{-2025})$ must be manipulated to $-2025\ln(2025)$ via log laws to allow for computation.

Question 5 Solution

Firstly, find the proportional relationship between λ and t.

$$N_{\text{t}} = N_0 e^{-\lambda t}$$
$$\frac{N_{\text{t}}}{N_0} = e^{-\lambda t}$$
$$-\lambda t = \ln\left(\frac{N_{\text{t}}}{N_0}\right)$$
$$\lambda = -\frac{\ln\left(\frac{N_{\text{t}}}{N_0}\right)}{t}$$
$$\lambda \propto \frac{1}{t}$$

Initially, it may seem as if $\lambda \propto -\frac{1}{t}$, which would imply that D is the answer. However, it is crucial to realise that $\ln\left(\frac{N_{\text{t}}}{N_0}\right)$ will always be negative. This is because $\ln(a)$ is negative for all $a < 1$, and since N_{t} will always be less than N_0, $\frac{N_{\text{t}}}{N_0}$ will always be less than 1, hence $\ln\left(\frac{N_{\text{t}}}{N_0}\right)$ will always be negative. Therefore, $\lambda \propto \frac{1}{t}$, meaning the answer is C.

Question 6 Solution

Firstly, write Emmy's method in mathematical terms:

$$2^{-\frac{t}{t_{\frac{1}{2}}}} = \frac{N_{\mathrm{t}}}{N_0}$$

Now we can begin at the equation $N_{\mathrm{t}} = N_0 e^{-\lambda t}$, and aim to end at $2^{-\frac{t}{t_{\frac{1}{2}}}} = \frac{N_{\mathrm{t}}}{N_0}$.

$$N_{\mathrm{t}} = N_0 e^{-\lambda t}$$
$$\frac{N_{\mathrm{t}}}{N_0} = e^{-\lambda t}$$

$$\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}}$$

$$\therefore \frac{N_{\mathrm{t}}}{N_0} = e^{-\frac{\ln(2) \times t}{t_{\frac{1}{2}}}}$$
$$\frac{N_{\mathrm{t}}}{N_0} = \left(e^{\ln(2)}\right)^{-\frac{t}{t_{\frac{1}{2}}}}$$
$$\frac{N_{\mathrm{t}}}{N_0} = 2^{-\frac{t}{t_{\frac{1}{2}}}}$$

Thus, Emmy's method can be mathematically proved from $N_{\mathrm{t}} = N_0 e^{-\lambda t}$.

Nuclear Energy SOLUTIONS

Question 1 Solution

Super criticality refers to the increasing rate of fission in the system, leading to an uncontrolled reaction, and hence leading to a meltdown. To prevent a meltdown, the rate of fission must be reduced. This rules out D as the answer, as adding fuel elements will increase the rate of fission. A cannot be the answer, as removing the coolant will only accelerate overheating. C may reduce the effects of an uncontrolled reaction, but adding coolant alone is not enough to handle an uncontrolled reaction. Instead, B is a much better answer as it will prevent the fast neutrons from being slowed to thermal neutrons, and because fast neutrons are hundreds of times less likely to be absorbed by a Uranium-235 nucleus, the rate of fission will decrease, preventing a meltdown. Since B is the only answer that actively slows the rate of fission, B is the correct answer.

Question 2 Solution

Firstly, we know the following information by our given periodic table.

$$^{237}_{93}\mathrm{Np} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{?}_{?}\mathrm{X} + 3\,{}^{1}_{0}\mathrm{n}$$

Now, the number of protons for this equation, or in other words the sum of the atomic numbers must be equal on both sides. Neptunium has 93 protons, and Barium has 56 protons. Therefore, the unknown nuclide has 37 protons, meaning it is a Rubidium nuclide.

$$^{237}_{93}\mathrm{Np} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{?}_{37}\mathrm{Rb} + 3\,{}^{1}_{0}\mathrm{n}$$

To find the mass number of the Rubidium nuclide, see the Neptunium nuclide began with a mass number of 237 and an additional neutron was absorbed, meaning 238 nucleons were present. 141 of those nucleons created the Barium nuclide, and 3 of those nucleons were emitted neutrons, leaving 94 nucleons to create the Rubidium.

$$^{237}_{93}\mathrm{Np} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{94}_{37}\mathrm{Rb} + 3\,{}^{1}_{0}\mathrm{n}$$

Thus, the other nuclide is Rubidium-94.

Question 3 Solution

Criteria	Marks
• Provides an advantage of fusion energy • Mathematically supports advantage	2
• Provides an advantage of fusion energy	1

Sample answer:
Although each fission reaction releases more energy than each fusion reaction, the energy released by fusion reactions are much more nucleon-efficient. The fission reaction shown requires 236 nucleons to produce 173 MeV, giving a rate of approximately 0.73 MeV nucleon^{-1}. However, a fusion reaction requires 5 nucleons to produce 17.6 MeV, giving a rate of 3.52 MeV nucleon^{-1}, almost 5 times higher than fission reactions. Since nucleons make up the majority of mass, fusion provides an advantage over fission of harnessing more energy per unit mass of fuel.

Question 4 Solution

Criteria	Marks
• Links the increase in stability to release of energy • Refers specifically to diagram	2
• Links the increase in stability to release of energy	1

Sample answer:
The diagram above shows Iron-56 to have the highest binding energy per nucleon, meaning it is the most stable. Fusion reactions occur when the number of nucleons of the reactants are below 56, and fission reactions occur when the number of nucleons of the reactants are above 56, leading to the end products of both fission and fusion reactions being more stable and therefore to have a higher total binding energy. This higher total binding energy corresponds to a lower mass, and therefore the mass defect is released as energy by Einstein's equation, $E = mc^2$.

Question 5 Solution

Criteria	Marks
• Identifies the representations of the bricks and mortar • Provides a disadvantage of the model	3
• Identifies the representations of the bricks and mortar	2
• Identifies the representation of the bricks or the mortar	1

Sample answer:
Emmy's analogy represents fission reactions as the individual bricks show the nucleons, initially being bound together in a single nucleus to later being separated between 2 nuclei. The mortar represents the binding energy that initially holds the nucleons together to form a single nucleus, then being released in the form of energy after the nucleus undergoes fission, additionally representing the mass defect as it itself weighs 0.5 kg. A disadvantage exists in this model as the analogy cannot be applied to fusion reactions, as mortar would be required to fuse the bricks, representing nuclei together, implying a gain in mass in fusion reactions which is inaccurate.

Question 6 Solution

Criteria	Marks
• Provides structure and function of fuel rods • Provides structure and function of moderator • Provides structure and function of coolant • Provides structure and function of control rods • Includes a relevant nuclear equation with its produced energy	9
• Satisfies 4 of the 9 mark criteria items	8-7
• Satisfies 3 of the 9 mark criteria items	6-5
• Satisfies 2 of the 9 mark criteria items	4-3
• Satisfies 1 of the 9 mark criteria items	2-1

Sample answer is given on the next page.

A nuclear reactor must use fuel rods, which contain fissile material, commonly enriched Uranium-235, in a cylindrical form that are stored in the core of the reactor so that the system has above critical mass so that a self sustaining reaction can take place. The fuel rods provide the fissile material where the fission reactions will take place, resulting in an example reaction as shown:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n} + \mathrm{Energy}(173\,\mathrm{MeV})$$

Due to the daughter nuclides being more stable than the parent nuclide, binding energy is released as energy in the reaction in the form of kinetic energy, and heat energy which will be absorbed by the coolant as explained later. However, the emitted fast neutrons have a kinetic energy of about 1-3 MeV, which is too high for it to be absorbed in another U-235 nucleus to continue the chain reaction. To slow the neutrons, nuclear reactors use moderators, typically in the form of deuterium oxide, or heavy water. Heavy water, or D_2O is H_2O however instead of the hydrogen nuclei ($^{1}_{1}\mathrm{H}$) being simply a single proton, an isotope called deuterium ($^{2}_{1}\mathrm{H}$), where the nucleus is composed of a proton and a neutron is used to form the water molecule instead, resulting in a smaller neutron cross section, reducing the likelihood of an absorption of neutrons. The moderator slows the neutron via inelastic collisions, resulting in the fast neutron losing its kinetic energy, dropping from 1-3 MeV to 0.025 eV. Such thermal neutrons are hundreds of times more likely to be absorbed by U-235 nuclei, continuing the chain reaction. However, as each fission reaction requires only one neutron to be absorbed, and can emit 2-3 neutrons, an exponential increase in activity can occur, leading to an uncontrolled fission reaction. To prevent this from occurring, control rods, typically made of hafnium due to their avid absorption of neutrons, are placed as rods in between fuel rods to absorb excess neutrons, allowing for fission to occur at a controlled pace. Additionally in modern times, such control rods are engineered so that in the case of an emergency shutdown, or SCRAM, the control rods will automatically insert themselves between fuel rods to immediately stop further fission reactions by gravity. Finally, in order to effectively harness the heat energy that is released from fission reactions, coolant is used, usually H_2O, serving a double purpose of preventing the components of a nuclear reactor from overheating, and also to drive a steam turbine to generate electricity. Thus, by the fuel rods allowing for fission reactions to take place, the moderators slowing the fast neutrons to thermal neutrons for absorption, the control rods allowing for absorbing excess neutrons to prevent an uncontrolled reaction, and coolant that harnesses the heat energy produced to electrical energy, a nuclear fission reactor wields the strong force to produce energy.

Mass-Energy Equivalence SOLUTIONS

Question 1 Solution

If nucleus A has a lower total binding energy than nucleus B, it means that less energy will be required to separate nucleus A into its nucleons than nucleus B, and that the mass defect of nucleus A will be less than nucleus B (because the binding energy is proportional to the mass defect as shown in $E = mc^2 \implies E_{\text{binding}} \propto m_{\text{defect}}$. Hence, the answer is C.

Question 2 Solution

Find the mass loss of the reaction to find the energy released.

Data List:

- $m_{\text{i}} = 2(12.000)$
  $= 24.000\,\text{u}$
- $m_{\text{f}} = 19.992 + 4.003$
  $= 23.995\,\text{u}$
- $E = ?\,\text{MeV}$

Solution:

$$\begin{aligned} m_{\text{loss}} &= m_{\text{i}} - m_{\text{f}} \\ &= (24.000) - (23.995) \\ &= 0.005\,\text{u} \end{aligned}$$

$$\begin{aligned} E &= m_{\text{loss}} \times 931.5 \\ &= (0.005) \times 931.5 \\ &= 4.6575 \\ &= 4.658\,\text{MeV} \end{aligned}$$

Question 3 Solution

In this reaction, a Hydrogen-2 nucleus is separated into its nucleons, a proton and a neutron, after absorbing a gamma particle. Because it is given that the binding energy is 1.714 MeV, it can be known that 1.714 MeV of energy has been used (absorbed) to separate the original nucleus into its nucleons. Since the only energy the Hydrogen-2 nucleus could have absorbed was the gamma particle's energy, it can be known that energy has been absorbed, and the gamma particle had an energy of 1.714 MeV.

Question 4 Solution

Firstly, find the energy released by a single alpha decay of Polonium-210.
($^{210}_{84}\text{Po} \longrightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$)

Data List:

- $m_\text{i} = 209.983\,\text{u}$
- $m_\text{f} = 205.974 + 4.003$
  $= 209.977\,\text{u}$
- $E_\text{per decay} = ?\,\text{MeV}$

Solution:

$$\begin{aligned}
E_\text{per decay} &= m_\text{loss} \times 931.5 \\
&= (m_\text{i} - m_\text{f}) \times 931.5 \\
&= ((209.983) - (209.977)) \times 931.5 \\
&= (0.014) \times 931.5 \\
&= 13.041\,\text{MeV}
\end{aligned}$$

Now that it is known that each alpha decay of a Polonium-210 atom releases 13.041 MeV, we must find the number of atoms that have decayed.

Data List:

- $N_0 = 2.868 \times 10^{23}$
- $t_{\frac{1}{2}} = 138.376\,\text{d}$
- $t = 7\,\text{d}$
- $E_\text{released} = ?\,\text{J}$

Solution:

$$\begin{aligned}
N_\text{t} &= N_0 e^{-\lambda t} \\
\lambda &= \frac{\ln(2)}{t_{\frac{1}{2}}} \\
\therefore N_\text{t} &= N_0 e^{-\frac{t \ln(2)}{t_{\frac{1}{2}}}} \\
N_\text{t} &= N_0 2^{-\frac{t}{t_{\frac{1}{2}}}} \\
N_\text{t} &= (2.868 \times 10^{23}) 2^{-\frac{7}{138.376}} \\
N_\text{t} &= 2.769\ldots \times 10^{23}
\end{aligned}$$

$$\begin{aligned}
\therefore N_\text{decayed} &= N_0 - N_\text{t} \\
&= 2.868 \times 10^{23} - 2.769\ldots \times 10^{23} \\
&= 9.882\ldots \times 10^{21}
\end{aligned}$$

$$\begin{aligned}
\therefore E_\text{released} &= N_\text{decayed} \times E_\text{per decay} \\
&= 9.882\ldots \times 10^{21} \times 13.041 \\
&= 1.228\ldots \times 10^{23}\,\text{MeV} \\
&= 1.228\ldots \times 10^{29}\,\text{eV} \\
&= 2.064\ldots \times 10^{10}\,\text{J} \\
&= 2.065 \times 10^{10}\,\text{J}
\end{aligned}$$

Question 5 Solution

Since momentum must be conserved:

$$
\begin{aligned}
p_{\mathrm{A}} + p_{\mathrm{B}} &= 0 \\
p_{\mathrm{A}} &= -p_{\mathrm{B}} \\
m_{\mathrm{A}} v_{\mathrm{A}} &= -m_{\mathrm{B}} v_{\mathrm{B}} \\
n m_{\mathrm{B}} v_{\mathrm{A}} &= -m_{\mathrm{B}} v_{\mathrm{B}} \\
n v_{\mathrm{A}} &= -v_{\mathrm{B}} \\
v_{\mathrm{B}} &= -n v_{\mathrm{A}}
\end{aligned}
$$

So to find how many times the kinetic energy of nucleus B is of nucleus A:

$$
\begin{aligned}
K.E._{\mathrm{B}} \div K.E._{\mathrm{A}} &= \frac{m_{\mathrm{B}} v_{\mathrm{B}}^2}{2} \div \frac{m_{\mathrm{A}} v_{\mathrm{A}}^2}{2} \\
&= \frac{m_{\mathrm{B}} (-n v_{\mathrm{A}})^2}{2} \times \frac{2}{(n m_{\mathrm{B}}) v_{\mathrm{A}}^2} \\
&= \frac{n^2 m_{\mathrm{B}} v_{\mathrm{A}}^2}{2} \times \frac{2}{n m_{\mathrm{B}} v_{\mathrm{A}}^2} \\
&= n
\end{aligned}
$$

Question 6 Solution

Firstly, find the amount of energy each reaction gives.

Data List:

- $m_\text{i} = 2.0141018 + 3.0160493$
  $= 5.0301511\,\text{u}$
- $m_\text{f} = 4.0026032 + (\frac{1.675 \times 10^{-27}}{1.661 \times 10^{-27}})$
  $= 5.011\ldots\,\text{u}$
- $E_\text{reaction} = ?\,\text{J}$

Solution:

$$\begin{aligned}
E_\text{reaction} &= m_\text{loss} \times 931.5 \\
&= (m_\text{i} - m_\text{f}) \times 931.5 \\
&= ((5.0301511) - (5.011\ldots)) \times 931.5 \\
&= 0.019\ldots \times 931.5 \\
&= 17.809\ldots\ \text{MeV} \\
&= 1.780\ldots \times 10^{7}\,\text{eV} \\
&= 2.853\ldots \times 10^{-12}\,\text{J}
\end{aligned}$$

Now divide the amount of energy that would be used in the 50-day voyage by the energy given by each reaction to find how many reactions occur.

Data List:

- $E_\text{reaction} = 2.853\ldots \times 10^{-12}\,\text{J}$
- $E_\text{day} = 9 \times 10^{12}\,\text{J}$
- $n_\text{reactions} = ?$

Solution:

$$\begin{aligned}
n_\text{reactions} &= \frac{E_\text{required}}{E_\text{reaction}} \\
&= \frac{50 \times E_\text{day}}{E_\text{reaction}} \\
&= \frac{50 \times (9 \times 10^{12})}{2.853\ldots \times 10^{-12}} \\
&= 1.577\ldots \times 10^{26}
\end{aligned}$$

Now, because we know the initial mass of each reaction, the mass fused can be found by multiplying the initial mass per reaction by the number of reactions that occur.

$$\begin{aligned}
m_\text{fused} &= m_\text{i} \times n_\text{reactions} \\
&= (5.0301511) \times (1.577\ldots \times 10^{26}) \\
&= 7.933\ldots \times 10^{26}\,\text{u} \\
&= 1.317\ldots\ \text{kg} \\
&= 1.32\,\text{kg}
\end{aligned}$$

Deep Inside the Atom SOLUTIONS

Standard Model of Matter SOLUTIONS

Question 1 Solution

Neutrons are made of two down quarks and one up quark (udd), and protons are made of one down quark and two up quarks (uud).

Question 2 Solution

Leptons are elementary particles; thus, the answer is C. Hadrons are by definition made of at least two quarks, baryons are made of an odd number of quarks that is greater than 1, and mesons are made of a quark and an anti-quark.

Question 3 Solution

The particle with the quark composition uud must be a proton, and the particle with the quark composition udd must be a neutron. Thus, the proton becomes a neutron through beta positive decay. The beta positive decay from a proton to a neutron is as follows:

$$^{1}_{1}\text{p} \longrightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\text{e} + \nu$$

Therefore, the two other particles that are produced in this process are the positron and the neutrino.

Question 4 Solution

Criteria	Marks
• Identifies quark composition of neutron • Identifies quark composition of proton • Identifies electron as a lepton • Talks of gluons as a carrier of the strong force between quarks.	4
• 3 of the above.	3
• 2 of the above.	2
• 1 of the above.	1

Sample answer:
The neutral helium atom is composed of 2 protons, 2 neutrons and 2 electrons. A proton is a baryon consisting of three fermions: 2 up quarks and 1 down quark (uud). The neutron is a baryon, similarly consisting of three fermions: 1 up quark and 2 down quarks (udd). The quarks of both the neutron and the proton are held together by the strong force that is mediated by the gluon, which is a boson. The electron is a lepton, which is an elementary particle that can be isolated.

Question 5 Solution

It should be noted that the charge of the up, charm and top quarks are all of charge $+\frac{2}{3}$ and the charge of the down, strange and bottom quarks are all of charge $-\frac{1}{3}$.

$$q_{\text{pentaquark}} = q_u + q_u + q_d + q_c + q_{\bar{c}}$$
$$q_{\text{pentaquark}} = +\frac{2}{3} + \frac{2}{3} - \frac{1}{3} + \frac{2}{3} - \frac{2}{3}$$
$$q_{\text{pentaquark}} = +1$$

Question 6 Solution

In the reactants of the reaction, since the neutron is composed of udd and the anti neutron is composed of $\bar{u}\bar{d}\bar{d}$ and the reaction is balanced, both the reactants and products will have 1 up quark, 2 down quarks, 1 anti up quark and 2 anti down quarks. The charges of each quark are given below

Quark	Charge
Up quark	$+\frac{2}{3}$
Anti up quark	$-\frac{2}{3}$
Down quark	$-\frac{1}{3}$
Anti down quark	$+\frac{1}{3}$

As there are three pions (quark anti-quark pair) of charge +1, 0, and -1, the quarks must be arranged so that there are three quark anti-quark pairs of charge +1, 0 and -1. The only possible combination is if the π^+ has a structure of $u\bar{d}$, the π^- has a structure of $\bar{u}d$, and by elimination, the structure of the π^0 is $d\bar{d}$.

Particle Accelerators SOLUTIONS

Question 1 Solution

Criteria	Marks
• Provides a similarity in their operation • Provides a difference in their operation	2
• Provides a similarity OR difference in their operation	1

Sample answer:
The operation of the NSLS and the Cyclotron of the University of California are similar in the way that both use the electric field to increase the speed of particles and use the magnetic field to maintain a circular path for accelerating particles. However, the NSLS differs from the 60-inch Cyclotron as the magnetic field of the NSLS increases as the particle's speed increases to maintain a circular path of a constant radius, allowing for possibly greater speeds of the particle.

Question 2 Solution

$$\begin{aligned} F_c &= F_B \\ \frac{mv^2}{r} &= qvB \\ \frac{mv}{r} &= qB \\ \text{Now } v &= \frac{2\pi r}{T} \\ v &= 2\pi r f \\ \therefore \frac{m \times 2\pi r f}{r} &= qB \\ f &= \frac{qB}{2\pi m} \end{aligned}$$

Question 3 Solution

Criteria	Marks
• Utilises a specific example experiment • Explains how the experiment has contributed to the understanding of a fundamental particle	3
• Utilises a specific example experiment • Links the experiment to a fundamental particle	2
• Provides a specific example experiment	1

Sample answer:
The 1968 SLAC experiments used high-energy electron-proton scattering to reveal the proton's internal structure. The SLAC was used to direct high-energy beams of electrons towards protons, in which the scattering patterns provided the first direct evidence for quarks as fractional-charge constituents of nucleons, thus leading to the confirmation of the existence of quarks as fundamental particles.

Question 4 Solution

Criteria	Marks
• Describes the unique high-energy environments in particle accelerators • Utilises an additional scientific discovery by particle accelerators	3
• Identifies the high-energy environments in particle accelerators • Utilises an additional scientific discovery by particle accelerators	2
• Provides some relevant information	1

Sample answer:
Scientific discoveries such as the Higgs boson are impossible without particle accelerators, due to such exotic massive particles only existing in extreme energy environments that can only be reproduced to analyse in particle accelerators. This can also be seen in the discovery of the top quark, as due to its mass of 172.5 GeV, its discovery was only possible due to the extremely high energy conditions that can only be reproduced to analyse in particle accelerators (such as Fermilab's Tevatron).

Question 5 Solution

Criteria	Marks
• Identifies the true component of particle accelerators being essential to scientific advancement • Supports true component with example • Identifies the false component of particle accelerators as having no disadvantages • Provides a disadvantage of particle accelerators	4
• 2-3 of the above	2-3
• Provides some relevant information	1

Sample answer:
The statement is true in declaring that particle accelerators have been essential for advancements in the field of science. For example, discoveries of exotic particles such as the Higgs boson would have been practically impossible without the high-energy environment that can only be reproduced to analyse in particle accelerators such as the LHC. However, the statement is false in declaring particle accelerators have no disadvantages, as multiple difficulties present themselves in working with particle accelerators, such as their extreme cost to produce, maintain and run, or the technical complexity requiring highly skilled personnel and advanced infrastructure, limiting accessibility.

Question 6 Solution

Criteria	Marks
• Utilises 2 specific example experiments • Demonstrates with both examples how each experiment has led to the validation of a scientific theory, leading to developments in the Standard Model	7
• Satisfies above criteria, however, only demonstrates how experiments have advanced the Standard Model	5-6
• Satisfies above criteria, however, only utilises 1 specific example experiment	3-4
• Provides some relevant information	1-2

Sample answer provided on the following page.

The Standard Model of matter is a framework that describes the fundamental particles and force carriers that have been both theorised and experimentally confirmed. In its development, particle accelerators have played a crucial role in validating the existence of various elementary particles predicted by theory.

Before 1968, quarks were purely theoretical mathematical constructs to explain the behaviour of nucleons such as protons and neutrons. Due to the lack of an experimental basis, the proposal of quarks was met with scepticism. However, in 1968 at the Stanford Linear Accelerator Centre (SLAC), electron-proton scattering experiments were carried out. The scattering of the electrons unexpectedly showed some electrons were significantly deflected, while others experienced only minimal deflection. These results indicated the presence of smaller point-like particles within the proton. Further analysis of these scattering results led to the confirmation of quarks as the constituent particles that make up a proton, leading to the development of the Standard Model to include quarks as elementary fermions.

Another significant validation came in 2012, when the Higgs boson—first proposed by Peter Higgs in 1964 as part of a mechanism to explain how particles acquire mass—was experimentally confirmed. Using the Large Hadron Collider (LHC), scientists observed the results of high-energy proton-proton collisions, which revealed a particle consistent with the predicted properties of the Higgs boson. Its discovery provided critical confirmation of the Higgs mechanism and led to its inclusion in the Standard Model as an elementary scalar boson.

Thus, particle accelerators have been instrumental in validating theoretical predictions and advancing the development of the Standard Model of matter.

Fourth Edition

Glenn Y.W. Kim

Foundational Physics for the HSC